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The Tiny (g-2) Muon Wobble from Small-μ Supersymmetry

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Abstract

A new measurement of the muon anomalous magnetic moment has been recently reported by the Fermilab Muon g-2 collaboration and shows a $4.2\,\sigma$ departure from the most precise and reliable calculation of this quantity in the Standard Model. Assuming that this discrepancy is due to new physics, we consider its relation with other potential anomalies, especially in the muon sector, as well as clues from the early universe. We comment on new physics solutions discussed extensively in the literature in the past decades, to finally concentrate on a simple supersymmetric model that also provides a dark matter explanation. We show results for an interesting region of supersymmetric parameter space that can be probed at the high luminosity LHC and future colliders, while leading to values of $(g_\mu - 2)$ consistent with the Fermilab and Brookhaven $(g_\mu - 2)$ measurements. Such a parameter region can simultaneously realize a Bino-like dark matter candidate compatible with direct detection constraints for small to moderate values of the Higgsino mass parameter $|\mu|$.

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I. INTRODUCTION

The Standard Model (SM) of particle physics has built its reputation on decades of measurements at experiments around the world that testify to its validity. With the discovery of the Higgs boson almost a decade ago [1, 2] all SM particles have been observed and the mechanism that gives mass to the SM particles, with the possible exception of the neutrinos, has been established. Nonetheless, we know that physics beyond the SM (BSM) is required to explain the nature of dark matter (DM) and the source of the observed matter-antimatter asymmetry. Furthermore, an understanding of some features of the SM such as the hierarchy of the fermion masses or the stability of the electroweak vacuum, is lacking.

The direct discovery of new particles pointing towards new forces or new symmetries in nature will be the most striking and conclusive evidence of BSM physics. However, it may well be the case that BSM particles lie beyond our present experimental reach in mass and/or interaction strength, and that clues for new physics may first come from results for precision observables that depart from their SM expectations. With that in mind, since the discovery of the Higgs boson, we are straining our resources and capabilities to measure the properties of the Higgs boson to higher and higher accuracy, and flavor and electroweak physics experiments at the LHC and elsewhere are pursuing a complementary broad program of precision measurements. Breakthroughs in our understanding of what lies beyond the SM could occur at any time.

Recently, new results of measurements involving muons have been reported. The LHCb experiment has reported new values of the decay rate of B-mesons to a kaon and a pair of muons compared to the decay into a kaon and electrons [3], providing evidence at the $3\,\sigma$-level of the violation of lepton universality. This so-called R_K anomaly joins the ranks of previously reported anomalies involving heavy-flavor quarks such as the bottom quark forward-backward asymmetry at LEP [4, 5], and measurements of meson decays at the LHC and B-factories such as R_{K^*} [6–8] and $R_{D^{(*)}}$ [9–14]. The Fermilab Muon (g-2) experiment has just reported a new measurement of the anomalous magnetic moment of the muon, $a_\mu \equiv (g_\mu - 2)/2$. The SM prediction of a_μ is known with the remarkable relative precision of 4×10^{-7}, $a_\mu^{\rm SM} = 116\ 591\ 810(43) \times 10^{-11}$ [15–35]. From the new Fermilab Muon (g-2) experiment, the measured value is $a_\mu^{\rm exp,\ FNAL} = 116\ 592\ 040(54) \times 10^{-11}$ [36], which combined with the previous E821 result $a_\mu^{\rm exp,\ E821} = 116\ 592\ 089(63) \times 10^{-11}$ [37], yields a value $a_\mu^{\rm exp} = 116\ 592\ 061(41) \times 10^{-11}$.

An important point when considering the tension between experimental results and the SM predictions are the current limitations on theoretical tools in computing the hadronic vacuum polarization (HVP) contribution to $a_\mu^{\rm SM}$, which is governed by the strong interaction and is particularly challenging to calculate from first principles. The most accurate result of the HVP contribution is based on a data-driven result, extracting its value from precise and reliable low-energy ($e^+e^- \to$ hadrons) cross section measurements via dispersion theory. Assuming no contribution from new physics to the low energy processes and conservatively accounting for experimental errors, this yields a value $a_\mu^{\rm HVP} = 685.4(4.0) \times 10^{-10}$ [15, 20–26], implying an uncertainty of 0.6 % in this contribution.[1] The SM prediction for the anomalous magnetic moment of the muon and the measured value then differ by $4.2\,\sigma$,

$$\Delta a_\mu \equiv (a_\mu^{\rm exp} - a_\mu^{\rm SM}) = (251 \pm 59) \times 10^{-11}\,. \tag{1}$$

It is imperative to ask what these anomalies may imply for new physics. The most relevant questions that come to mind are: Can the a_μ and $R_{K^{(*)}}$ anomalies be explained by the same BSM physics? Can they give guidance about the nature of DM? Are they related to cosmological discrepancies? How constrained are the possible solutions by other experimental searches? What are future experimental prospects for the possible solutions?

In Sec. II we provide a brief overview of the many models which have been previously proposed in the literature to explain the $(g_\mu - 2)$ anomaly and consider their impact on other possible anomalies and on unresolved questions of the SM. Then, in Sec. III, we discuss a supersymmetric solution in the most simplistic supersymmetric model at hand, the Minimal Supersymmetric Standard Model (MSSM). We focus on a region of the parameter space of the MSSM where the $(g_\mu - 2)$ anomaly can be realized simultaneously with a viable DM candidate. We show that in the region of moderate $|\mu|$ and moderate-to-large values of $\tan\beta$, a Bino-like DM candidate can be realized in the proximity of *blind spots* (that require $\mu M_1 < 0$) for spin independent direct detection experiments [43]. In this way, our MSSM scenario explores a different region of parameter space than the one considered in the study

[1] The HVP contribution has recently been computed in lattice QCD, yielding a higher value of $a_\mu^{\rm HVP} = 708.7(5.3) \times 10^{-10}$ [38]. Given the high complexity of this calculation, independent lattice calculations with commiserate precision are needed before confronting this result with the well tested data-driven one. We stress that if a larger value of the HVP contribution were confirmed, which would (partially) explain the $(g_\mu - 2)$ anomaly, new physics contributions will be needed to bring theory and measurements of ($e^+e^- \to$ hadrons) in agreement [39–42].

of Refs. [44, 45], which considers regions of large μ as a way to accommodate current SIDD bounds. Finally, we summarize and conclude in Sec. IV.

II. $(g_\mu - 2)$ CONNECTIONS TO COSMIC PUZZLES AND THE LHC

In order to bridge the gap between the SM prediction and the measured value for the anomalous magnetic moment of the muon, a BSM contribution of order $\Delta a_\mu = (20\text{–}30) \times 10^{-10}$ is needed. Taking the a_μ anomaly as a guidance for new physics, it is natural to ask how it can be connected to other anomalies, specially those in the muon sector, or to solving puzzles of our universe's early history. There are two broad classes of solutions to the $(g_\mu - 2)$ anomaly that may be considered in the light of the above:

- New relatively light particles with small couplings to muons, typically featuring particles with $\mathcal{O}(100)\,\text{MeV}$ masses and $\mathcal{O}(10^{-3})$ couplings to muons. Examples of such models we will discuss here are new (light) scalars and new (light) (Z') vector bosons. These new light particles may have left important clues in the cosmos.

- New heavy fermions or scalars (possibly accompanied by additional new particles), as well as leptoquark particles, with larger couplings to muons. Similar solutions appear also in supersymmetric extensions of the SM that we shall discuss separately in some detail in Sec. III. In addition, new gauge symmetries, spontaneously broken at low energies, can induce Z' vector bosons with masses comparable to the electroweak scale and $\mathcal{O}(1)$ couplings to muons. These types of new particles can be sought for at the LHC and other terrestrial experiments.

The most recent LHCb measurement [3], $R_K = \text{BR}(B \to K\mu^+\mu^-)/\text{BR}(B \to Ke^+e^-) = 0.846^{+0.044}_{-0.041}$ in the kinematic regime of $1.1\,\text{GeV}^2 \leq q^2 \leq 6.0\,\text{GeV}^2$ implies a violation of lepton universality and differs from the SM expectation at the $3.1\,\sigma$ level. Since R_K also involves muons, it naturally appears related to the $(g_\mu - 2)$ anomaly. However, as we shall discuss, it is hard to simultaneously fit both R_K and $(g_\mu - 2)$.

Scalar solutions: This is perhaps the simplest scenario for the explanation of the observed Δa_μ. A scalar particle, with mass $\lesssim 200\,\text{MeV}$ and couplings to muons of similar size as the corresponding SM-Higgs coupling, can lead to a satisfactory explanation of Δa_μ [46–51]. One can construct models with such a scalar particle and suppressed couplings to other leptons or quarks in a straightforward way [51]. Alternatively, one can construct models with appropriate values of the couplings of the new scalar to quarks to lead to an explanation of some flavor anomalies, for example the KOTO anomaly [52], but the constraints tend to be more severe and the model-building becomes more involved [53]. It is important to stress that it proves impossible to fully explain the R_K anomaly with scalars without violating $B_s \to \mu^+\mu^-$ measurements [54]; see, for example, Ref. [55].

A pseudoscalar particle may also lead to an explanation of Δa_μ, provided it couples not only to muons, but also to photons. The typical example are axion-like particles [56, 57], although obtaining the proper Δa_μ requires a delicate interplay between the muon and photon couplings.[2] Alternatively, a positive contribution to a_μ can arise from a two loop Barr-Zee diagram mediated by the pseudoscalar couplings to heavier quarks and leptons [59].

Fermionic solutions: Another interesting solution occurs in the case of vector-like leptons, which may induce a contribution to a_μ via gauge boson and Higgs mediated interactions [60, 61]. Note that the mixing between the SM leptons and the new heavy leptons must be carefully controlled to prevent dangerous flavor-changing neutral currents in the lepton sector. A recent analysis shows that consistency with the measured values of Δa_μ may be obtained for vector-like leptons with masses of the order of a few TeV [62].

Leptoquark solutions: This is one of the most interesting solutions to Δa_μ, since it can also lead to an explanation of the R_K anomaly; see, for example, Refs. [63–66]. A directly related and particularly attractive realization arises in R-parity violating supersymmetry, which enables the same type of interactions as a leptoquark theory; see, for example, Ref. [67]. This solution requires the scalar partner of the right-handed bottom quark to have masses of a few TeV, which may be tested at future LHC runs. Similar to the vector-like lepton scenarios, a careful choice of the leptoquark couplings is necessary to avoid flavor-changing neutral currents. This tuning may be the result of symmetries [66], and is perhaps the least attractive feature of such scenarios.

Gauge boson solutions: New gauge bosons coupled to muons are an attractive solution to the a_μ anomaly, since they can be incorporated in an anomaly-free framework that can also lead to an explanation of the $R_{K^{(*)}}$ anomalies. Of particular interest is the gauged $(L_\mu - L_\tau)$ scenario [68], since it avoids the coupling to electrons.[3] The $R_{K^{(*)}}$ anomalies

[2] A similar mechanism applies for $(g_e - 2)$ in the case of the QCD axion; see, for instance, Ref. [58]

[3] Models with $(L_\mu + L_\tau)$ give an intriguing connection to a novel mechanism of electroweak baryogenesis with

may be explained by the addition of vector-like quarks that mix with the second and third generation SM quarks [71–73], connecting the $(L_\mu - L_\tau)$ gauge boson to baryons. A common explanation of both $R_{K^{(*)}}$ and a_μ is, however, strongly constrained by neutrino trident bounds on Z' bosons coupled to muons [74–76].[4] In addition, bounds from BaBar [79] and CMS [80] from $[e^+e^-/pp \to \mu^+\mu^- + (Z' \to \mu^+\mu^-)]$ rule out the values of the new gauge coupling which could explain the observed value of a_μ for $m_{Z'} \geq 2m_\mu \simeq 210\,\mathrm{MeV}$. Due to these experimental constraints, explaining the Δa_μ anomaly with a light new gauge boson requires $m_{Z'} \lesssim 200\,\mathrm{MeV}$. Explanations of the flavor anomalies require larger gauge boson masses, preventing simultaneous explanations of $R_{K^{(*)}}$ and a_μ.

It is interesting to note that explanations of the $(g_\mu - 2)$ anomaly via gauged $(L_\mu - L_\tau)$ may have a relation to some of the cosmological puzzles, in particular the tensions of the late and early time determinations of the Hubble constant, H_0 [78, 81]. In the $m_{Z'} \sim 10\,\mathrm{MeV}$ region, the effective number of degrees of freedom can be enhanced by $\Delta N_{\mathrm{eff}} \approx 0.2$, alleviating the H_0-tension. Note that constraints from solar neutrino scattering in Borexino [78, 82, 83] and ΔN_{eff} bounds [81] rule out the couplings preferred by the a_μ anomaly for $m_{Z'} \lesssim 5\,\mathrm{MeV}$.

Before considering minimal supersymmetric scenarios for the $(g_\mu - 2)$ anomaly in some detail, let's summarize the discussion above as follows: 1) All the above solutions, with a broad range of masses and couplings of the new particles, can readily explain the $(g_\mu - 2)$ anomaly, but it is difficult to simultaneously accommodate the $R_{K^{(*)}}$ anomalies. This difficulty mainly arises from experimental constraints. In the rare cases where both solutions can be accommodated simultaneously, additional requirements are necessary for tightly connecting them. 2) In most scenarios, a DM candidate can be included in the model (with different levels of complexity). However, there does not appear to be a compelling connection offering a unique guidance for model building. On the other hand, in low-energy SUSY models with R-Parity conservation, an explanation of the $(g_\mu - 2)$ anomaly is naturally connected to the presence a DM candidate and other new particles within the reach of the (HL-)LHC and future colliders. We explore this possibility in its simplest realization in the next section.

CP-violation triggered in a dark sector that allows for a suitable DM candidate [69, 70]. Unfortunately, solutions to $(g_\mu - 2)$ in this appealing scenario are ruled out by $(B \to K\mu^+\mu^-)$ constraints due to contributions from the anomalous WWZ' coupling.

[4] There are also bounds from Coherent ν-Nucleus Scattering (CEνNS), although these are not yet competitive with the bounds from neutrino trident processes [77, 78].

III. TINY $(g_\mu - 2)$ MUON WOBBLE WITH SMALL $|\mu|$ IN THE MSSM

Supersymmetric extensions of the SM remain among the most compelling BSM scenarios [84–86], not least because in supersymmetric theories the stability of the Higgs mass parameter under quantum corrections can be ensured. In minimal supersymmetric extensions of the SM, the SM-like Higgs is naturally light [87–97] and the corrections to electroweak precision as well as flavor observables tend to be small, leading to good agreement with observations. Supersymmetric extensions can also lead to gauge coupling unification and provide a natural DM candidate, namely the lightest neutralino.

In this section, we discuss the regions of parameter space of the Minimal Supersymmetric Standard Model (MSSM) [84–86] where the $(g_\mu - 2)$ anomaly can be simultaneously realized with a viable DM candidate. Related recent (but prior to the publication of the Fermilab Muon (g-2) result) studies can, for example, be found in Refs. [44, 45, 98–100]. One crucial difference in the region of parameter space we study here compared to the very recent work in Refs. [44, 45] is that we show how the experimentally observed value of a_μ can be explained in the MSSM together with a viable DM candidate for moderate (absolute) values of the Higgsino mass parameter $|\mu| \lesssim 500\,\mathrm{GeV}$. In this region of parameter space, a Bino-like neutralino can be an excellent DM candidate if its (spin independent) direct detection cross section is suppressed by the so-called blind spot cancellations [43], which require μ and the Bino mass parameter, M_1, to have opposite sign.

A. Δa_μ and Direct Dark Matter Detection Constraints

The MSSM contributions to a_μ have been discussed extensively in the literature, see, for example, Refs. [100–107]. The most important contributions arise via chargino-sneutrino and neutralino-smuon loops, approximately described by [100]

$$a_\mu^{\widetilde{\chi}^\pm - \widetilde{\nu}_\mu} \simeq \frac{\alpha m_\mu^2 \mu M_2 \tan\beta}{4\pi \sin^2\theta_W m_{\widetilde{\nu}_\mu}^2} \left[\frac{f_{\chi^\pm}\left(M_2^2/m_{\widetilde{\nu}_\mu}^2\right) - f_{\chi^\pm}\left(\mu^2/m_{\widetilde{\nu}_\mu}^2\right)}{M_2^2 - \mu^2} \right], \tag{2}$$

$$a_\mu^{\widetilde{\chi}^0 - \widetilde{\mu}} \simeq \frac{\alpha m_\mu^2 M_1 \left(\mu\tan\beta - A_\mu\right)}{4\pi \cos^2\theta_W \left(m_{\widetilde{\mu}_R}^2 - m_{\widetilde{\mu}_L}^2\right)} \left[\frac{f_{\chi^0}\left(M_1^2/m_{\widetilde{\mu}_R}^2\right)}{m_{\widetilde{\mu}_R}^2} - \frac{f_{\chi^0}\left(M_1^2/m_{\widetilde{\mu}_L}^2\right)}{m_{\widetilde{\mu}_L}^2} \right], \tag{3}$$

where M_2 is the Wino mass parameter and $m_{\widetilde{f}}$ are the scalar particle $\widetilde{f}$ masses, with the loop functions

$$f_{\chi^\pm}(x) = \frac{x^2 - 4x + 3 + 2\ln(x)}{(1-x)^3}\,, \tag{4}$$

$$f_{\chi^0}(x) = \frac{x^2 - 1 - 2x\ln(x)}{(1-x)^3}\,; \tag{5}$$

see Refs. [104, 107] for the full (one-loop) expressions. It is interesting to note that these two contributions can be of the some order of magnitude: The chargino-sneutrino contribution is proportional to Higgsino-Wino mixing which can be sizeable, but suppressed by the smallness of the Higgsino-sneutrino-muon coupling which is proportional to the muon Yukawa coupling, $\propto m_\mu \tan\beta/v$ with the SM Higgs vacuum expectation value v. The neutralino-smuon contribution, on the other hand, arises via muon-smuon-neutralino vertices which are proportional to the gauge couplings, but is suppressed by the small smuon left-right mixing, $\propto m_\mu(\mu\tan\beta - A_\mu)/(m^2_{\widetilde{\mu}_R} - m^2_{\widetilde{\mu}_L})$. Regarding corrections beyond one-loop [108, 109], the most relevant contribution is associated with corrections to the muon Yukawa coupling, Δ_μ. These corrections become relevant at large values of $\mu\tan\beta$ and can be re-summed at all orders of perturbation theory [110]. While these corrections lead to small modifications of a_μ, they do not change the overall dependence of Δa_μ on the masses of the supersymmetric particles.

From Eqs. (2)–(3) we can observe that the sign of the MSSM contributions to a_μ depend sensitively on the relative signs of the gaugino masses M_1 and M_2 and the Higgsino mass parameter μ. As we will discuss shortly, a DM candidate compatible with the current null-results from direct detection experiments can be realized for $|\mu| \lesssim 500\,\mathrm{GeV}$ if M_1 and μ have opposite signs. For this combinations of signs, the contribution from the neutralino-smuon loop to a_μ will be negative, $a_\mu^{\widetilde{\chi}^0-\widetilde{\mu}} < 0$. Since the measured value of a_μ is *larger* than the SM prediction by $\Delta a_\mu \simeq 25 \times 10^{-10}$, we require the chargino-sneutrino contribution to be positive and larger than the neutralino-smuon contribution. This can be realized if M_2 has the same sign as μ and if $|M_2|$ is of similar size as $|\mu|$ and the soft smuon masses. In the regime of moderate or large values of $\tan\beta$, and assuming all weakly interacting sparticles have masses of the same order, $\widetilde{m}$, one obtains approximately

$$\Delta a_\mu \simeq 1.3 \times 10^{-9} \tan\beta \times \left(\frac{100\ \mathrm{GeV}}{\widetilde{m}}\right)^2\,, \tag{6}$$

The factor 1.3 reduces to values closer to 1 if M_1 and M_2 have opposite signs. This implies that for values of $\tan\beta \simeq 10$, sparticles with masses $\widetilde{m} \sim 200\,\mathrm{GeV}$ can lead to an explanation of the observed Δa_μ anomaly, while for $\tan\beta = 60$, the characteristic scale of the weakly interacting sparticle masses may be as large as $\widetilde{m} \sim 500\,\mathrm{GeV}$.

The range of $\tan\beta$ and of sparticle masses consistent with the observed Δa_μ has implications on the DM properties. We will concentrate on DM candidates with masses comparable to the weak scale such that the thermal DM relic density reproduces the observed value. In the MSSM, DM candidates in this mass range can be realized if the lightest supersymmetric particle is an almost-pure Bino, $m_\chi \simeq |M_1|$.

For the moderate-to-large values of $\tan\beta$ required explain the $(g_\mu - 2)$ anomaly, the spin independent direct detection (SIDD) amplitude for the scattering of DM with nuclei (N) is proportional to

$$\mathcal{M}_p^{\mathrm{SI}} \propto \frac{v}{\mu^2}\left[2\frac{(M_1 + \mu\sin 2\beta)}{m_h^2} - \frac{\mu\cos 2\beta}{m_H^2}\tan\beta\right], \tag{7}$$

where m_h and m_H are the masses of the SM-like and the new heavy Higgs. We see that the SIDD amplitude depends in a crucial way on the sizes and signs of M_1 and μ. There are two options to lower the SIDD amplitude: For large values of $|\mu|$, the Higgsino components of the DM candidate become small and the SIDD amplitude is suppressed. Alternatively, the terms inside the brackets in Eq. (7) can cancel, leading to a suppression of the SIDD amplitude. The latter option is particularly interesting since it allows $|\mu|$ to remain of the order of the electroweak scale; see, for example Ref. [111] for a recent discussion of naturalness and the connection with direct detection bounds.

Regarding the first term in Eq. (7), if $M_1 \simeq -\mu\sin 2\beta$, the contributions of the Higgsino-up and the Higgsino-down admixtures to the $(\chi\chi h)$ interaction cancel. The second term is the contribution to the $(\chi N \to \chi N)$ amplitude arising from the t-channel exchange of the non-SM-like heavy Higgs boson H. The *generalized blind spot* condition for SIDD cross section of a Bino-like DM candidate is then [43]

$$\frac{2\,(M_1 + \mu\sin 2\beta)}{m_h^2} \approx \frac{\mu\tan\beta\cos 2\beta}{m_H^2}\,. \tag{8}$$

If the condition in Eq. (8) is satisfied, the amplitudes mediated by h and by H exchange interfere destructively, suppressing the SIDD cross section; a property that also holds at the one-loop level [112]. Hence, if the neutralino is mostly Bino-like, for a given value of $|\mu|$ and

M_1, the cross section is suppressed (enhanced) if μ and M_1 have opposite (the same) sign.[5]

The value of the heavy Higgs boson mass plays an important role in the blind-spot cancellation. In the presence of light electroweakinos, the current LHC bounds on m_H coming from searches for heavy Higgs bosons decaying into τ-leptons [113–116] can be approximated by

$$m_H \geq 250 \text{ GeV} \times \sqrt{\tan\beta} \sim 2\, m_h \sqrt{\tan\beta}\ . \tag{9}$$

For values of m_H close to this bound, the SIDD amplitude is proportional to

$$\mathcal{M}_p^{\rm SI} \propto \frac{2}{m_h^2}\frac{M_1 v}{\mu^2}\left[1 + \frac{\mu}{2M_1}\left(\frac{4}{\tan\beta} + \frac{1}{4}\right)\right]. \tag{10}$$

To exemplify the relevance of the relative sign and size of μ and M_1, consider $\mathcal{M}_p^{\rm SI}$ for $\tan\beta = 16$. As a reference value for the SIDD amplitude, let us set $\mu \simeq -M_1$. Keeping M_1 fixed, but increasing the value of μ to $\mu \simeq -2M_1$, the value of $\mathcal{M}_p^{\rm SI}$ becomes a factor of $\approx 1/6$ smaller. Let us compare this to the situation if μ and M_1 to have the same sign. First, we can note that for $\mu = M_1$, the SIDD amplitude is almost a factor 2 larger than for $\mu = -M_1$. Furthermore, in order to obtain a reduction of $\mathcal{M}_p^{\rm SI}$ by a factor of 1/6, one would have had to raise the value of μ from $\mu \sim M_1$ to $\mu \sim 4M_1$. This exemplifies that obtaining SIDD cross sections compatible with experimental limits either requires $(\mu M_1) < 0$ (blind spot solution) or, to compensate for a positive sign of this product, one must sufficiently enhance the ratio μ/M_1 (large μ solution).

The spin dependent (SD) interactions are instead dominated by Z-exchange, and can only be suppressed by lowering the Higgsino component of the lightest neutralino. At moderate or large values of $\tan\beta$, the amplitude for SD interactions is proportional to [99]

$$\mathcal{M}^{\rm SD} \propto \left(\frac{v}{\mu}\right)^2 \cos 2\beta\ . \tag{11}$$

Comparison with the results from direct detection experiments [117–120] leads to an approximate bound on μ,

$$|\mu| \gtrsim 300\,\text{GeV}\ , \tag{12}$$

with a mild dependence on M_1.

To summarize this discussion, we show the qualitative behavior of the direct detection cross sections in Figs. 1 and 2 in the M_1–μ plane. We use approximate analytic expressions

[5] Note that $\cos(2\beta) = (1 - \tan^2\beta)/(1 + \tan^2\beta) \simeq -1$ for moderate-to-large values of $\tan\beta$.

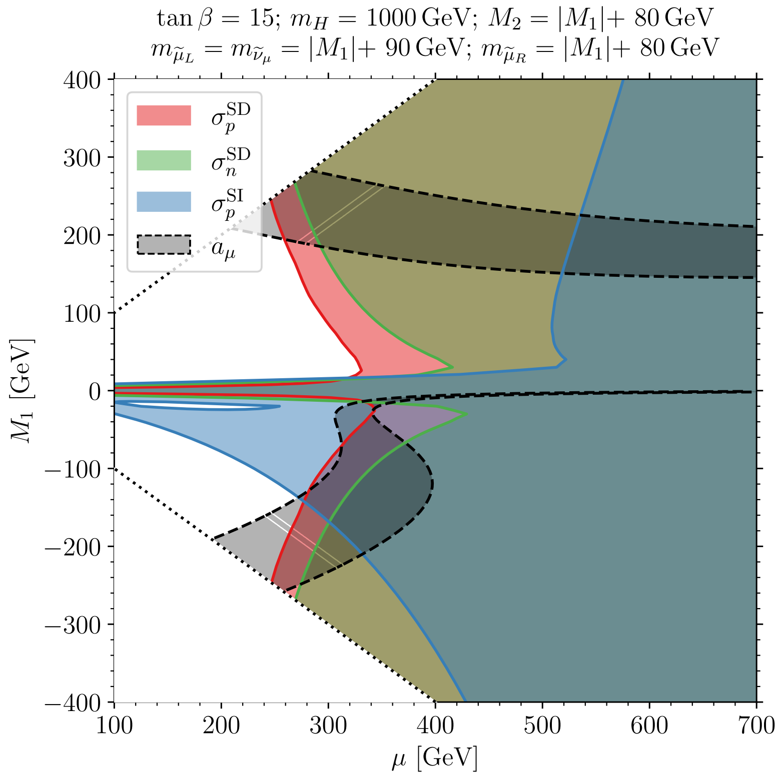

FIG. 1. Approximate bounds on the values of μ and M_1 coming from Δa_μ and DM direct detection constraints for $\tan\beta = 15$ and values of the slepton, Higgs and Wino mass parameters that leads to consistency with LHC constraints. The areas shaded in the respective colors are consistent with the current SI and SD direct detection bounds, and in the gray areas bounded by the dashed black lines we find a MSSM contribution $\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}$, explaining the value observed by the Fermilab and Brookhaven Muon (g-2) experiments. The two darkest gray areas denote the preferred region of parameter space.

for the cross sections and set the masses of the heavy Higgs boson and $\tan\beta$ to characteristic values. The values of M_2 and the slepton masses have been chosen to avoid current constraints from slepton and chargino searches at the LHC, see, for example, Refs. [121–131]. The regions shaded in the different colors denote the region allowed by current direct detection constraints on the SD-proton [117, 120], SD-neutron [118, 119], and SI [132–135] scattering cross section. We see that whereas the SD constraints provide an approximately symmetric lower bound on μ, due to the SI constraints the values of $|\mu|$ need to be significantly larger for positive $\mu \times M_1$ than for negative $\mu \times M_1$. We also show the region

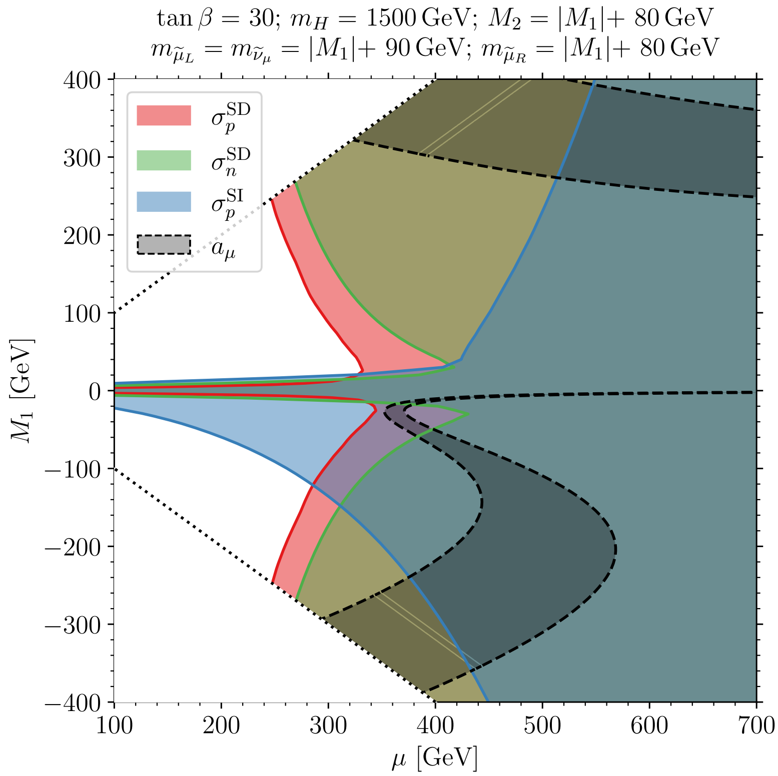

FIG. 2. Same as Fig. 1 but for $\tan\beta = 30$.

where the MSSM contribution explains the $(g_\mu - 2)$ anomaly in Figs. 1 and 2 with the gray shade bounded by the dashed black line. The shape of the region preferred by Δa_μ may be understood from the interplay between the Bino- and Wino-mediated contributions. For large values of $|\mu|$, the Bino contribution tends to be the most relevant one. If one considers positive values of $\mu \times M_1$, it gives a positive contribution to Δa_μ which can account for the $(g_\mu - 2)$ anomaly for sufficiently large values of $\tan\beta$. However, for smaller values of $|\mu|$ and negative values of $\mu \times M_1$, as required by the blind spot solution, the Bino contribution tends to be subdominant and neither has the sign nor the magnitude to account for the $(g_\mu - 2)$ anomaly. If anything, depending on the sign of $M_1 \times M_2$, it will partially cancel the Wino contribution to Δa_μ. For smaller values of $|\mu|$, an explanation for the $(g_\mu - 2)$ anomaly requires a Wino-mediated contribution enabled by $\mu \times M_2 > 0$ and moderate values of M_2. We stress that our preferred region is the intersection of the a_μ and the direct detection contours, corresponding to the regions with the darkest shade.

B. Δa_μ and the DM Relic Density

For a Bino-like DM candidate with mass in the few hundred GeV range, the observed relic density can be realized via thermal production through different mechanisms, such as co-annihilation with sleptons or charginos [136–141], t-channel annihilation via light left-right mixed staus [142] or smuons [143], or resonant s-channel annihilation [139, 140]. In Tables I and II we present a few benchmark scenarios which simultaneously accommodate the $(g_\mu - 2)$ anomaly and a viable DM candidate. All of them are consistent with the observed relic density, the observed value of Δa_μ, and satisfy the LHC constraints as well as constraints from direct detection. Bounds from Higgsino and Wino pair production depend on a careful consideration of the decay branching ratios [144, 145]. For the aim of this work, however, we have consider a compressed spectrum, for which the Wino production constraints are weakened. The results for the spectrum, Δa_μ, relic density as well as the SI and SD cross sections have been obtained with `Micromegas v 5.2.7.a` [146–148].

- The most natural benchmark is that associated with a light smuon. The proper relic density may be obtained by co-annihilation of the lightest neutralino with the muon sneutrino. The benchmark BMSM gives a representation of such a possibility.

- Another similar solution is associated with the co-annihilation of a light stau with the lightest neutralino, something that happens naturally at large values of $\tan\beta$, where the lightest stau is pushed to masses lower than those of the sneutrinos. BMS1 gives a representative spectrum consistent with such a possibility.

- A light neutralino can also annihilate via the interchange of a t-channel stau. BMS2 gives a representation of this possibility.

- BMS3 represents a case of mixed co-annihilation with charginos and staus.

Although the mechanism controlling the relic density is different for the different benchmark points, all of them present similar characteristics. They feature masses of weakly interacting sparticles masses lower than about 500 GeV and values of $\tan\beta$ of the order of a few 10's, leading to values of Δa_μ in the desired range. All of them have negative values of $\mu \times M_1$ and positive values of μM_2 as discussed above.

	BMSM	BMS1	BMS2	BMS3
M_1 [GeV]	-290	-234	-96	-175
M_2 [GeV]	350	280	212	210
μ [GeV]	500	460	350	355
$M_L^{1,2}$ [GeV]	300	400	272	275
M_L^3 [GeV]	500	300	272	275
$M_R^{1,2}$ [GeV]	300	300	112	190
M_R^3 [GeV]	500	300	112	190
M_A [GeV]	1800	1800	1500	1000
$\tan\beta$	40	40	25	15

	BMSM	BMS1	BMS2	BMS3
m_χ [GeV]	287.0	231.5	92.6	172.5
$m_{\tilde{\tau}_1}$ [GeV]	464.9	241.8	104.6	189.2
$m_{\tilde{\mu}_1}$ [GeV]	303.2	395.0	120.4	195.0
$m_{\tilde{\nu}_\tau}$ [GeV]	496.0	293.3	264.5	267.7
$m_{\tilde{\nu}_\mu}$ [GeV]	293.3	395.0	264.6	267.7
$m_{\chi_1^\pm}$ [GeV]	334.5	267.5	208.2	193.7
$\Delta a_\mu\ 10^9$	2.43	2.98	2.66	2.06
$\Omega_{\rm DM}h^2$	0.116	0.120	0.118	0.121
$\sigma_p^{\rm SI}$ [10^{-10} pb]	2.01	1.26	0.11	0.98
$\sigma_p^{\rm SD}$ [10^{-6} pb]	4.67	5.27	10.1	13.8
$\sigma_n^{\rm SI}$ [10^{-10} pb]	2.00	1.24	0.11	0.95
$\sigma_n^{\rm SD}$ [10^{-6} pb]	3.77	4.23	7.9	10.9

TABLE I. Values of the MSSM parameters, mass spectrum and quantities relevant for dark matter and $(g_\mu - 2)$ for the case of Bino-like DM co-annihilating with a muon sneutrino (BMSM), co-annihilating with a light stau (BMS1), annihilating via stau mediated t-channel (BMS2) and co-annihilating with staus and charginos (BMS3).

In Table II we present benchmarks for which the sleptons do not give a relevant contribution to the thermal production of the DM relic density:

- The lightest neutralino may co-annihilate with the lightest chargino, due to the small mass difference. The LHC bounds may be avoided due to the compressed spectrum. The benchmark BMW represents such a possibility.

- The lightest neutralino can acquire the proper relic density via resonant s-channel annihilation. BMH1 and BMH2 represent such a possibility.

The proper thermal relic density may be obtained via resonant s-channel annihilation mediated by either the Z, the SM-like Higgs (h) or the heavy Higgs bosons, A and H. For the values of $\tan\beta$ necessary to enhance Δa_μ, the bounds on the heavy Higgs bosons become very strong, implying a heavy spectrum. Indeed, using the bounds on m_H provided in Eq. (9) and the approximate expression for Δa_μ, Eq. (6), and assuming that all the weakly

	BMW	BMH1	BMH2
M_1 [GeV]	-227	-62.	62.4
M_2 [GeV]	260	140.	130.
μ [GeV]	450	320	500.
$M_L^{1,2}$ [GeV]	332	720	720
M_L^3 [GeV]	332	720	720
$M_R^{1,2}$ [GeV]	330	720	720
M_R^3 [GeV]	330	720	720
M_A [GeV]	1500	2000	2500
$\tan\beta$	25	40	45

	BMW	BMH1	BMH2
m_χ (m_h) [GeV]	224.6	60.1 (124.8)	60.8 (124.9)
$m_{\tilde{\tau}_1}$ [GeV]	303.3	708.1	693.1
$m_{\tilde{\mu}_1}$ [GeV]	332.9	721.4	721.4
$m_{\tilde{\nu}_\tau}$ [GeV]	325.9	717.2	717.2
$m_{\tilde{\nu}_\mu}$ [GeV]	325.9	717.2	717.2
$m_{\chi_1^\pm}$ [GeV]	247.9	140.2	135.6
Δa_μ 10^9	2.13	2.38	2.11
$\Omega_{\rm DM}h^2$	0.117	0.116	0.121
$\sigma_p^{\rm SI}$ [10^{-10} pb]	1.20	3.6 10^{-3}	0.37
$\sigma_p^{\rm SD}$ [10^{-6} pb]	5.7	13.1	2.4
$\sigma_n^{\rm SI}$ [10^{-10} pb]	1.19	5.2 10^{-3}	0.39
$\sigma_n^{\rm SD}$ [10^{-6} pb]	4.6	10.2	1.9

TABLE II. Same as Table I but for the case of co-annihilation with a Wino (BMW) and resonant s-channel annihilation via the SM-like Higgs boson (BMH1 and BMH2). For BMH1 and BMH2 we also provide the mass of the SM-like Higgs boson m_h between brackets.

interacting sparticles have masses close to $m_H/2$, the maximal value for Δa_μ that may be obtained is

$$\Delta a_\mu \simeq 10^{-9}\tan\beta\frac{4}{m_H^2}(100\ \text{GeV})^2 \lesssim 7\times 10^{-10}, \tag{13}$$

which is a factor of a few smaller than the observed anomaly. Therefore, we shall not discuss this particular solution further.

Regarding the resonant s-channel annihilation via the Z-boson, it presents similar characteristics to resonant annihilation mediated by the SM-like Higgs, h. We therefore present two example of the latter case. The two examples are related to the fact that, for such small values of $M_1 \simeq 60\,\text{GeV}$, values of $|\mu| \lesssim 500\,\text{GeV}$ may lead to the desired suppression of the cross section for either sign of μ. This follows, for instance, from Eq. (7), from where we also observe that for positive values of $\mu \times M1$, values of m_H significantly larger than the current experimental bounds are preferred. Observe that in the case of BMH1 and BMH2 we chose the sleptons to be heavy to avoid the bounds from the LHC, hence, the proper values of Δa_μ require relatively large values of $\tan\beta$. An extended discussion of the region of parameters consistent with Δa_μ for these s-channel annihilation are presented in Ref. [99].

Let us finally note that while we have chosen benchmark points compatible with current bounds on heavy Higgs bosons, sleptons, and charginos from the LHC, these models are within the reach of future runs of the (HL)-LHC and, in the case of sleptons and charginos, future lepton colliders, see, for example, Refs. [149–161].

IV. SUMMARY AND CONCLUSIONS

In this article we present several possible extensions of the Standard Model that can lead to an explanation of the value of Δa_μ measured at the Fermilab and Brookhaven experiments. While the simplest explanation is just the addition of a scalar particle, one can also rely on new gauge bosons, vector-like fermions or leptoquark models. The leptoquark (or R-parity violating supersymmetry) solution seems to be interesting since it can accommodate not only the values of Δa_μ, but can also lead to an explanation of the flavor anomalies, although at the prize of a delicate tuning between the couplings of the leptoquarks to quarks and leptons.

This work puts most emphasis on a solution based on the Minimal Supersymmetric extension of the Standard Model, in which, although one cannot address the flavor anomalies, one can find solutions leading to a compelling DM explanation. In particular, we discuss the conditions that are required to be consistent with the observed Δa_μ, existing direct dark matter detection constraints, and the bounds from the LHC on new Higgs bosons and supersymmetric particles. We stress the importance of negative values of $\mu \times M_1$ to satisfy the direct detection constraints for small values of $|\mu|$ and consider the impact of this condition on the resulting values of Δa_μ. In general, the measured values of Δa_μ are consistent with moderate or large values of $\tan\beta$ and light electroweak interacting supersymmetric particles. The observed relic density may be produced via a wide range of different mechanisms, including co-annihilation with sleptons, t-channel annihilation mediated by staus, or resonant s-channel annihilation mediated by the Standard Model-like Higgs or Z bosons. The resonant s-channel annihilation mediated by heavy Higgs bosons, on the other hand, tends to require heavy supersymmetric particles leading to values of Δa_μ that are smaller than those recently observed by the Fermilab Muon (g-2) experiment.

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