

STUDYING STUDENT ACTIONS WITH DYNAMIC TEXTBOOKS IN UNIVERSITY SETTINGS: THE LOG AS RESEARCH INSTRUMENT

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Textbooks are common in university mathematics education, but compared to K-12 education, there is little research on their use. This scarcity of research might be due to the difficulty in gathering reliable data. In some studies one-on-one interviews are conducted in lab-like settings in which students are observed as they use their textbooks. In others, surveys are used that ask students to report on various activities that can be done with the textbooks. Both approaches are informative, but insufficient if the goal is to understand student textbook use when hundreds of students use them in real time. The availability of digital textbooks in course management systems facilitates collecting viewing data from hundreds of students that helps describe browsing patterns and time spent; yet viewing as a way of using a textbook is insufficient for characterizing reading strategies or the comprehension that such reading elicits. In this essay, we present an account of two developmental processes. First, that of a tool, the log, that we used to gather data from large numbers of university students about their *actions* with a digital textbook. Second, that of our understanding of the mediating role of the log in the activity of gathering the data.

To frame the developmental process of the log as a research instrument, we use Rabardel and colleagues' instrumental approach. They state that the material artifact (*e.g.*, the log, the textbook) can be acted on in certain ways by its users, and distinguish between users who are aware of the kinds of tasks the material artifact supports and users to whom the material artifact is still empty of meaning (Rabardel & Waern, 2003). The material artifact becomes an 'instrument' only after the user attaches a scheme of use for a particular task to the artifact; that is, when the user gains insights into, and puts in practice, the artifact's implicit potential. As we reflected on how the log evolved, we analyzed the schemes of use we assigned to it for gathering data on student actions with textbooks. One conceptual scheme of use involved the mediational role of the log, which we observed by identifying the relationships between researcher and methods. Before elaborating on what we learned about student use through this lens and describing how we theorized the evolution of the log, we present several definitions.

The 'log' is a short survey that contains a combination of fact-based questions (*e.g.*, textbook used, topics covered,

week of the term) and reflective prompts (*e.g.*, whether a particular activity was a major focus of the lesson or not; what did participants do while reading) that is sent periodically to participants to gather their immediate recollections and reflections of the activity at hand.

Our study focused on a type of digital textbook that we call 'dynamic', an open-source (code available to users) and open-access (viewing and printing possible) textbook that is available in HTML and PDF formats. When viewable in HTML, the user can interact with computational cells (windows with executable and modifiable Python code, executable within the programming language Sage), expand or hide examples or sections by clicking on them, and take advantage of search or adaptive features to ease access. More importantly, because they are open-access, the textbooks can be distributed for free to the students, and because they are open-source, the authors can make modifications that are immediately available to users. The textbooks are authored in PreTeXt [1], a markup language that codifies the structure of textbooks so that the textbook can be created and viewed in multiple formats including Braille. The codified structure facilitates the tracking of users' minute-by-minute viewing of an HTML textbook. Such tracking generates massive real-time viewing data sets that provided us with a starting point to gather information about student actions with the textbook.

As we progressed through four phases of data collection we learned more about the log and its connection to the nature of our research object: to collect data that would allow us to reconstruct student actions with the dynamic textbooks. These actions could, in the future, be used to establish a connection between reading strategies and comprehension of mathematical content in the textbooks. By the fourth phase, student responses included reflective and nuanced actions with the textbook that enabled us to also discern reading strategies: Students said they "reread definitions, examples, and proofs to gain better knowledge about certain theories and basic concepts". They searched for definitions, found "as many examples for every axiom/postulate", drew "diagrams and pictures to display statements geometrically or visually", thought "of [their] own examples and then work[ed] with [their] examples to see if [they] could be consistent",

“look[ed] up the homework problems”, “use[d] the textbook as a secondary reference”, made “a note of things [they] have trouble remembering”, “read the main point and then [...] view[ed] the problems and attempt[ed] them”. A few students also said they did not use their textbooks. How were we able to obtain these responses that allow us to reconstruct student actions? How did the research process unfold as we were pursuing this goal? In the next section, we present the theoretical analysis of the evolution of the log as a research instrument that helps in answering these questions.

Theorizing the nature of the log

To become a research instrument, the log requires that the researcher assign it a scheme of use that is tied to the researcher’s goal, namely gathering data on student actions with dynamic textbooks. In our case, we conceptualized the logs as mediators of student actions with the dynamic textbooks; as such, we anticipated several mediations (see Figure 1).

Arrow 1 depicts the mediation between us (researchers) and the students via logs so that the object of the research activity (gathering data on student actions with textbooks) is satisfied. In this mediation, the log needed to bridge our relationship with the students so they responded to us in the intended ways (*i.e.*, ways that could be used to interpret their actions with their textbooks, so we can infer use). We designed log questions that included areas of inquiry about the object of the research activity, that is, questions that we anticipated would jog students’ memory and attention so they would respond by describing their actions with the textbooks. Note that in such a system of mediations, the log also acts as a mediator between the students and the textbooks because it supports students’ reflection on their actions—this mediation is not depicted in the diagram, as the focus of this essay is on the method that promoted the researcher-student relationship towards the desired object of the research.

Arrow 2 illustrates the mediational role of the log as the researchers make sense of the student responses. We interpreted student responses in terms of what the log questions communicated to the students; after collecting the data, we realized that student reflection on their actions was a high-level cognitive process that students needed to engage in, and practice, in order to offer responses that would allow for an in-depth analysis of their actions.

Arrow 3 showcases the mediation of the log towards ourselves; we learned how the logs worked with the students and modified our thinking about the reach and capacity of the logs. We also enriched the logs with viewing data of textbook use, and satisfied our research object. While these mediations are implicit in any research enterprise, in our case their importance was heightened because we had a large number of participants that were not physically nearby. It was this specific aspect, and the object to collect data that would allow us to reconstruct student actions with dynamic textbooks, that led us to realize that the log was going to be a key instrument for our purposes; but this realization came in various phases. In the analysis we present next, we show how the logs mediated the researcher-student relationship and how in each phase the mediations shaped our intentions for the data collection as students responded to the logs and

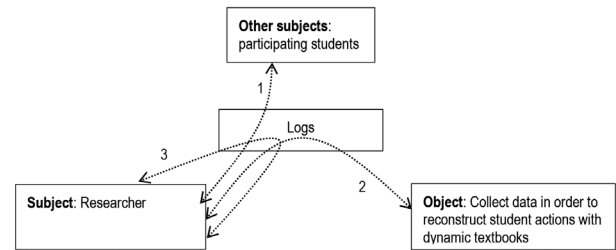


Figure 1. Possible mediations between the logs and the researcher, the students, and the object of activity (collect data in order to reconstruct student actions with dynamic textbooks). Dotted arrows represent possible mediations between the subject and (1) the students, (2) the object, and (3) self.

as we continued our efforts to get access to student actions with their dynamic textbooks using the questions in the logs.

The developmental process of the log as research instrument

Our data were collected over three semesters, from seven sections of linear algebra and three sections of abstract algebra, taught by eight different instructors (two taught their course twice) located at eight different cities. In total, 175 students were involved. Each course used a dynamic textbook [2]. We gathered about 730 log responses from the students. We describe next our revision process in four phases, as we were working towards a reconstruction of student actions with their dynamic textbook, together with our interpretation of the mediations.

First phase: prompting journaling with highlighted examples

The research activity entails a bidirectional relationship between us and the students mediated by the log. Initially, we thought about the periodicity of the log as giving us information about textbook use from the students so we could monitor changes over the semester and give students a chance to think through their use of the textbook. In this phase, we attempted to recreate Rezat’s (2013) journaling strategy and asked students to highlight sections of their textbooks and journal about what sections they used and why (see Figure 2). The images we included in the prompt had highlighted sections, each of which had been labeled with numbers (from 1 to 3). The numbers, affixed to post-it notes, suggested reasons that a student may have had for highlighting the text; those reasons were adapted from existing research and augmented by undergraduate students in our research team. The figure includes also the log question that accompanied the image and paradigmatic student responses.

The student responses addressed the questions asked. The student actions with the textbooks were operationalized as: what textbook sections students used (*e.g.*, “Solving Systems of Linear Equations”) when they used them (“while preparing for class,” “doing homework”), and why (“going over questions,” “studying for exams,” to “reread all the sections”). In this phase, we encountered two issues that shed

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EXAMPLE 9

If $\vec{v}_1, \dots, \vec{v}_m$ is a basis of a subspace V of $\mathbb{R}^n$, and if $\vec{v}$ is a vector in V , how many solutions $c_1, \dots, c_m$ does the equation

$$\vec{v} = c_1 \vec{v}_1 + \dots + c_m \vec{v}_m$$

have?

Solution

There is at least one solution, since the vectors $\vec{v}_1, \dots, \vec{v}_m$ span V (that's part of the definition of a basis). Suppose we have two representations

$$\begin{aligned} \vec{v} &= c_1 \vec{v}_1 + \dots + c_m \vec{v}_m \\ &= d_1 \vec{v}_1 + \dots + d_m \vec{v}_m. \end{aligned}$$

By subtraction, we find

$$(c_1 - d_1) \vec{v}_1 + \dots + (c_m - d_m) \vec{v}_m = \vec{0},$$

a relation among the vectors $\vec{v}_1, \dots, \vec{v}_m$. Since the vectors $\vec{v}_1, \dots, \vec{v}_m$ are linearly independent, this must be the trivial relation, and we have $c_1 - d_1 = 0, \dots, c_m - d_m = 0$, or $c_1 = d_1, \dots, c_m = d_m$. It turns out that the two representations $\vec{v} = c_1 \vec{v}_1 + \dots + c_m \vec{v}_m$ and $\vec{v} = d_1 \vec{v}_1 + \dots + d_m \vec{v}_m$ are identical. We have shown that there is one and only one way to write $\vec{v}$ as a linear combination of the basis vectors $\vec{v}_1, \dots, \vec{v}_m$. ■

Let us summarize.

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Theorem 3.2.10

Basis and unique representation

Consider the vectors $\vec{v}_1, \dots, \vec{v}_m$ in a subspace V of $\mathbb{R}^n$.

The vectors $\vec{v}_1, \dots, \vec{v}_m$ form a basis of V if (and only if) every vector $\vec{v}$ in V can be expressed *uniquely* as a linear combination

$$\vec{v} = c_1 \vec{v}_1 + \dots + c_m \vec{v}_m.$$

(In Section 3.4, we will call the coefficients $c_1, \dots, c_m$ the *coordinates* of $\vec{v}$ with respect to the basis $\vec{v}_1, \dots, \vec{v}_m$.)

③

Proof In Example 9 we have shown only one part of Theorem 3.2.10; we still need to verify that the uniqueness of the representation $\vec{v} = c_1 \vec{v}_1 + \dots + c_m \vec{v}_m$ (for every $\vec{v}$ in V) implies that $\vec{v}_1, \dots, \vec{v}_m$ is a basis of V . Clearly, the vectors $\vec{v}_1, \dots, \vec{v}_m$ span V , since every $\vec{v}$ in V can be written as a linear combination of $\vec{v}_1, \dots, \vec{v}_m$.

To show the linear independence of vectors $\vec{v}_1, \dots, \vec{v}_m$, consider a relation $c_1 \vec{v}_1 + \dots + c_m \vec{v}_m = \vec{0}$. This relation is a representation of the zero vector as a linear combination of $\vec{v}_1, \dots, \vec{v}_m$. But this representation is unique, with $c_1 = \dots = c_m = 0$, so that $c_1 \vec{v}_1 + \dots + c_m \vec{v}_m = \vec{0}$ must be the trivial relation. We have shown that vectors $\vec{v}_1, \dots, \vec{v}_m$ are linearly independent. ■

Consider the plane $V = \text{im}(A) = \text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4)$ introduced in Example 4. (Take another look at Figure 4.)

Thm. used on
test / hw.
* IMPORTANT
②

Assigned
reading +
* Important
proof! ③

Log Question

The UTMOST Student Log asks you to comment on your textbook use from your most recent lesson.

Q1. Which parts of your textbook did you use for the most recent lesson in the course? The samples show how a student highlighted the textbook, indicating some of the reasons for doing so.

Q2. When did you reference these sections?

Q3. Why did you focus on these parts of the textbook?

Paradigmatic log response

Q1. I used the sections we were going over in class. From *Solving Systems of Linear Equations* to *Linear Combinations*.

Q2. While preparing for class, While doing homework, While studying for exams

Q3. [These sections] were a great help when going over questions as they were offered a slightly different approach [than] perhaps use used in class. Some of the questions that we went over were directly from the text including questions T20 and T21 in *Spanning Sets*. Before our first exam I reread all the sections that we had gone over prior

Figure 2. Prompt and paradigmatic responses for Phase 1— Student-Researcher mediation.

light into the mediational role of the log between the students and the researchers. First, the highlighting feature was cumbersome to use; and second, the prompt did not generate the reflective responses we were expecting. In the paradigmatic response, for example, it is difficult to know what does “used the sections” mean: did students *read* every single word? Or did they just scan the content? Likewise, when the students say that they “reread all the sections” did they focus on the examples but not the theorems or did they read the proofs? During an interview a researcher can probe students for further clarification. This is when we recognized that students needed to realize that they were being asked to reflect on their actions, and that they needed practice on how to reflect on this use through periodically responding to the logs.

Second phase: prompting differentiation by contextualizing use

To generate more specificity and detail in student responses we decided to ask students to reflect on two different situations when they used the textbook: when preparing for class and during class. We thought that having a contrast would allow them to generate different orientations they may have had towards the textbook and nudge them into noticing more nuanced distinctions. The prompts were as follows:

Q1: Which sections or features of your textbook did you use to *prepare for class* in your most recent lesson?

Q2: Which sections or features of your textbook did you use *during class* in your most recent lesson?

For each prompt, students were asked to include:

1. the names of the sections or the features;
2. which elements you used (*e.g.*, introduction, theorem, proof, example, problem, etc.);
3. how you used those elements.

Two paradigmatic responses were:

In section 2.2 I refreshed my memory with the Euclidean algorithm to find the GCD of two numbers. While working on the homework, I used the Sage Exercises to check my answers. It was helpful considering I messed up once or twice on a problem. I also looked at Theorem 2.15 to help with one of the proof exercises in my homework.

We talked about subgroups in section 3.3. We used definitions, analyzed examples and worked out problems. We also started to talk about transposition in section 5.1. We worked out problems with cycle notation and two-line notation.

As the paradigmatic responses illustrate, relative to the first phase, we obtained a wider range of responses and more details about textbook use, including reports of students not using the textbook, which suggested student honesty. Student responses included their actions with the textbooks, which were operationalized as: what textbook sections

students used (*e.g.*, in the paradigmatic response “section 2.2,” “section 3.3”), what textbook elements they used (“Sage Exercises,” “Theorem 2.15,” “definitions”), and why or when (*e.g.*, “to check my answers,” “to help with one of the proof exercises,” “in my homework”). Noticeably, in their responses to the question about preparing for class, students used ‘I’, but for during-class question responses they used plural pronouns (*e.g.*, ‘we’, ‘our’). Moreover, these responses only described the textbook sections and the textbook elements but not the students’ intentions.

In the responses, the word ‘used’ was prominent in the descriptions students provided and we found it difficult to infer what students meant. Does ‘using a theorem’ mean that the theorem was read or that it was part of a proof? When definitions were ‘used’, were they copied from their textbook verbatim in their notes or paraphrased? We realized that we needed to heighten students’ awareness of their actions when ‘using’ their textbooks and that the log needed to prompt students to turn their gaze onto those kinds of actions. To manage this process, we introduced a question that showed students the minute-by-minute individualized viewing data that was available for the HTML dynamic textbooks (‘heatmaps’). We also began to reflect on the role of the log as a mediation between us and our research object.

Third phase: prompting recall via real-time data on textbook use

A heatmap is an interactive, visual representation of the frequency of viewing of the various elements of the textbook (see Figure 3). Figure 3a represents the aggregated viewings of all users of one of our textbooks over a semester. It is a two-dimensional representation (textbook section viewed by time). The vertical axis lists all the sections of the textbook (*e.g.*, SLE: Systems of Linear Equations; V: Vectors). The horizontal axis at the top shows each day in the term. In the intersection, a colored rectangle indicates the frequency of viewings of the given section over the given day. Lighter colors represent a low frequency of viewing, while darker colors represent more frequent viewing of that section. Clicking on one of the rectangles opens up a new page (not presented here) showing a new map at the textbook subsection level for one full day. Clicking on any of the new rectangles opens up a map of individualized use (Figure 3b). These representations scroll left to right and up and down. Their latest iteration shows a label as users hover the cursor over a rectangle; the label shows the user’s ID, the name of the textbook section, and the time of the day when that section was viewed.

We intended for this information to facilitate students’ recall of what they were doing when interacting with their textbook on a particular day in the week. We selected a day with heavy textbook viewing as represented by nearly black rectangles in the heatmaps (Figure 3a). The prompt we used is:

A representation (a map) of how much students in your class used the textbook is here [a URL to the heatmap was provided]. Each rectangle shows textbook use, and corresponds to a textbook section (vertical axis) and a time (horizontal axis). Scroll to the right to see sets of rectangles in different colors; each color represents a

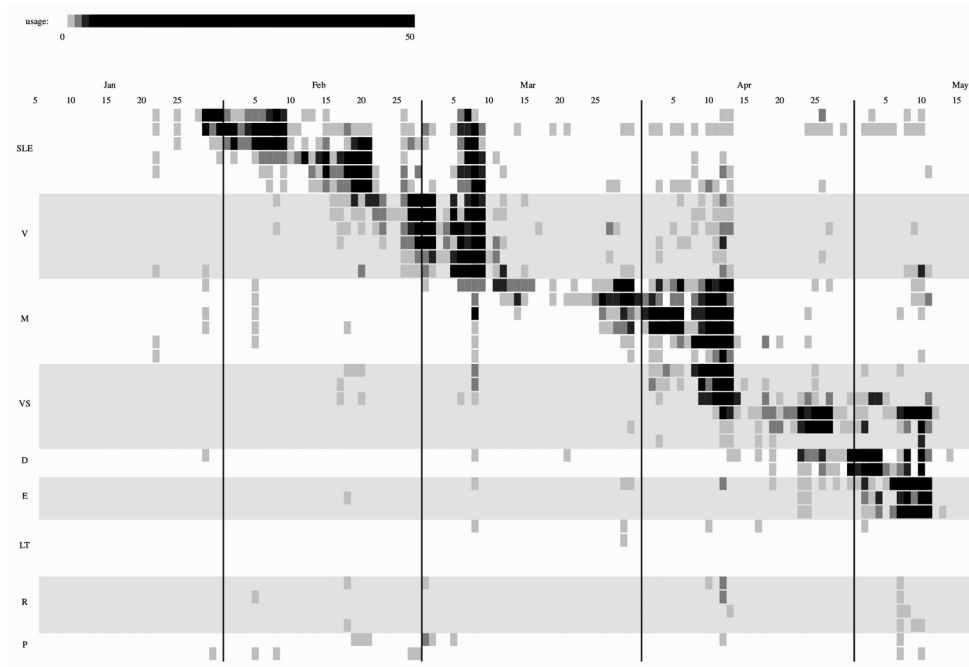


Figure 3a. Representation of user data for the full semester. The original map is in color (see cover).

different student. When you hover the cursor over a rectangle you will see the name of the section and the time when that section was opened. Please identify the color that represents you, and tell us what you were doing with a couple of sections represented by a couple of rectangles. Start your response by stating your color, the section, and the time shown in the rectangle.

With this prompt, we received responses that provided more context for the word ‘use’ than before, helping us learn how students were acting with the textbooks:

Sep 28 Thu, 12:24 PM: I only used the textbook for end of chapter problems. I completed the end of chapter problems for each section that the professor assigned, but for exam review, I skipped section SSLE because I

felt good about the material. In the end of chapter problems, I always click on the solutions after I work out my answer, and if there are theorems or proofs attached, I click on those as well to see what they are talking about.

In this paradigmatic response, the student explained how he or she “used the textbook for end of chapter problems” with the following: “for exam review, I skipped section SSLE because I felt good about the material”, “I always click on the solutions after I work out my answer”. In this phase, the student actions with the textbooks were operationalized as what textbook elements students used within section, when and why. The elements used were marked by emphasis on textbook elements (for example, in the paradigmatic response

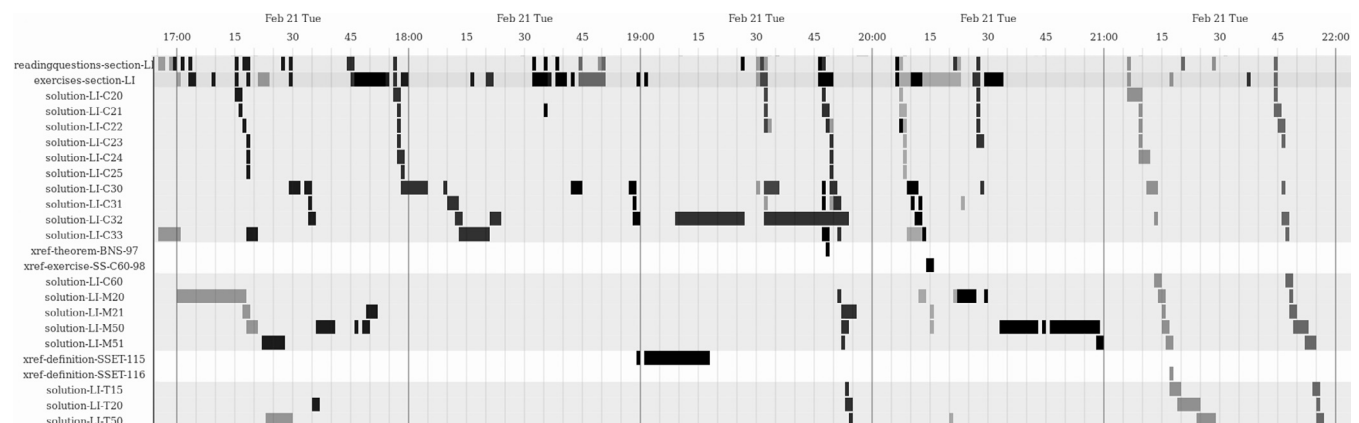
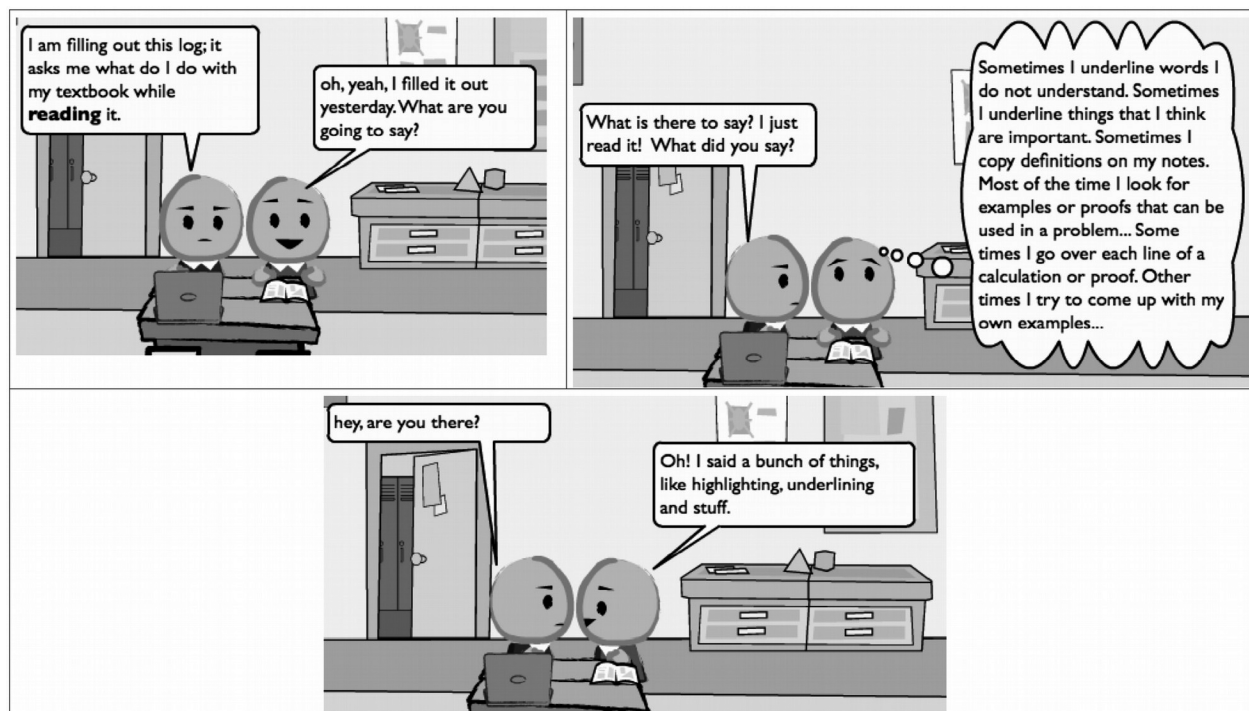


Figure 3b. Representation of user data for individual users. The original map is in color, with different colors representing different users.



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Log question:

What do you do when you read your textbook? (In combination with a heatmap as in Figure 3b.)

Paradigmatic log response

Jan 30 Tues, Chapter SLE [Systems of Linear Equations] 6-7 PM: I was looking over definitions and theorems to help me prepare my in-class quiz. I also needed some examples to look over to help me understand a topic in TSS [Types of Solutions Sets] and HSE [Homogeneous Systems of Equations]. Feb 14 Wed, Chapter SLE 10-11 PM: Like above, I needed to look at examples and definitions. Feb 19 Mon, Chapter M [Matrices] 6-9 PM: I was looking over example problems to help me prepare for my quiz, and I looked at definitions and theorems to help explain notation and solutions.

Figure 4. Prompt and paradigmatic response for Phase 4—Researcher-Researcher mediation.

“end of chapter problems”). When and why were marked by emphasis on storyline, *e.g.*, when: “for exam review”, why: “skipped section SSLE because [...]”. The phrase “after I work out my answer” also marks when, and the usual why offered in similar responses was to check correctness.

Simultaneously, we encountered two difficulties, related to the heatmaps. First, some students could not remember whether they had used the textbook on the given day, which we anticipated happening, because of the delay between sending the logs and students taking the time to respond to it. Second, some students were not able to identify themselves. Those constraints showcased the reflexive mediation of the log towards ourselves; the lag time between receiving a log and the day we asked them to reflect on it needed to be reduced and we needed to include individualized viewing patterns. We made these modifications (the tracking system now shows a user ID when hovering over an individual pattern).

Fourth phase: prompting story-telling with illustrations in the form of cartoons

In the fourth phase, we modified the prompt using cartoon characters to exemplify the responses we were looking for

and connected the prompt to individual heatmaps (see Figure 4) [3]. Cartoons, as a representation of reality, have both a serious and playful nature that we felt would better connect to our target population than, say, a video. It was also faster for us to produce a cartoon than a video, which was necessary given the quick turnaround we had for the data collection. Students in five sections answered the prompt in two logs (Log 1 and Log 5).

The answers to this prompt provided information about student actions with the textbook, operationalized as:

- the textbook element (*e.g.*, ‘definitions and theorems’) the student used along with the associated textbook chapter or subsection (*e.g.*, ‘subsection TSS’),
- the date and time when that textbook section was viewed, and
- why (*e.g.*, “to help explain notation and solutions,” “to help me understand”).

About half of the responses were similar to the paradigmatic response, suggesting that student actions with the

textbook were guided by the need to prepare for assessments and to complete the homework. The responses indicated also that students review definitions and examples, and use Sage and alternative resources, such as Khan Academy or Google. The later resources were not connected to the dynamic textbook (thus not reflected on representations of user data analytics) and that was a limitation in student storytelling. Not all students integrated external resources into their storylines of actions with textbooks, but the responses of those who did allowed us to look at the frequency of external resource used. On the other hand, regarding specific elements of the textbook, the log resulted in rich descriptions of student activity traced back to computer generated data and the opportunity for researchers to reconstruct student actions with the textbook.

In the third and the fourth phases, we were able to reflect on our own processes. The log allowed for a reflexive mediation between the research activity and ourselves as we developed a greater understanding of what the scheme of use of the log was. It acted as a mirror that fed back to us affordances and constraints of various attempts at accessing students' actions with the textbooks. With this process, we learned how the log could be used to gather data when other methods are not possible or difficult and also that the responses gathered could provide good access to the phenomenon at hand in large scale contexts. The heatmaps provided accuracy comparable to direct observations of textbook use. Moreover, mining information from heatmaps has an advantage over the information that can be garnered via direct observation, because user interaction with the dynamic textbooks generates higher volumes of data on actions at a lower cost. The cartoon, as a 'projective representation' (Chazan & Herbst, 2011) that immerses users in a reflective situation about how they act with their textbook, provided augmented information about the students' actions with the textbooks. Put another way, the cartoon projected the nature of the intended response onto the students. The combination of the heatmap and the cartoon explaining how to read and interpret the representation provided the combination that engaged students best in reflecting on their own actions with the textbook in real time.

Looking ahead

We believe that the knowledge gained from the development of the log as a research instrument could be transferred to the

development of other kinds of research instruments with digital technologies. As of this writing, we are experimenting with algorithms for natural language processing to synthesize student log responses, as we continue to accumulate data from students. We want to be able to quickly identify differences in responses when the students use a dynamic textbook and to identify differences that relate to the content of the textbook. We believe in our collective ingenuity as a community of researchers to tackle the methodological challenge of researchers reconstructing student activity with mathematics textbooks so we contribute to a new era of learning how digital textbooks can enhance the learning of mathematics.

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Notes

- [1] <https://pretextbook.org/>
- [2] Beezer, R. (2017) *A First Course in Linear Algebra*. Gig Harbour, WA: Congruent Press. Online at <http://linear.pugetsound.edu> and <http://linear.ups.edu/html/fcla.html> and Judson, T. (2017) *Abstract Algebra: Theory and Applications*. Ann Arbor, MI: Orthogonal Publishing L3C. Online at <http://abstract.pugetsound.edu> and <http://abstract.ups.edu/aata/>.
- [3] The cartoons were developed with the Depict tool in the LessonSketch environment: <https://www.lessonsketch.org/>

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Tiere	Fisch	Hund	Katze	Maus	Wellenritzen	Meeschweinchen	Kanarienvogel	Reptilien
Anzahl	18	30	25	6	10	12	15	4
Schüler								
%	15%	25%	20%	5%	8.33%	10.0%	12.5%	3.33%
grad	54	90	75	18	30	36	45	12