Proceedings of the 2020 IISE Annual Conference
L. Cromarty, R. Shirwaiker, P. Wang, eds.

A Two-Stage Stochastic Programming Approach for Enhancing Seismic Resilience of Water Pipe Networks

Abstract ID: 789009

Azam Boskabadi, PhD Candidate, Industrial, Manufacturing, and Systems Engineering Department, University of Texas at Arlington, Texas, USA

Dr. Jay Michael Rosenberger, Professor, Industrial, Manufacturing, and Systems Engineering Department, University of Texas at Arlington, Texas, USA

Dr. Mohsen Shahandashti, Assistant Professor, Civil Engineering Department, University of Texas at Arlington, Texas, USA

Abstract

Earthquakes are sudden and inevitable disasters that can cause enormous losses and suffering, and having accessible water is critically important for earthquake victims. To address this challenge, utility managers do preventive procedures on water pipes periodically to withstand future earthquake damage. The existing seismic vulnerability models usually consider simple methods to find the pipes to rehabilitate with highest priority. In this research, we develop an optimization approach to determine which water pipes to rehabilitate subject to a limited budget.

Keywords

Network Optimization, Two-Stage Stochastic Programming, Linear Approximation, Mixed Integer Non Linear Programming (MINLP), Decision Analysis.

1. Introduction

Earthquakes are sudden and inevitable disasters that can cause enormous losses and suffering. Preparing enough utility resources right after the earthquake is one of the most vital actions. Water, as the most important resource for keeping humans alive in these kinds of disasters, plays an important role. Historical data from past earthquakes show the importance of providing drinkable water right after an earthquake. For addressing this challenge, utility managers do a preventive procedure that repairs some of their water pipes periodically, but the important question is which pipes should be repaired since utility managers have limited budget. Existing seismic vulnerability models just consider simple methods to find pipes with highest priority [1].

In this research, we develop an optimization model that finds a best rehabilitation policy before an earthquake that maximizes expected service to the people right after the earthquake. Figure 1 shows the two-stage stochastic process considered in this research. In stage 1, an initial rehabilitation policy/decision is made subject to the limited budget. Then a hypothetical earthquake occurs and generates a random scenario that determines which pipes are broken. In stage 2, right after the earthquake, a recourse function determines the water flow in the unbroken pipes.

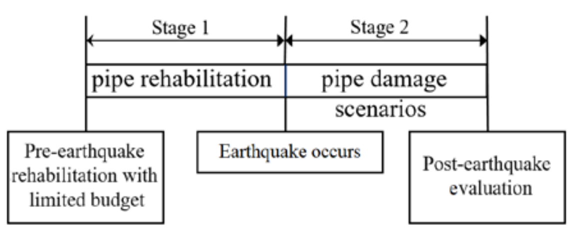

Figure 1: Two-Stage Stochastic Model

The contribution of this paper is as follows. We formulate the aforementioned optimization model as a two-stage stochastic mixed integer nonlinear program (MINLP). The MINLP model cannot be solved by commercial optimization software, like BARON even for problems with a very small number of scenarios. Consequently, we propose piecewise linear functions (PLF) to approximate the nonlinearity in the MINLP. Therefore, we formulate a mixed integer linear program (MILP) to approximate the MINLP. The optimization of the MILP is still challenging to solve, so we introduce a sequential algorithm to mitigate this computational issue and find bounds for the approximated optimal solution.

2. Model Description

2.1. Stochastic Program for Water Pipe Rehabilitation Problem

We introduce a stochastic program for water pipe rehabilitation problem with a recourse flow function to maximize the output flow after the earthquake.

Let the water pipe network be represented as a graph $G = (N, A)$, where N is a set of nodes, and A is a set of arcs/pipes. For each arc $(i,j) \in A$, let rehabilitation decision variable

$$x_{ij} = \begin{cases} 1; & \text{if pipe from node i to j is rehabilitated} \\ 0; & \text{otherwise} \end{cases}$$

In addition, let Ξ be a set of random scenarios in which each determines which pipes break according to a Monte Carlo Simulation. For each scenario $\xi \in \Xi$, let P^{ξ} be the probability that scenario ξ occurs. We can calculate the P^{ξ} as follows:

$$P^{\xi} = \frac{\text{frequency of special scenario happens}}{\text{all scenarios}} \tag{1}$$

Let a *loop* be a sequence of connected pipes that begins and ends with the same pipe. In network literature, a loop is usually referred to as a cycle, but in this research, we have elected to use the term loop to be consistent with the hydraulic literature. Before the earthquake, let K be the set of all loops in the network G. For each loop $k \in K$, let the loop variable O_k^{ξ} be

$$O_k^{\xi} = \begin{cases} 1; & \text{if all pipes in loop k are unbroken in scenario } \xi \\ 0; & \text{if at least one of the pipes in loop k is broken in scenario } \xi \end{cases}$$

We define binary parameter r_{ij}^{ξ} that is 1 when the pipe from node i to j breaks in scenario $\xi \in \Xi$ if unrehabilitated and 0 otherwise.

For each arc $(i,j) \in A$ and scenario $\xi \in \Xi$, let f_{ij}^{ξ} be the flows from node i to j in scenario $\xi \in \Xi$. In addition, we define NF_i^{ξ} to be the net flow (inflow/outflow) of node i in scenario $\xi \in \Xi$. Let N_t be the subset of N consisting of demand nodes and N_s be the subset of N that contains source nodes, Therefore,

$$NF_i^{\xi} = 0 \; if \; i \in N \backslash N_t \cup N_s, NF_i^{\xi} \geq 0 \; if \; i \in N_t, \text{and } NF_i^{\xi} \leq 0 \; if \; i \in N_s.$$

In addition, let l_{ij} be the cost of rehabilitating the pipe from node i to j, and let L be the rehabilitation budget. Moreover, let T_{ij} be a certain coefficient for each pipe that depends on the physical features of the pipe like its material and diameter, let ρ be an experimental constant, usually equal to 1.852, the hydraulic literature often defines the *pressure* in a pipe (i, j) to be $T_{ij} f_{ij}^{\xi\ \rho}$ [2]. In addition, let U_{ij} be the maximum possible flow in each pipe $(i, j) \in A$. Therefore, the *extensive form* of the stochastic programming model for the water pipe rehabilitation problem is formulated as:

$$\text{Max } ESSI_x = \sum_{\xi \in \Xi} (P^{\xi}) \left(\frac{\sum_{i \in N_t} NF_i^{\xi}}{\text{pre earthquake outflow}} \right) \tag{2}$$

$$\sum_{(i\ j) \in A} l_{ij}\, x_{ij} \leq L \tag{3}$$

$$\sum_{j \in N: (i\ j) \in A} f_{ij}^{\xi} - \sum_{j \in N: (j\ i) \in A} f_{ji}^{\xi} = NF_i^{\xi} \qquad \forall i \in N,\ \forall \xi \in \Xi \tag{4}$$

$$O_k^{\xi} = \prod_{(i\ j) \in A_k} (1 - (1 - x_{ij}) r_{ij}^{\xi}) \qquad \forall k \in K,\ \forall \xi \in \Xi \tag{5}$$

$$O_k^{\xi} \left(\sum_{(i\ j) \in A_k} T_{ij} f_{ij}^{\xi\ \rho} \right) = 0 \qquad \forall k \in K,\ \forall \xi \in \Xi \tag{6}$$

$$0 \leq f_{ij}^{\xi} \leq (1 - (1 - x_{ij}) r_{ij}^{\xi})\, U_{ij} \qquad \forall (i\ j) \in A, \forall \xi \in \Xi \tag{7}$$

$$f_{ij}^{\xi} f_{ji}^{\xi} = 0 \qquad \forall (i\ j) \in A,\ \forall \xi \in \Xi \tag{8}$$

$$x_{ij} \in \{0, 1\} \qquad \forall (i\ j) \in A \tag{9}$$

$$O_k^{\xi} \in \{0, 1\} \qquad \forall k \in K, \forall \xi \in \Xi \tag{10}$$

In this MINLP, the objective function (2) maximizes the Expected System Serviceability Index (ESSI). Constraint (3) is a knapsack constraint that restricts the cost of rehabilitation to be less than a predetermined budget. Constraint set (4) ensures that the difference between input and output flow at each node is equal to the supply or demand of that node, also referred to as *flow conservation constraints*. Constraint set (5) ensures that each loop exists if and only if all of its pipes are unbroken after the earthquake. Constraint set (6) ensures that each remaining loop, satisfies *pressure* *conservation constraints* with $\rho = 1.852$. Constraint set (7) defines the relationship among *x*, *r*, and *f*. It guarantees that if a pipe is rehabilitated before the earthquake (*x* = 1), the earthquake does not break it. On the other hand, if a pipe is not rehabilitated before the earthquake (*x* = 0), it's broken after the earthquake if $r^{\xi} = 1$. Consequently, the flow f on an arc in scenario ξ can be nonzero if x=1 or $r^{\xi} = 0$. Constraint set (8) makes sure that each flow is just in one direction in each pipe. Constraint sets (9) and (10) are integer restrictions.

We compute the maximum possible flow in each pipe $(i, j) \in A$ by solving a network flow problem as follows:

$$U_{ij} = \text{Max}\, f_{ij} \quad (11)$$

$$\text{s.t.: } \sum_{j \epsilon N:\,(i\,j) \epsilon A} f_{ij} - \sum_{j \epsilon N:(j\,i) \epsilon A} f_{ji} = NF_i \qquad \forall i \epsilon N,\ \forall \xi \in \Xi \quad (4)$$

$$f_{ij} \geq 0 \qquad \forall (i\,j) \in A \quad (7a)$$

In this model, the objective function (11) maximizes the flow in a given pipe, while the model considers the flow conservation constraint set that we described before.

2.2. Linear Approximation Model

Since the previously given stochastic MINLP is computationally intractable, even for a small number of scenarios, we formulate an approximation as an MILP. We have three nonlinear constraint sets, (5), (6) and (8). For linearization of constraint set (5), we introduce two new constraint sets that can substitute for constraint set (5):

$$O_k^{\xi} \leq 1 - \left(1 - x_{ij}\right) r_{ij}^{\xi} \qquad \forall k \in K, \forall (i\,j) \epsilon A_k,\ \forall \xi \in \Xi \quad (5a)$$

$$O_k^{\xi} \geq \sum_{\forall (i\,j) \epsilon A_k} 1 - \left(1 - x_{ij}\right) r_{ij}^{\xi} - (|A_k| - 1) \qquad \forall k \in K,\ \forall \xi \in \Xi \quad (5b)$$

Constraint set (6) can be linearized as follows [25], where *M* is a big positive constant:

$$-MO_k^{\xi} \leq \sum_{(i\,j) \epsilon A_k : r_{ij}=0} T_{ij} f_{ij}^{\xi\ \rho} - \sum_{(i\,j) \epsilon A_k : r_{ij}=0} T_{ij} f_{ji}^{\xi\ \rho} \leq MO_k^{\xi} \qquad \forall k \in K,\ \forall \xi \in \Xi \quad (6a)$$

Constraint set (6a) can be written as two constraint sets:

$$-MO_k^{\xi} \leq \sum_{(i\,j) \epsilon A_k : r_{ij}=0} T_{ij} f_{ij}^{\xi\ \rho} - \sum_{(i\,j) \epsilon A_k : r_{ij}=0} T_{ij} f_{ji}^{\xi\ \rho} \qquad \forall k \in K,\ \forall \xi \in \Xi \quad (6b)$$

$$\sum_{(i\,j) \epsilon A_k : r_{ij}=0} T_{ij} f_{ij}^{\xi\ \rho} - \sum_{(i\,j) \epsilon A_k : r_{ij}=0} T_{ij} f_{ji}^{\xi\ \rho} \leq MO_k^{\xi} \qquad \forall k \in K,\ \forall \xi \in \Xi \quad (6c)$$

However, we still have the term $f_{ji}^{\xi\ \rho}$ that should be linearized. By using Piecewise Linear Functions (PLF) [26], constraint sets (6b) and (6c) can be approximated. The term $f_{ij}^{\xi\ \rho}$ can be estimated by a PLF with *S* linear pieces. For each linear piece *s* = 1, …, *S*, let m_s be the slope, c_s be the intercept, (a'_s, b'_s) be the domain, and $w_{ij\,s}$ be an integer variable that indicates the flow of pipe (*i*,*j*) is in the domain of *s*. Hence, $f_{ij}^{\xi\ \rho}$ approximately satisfies the following conditions:

$$f_{ij}^{\xi\ \rho} \approx \left(m_1 f_{ij\,1}^{\xi} + c_1 w_{ij\,1}\right) + \left(m_2 f_{ij\,2}^{\xi} + c_2 w_{ij\,2}\right) + \cdots + \left(m_s f_{ij\,s}^{\xi} + c_s w_{ij\,s}\right) \quad (15)$$

$$\text{s.t.:} \quad 0 \leq f_{ij\,1}^{\xi} \leq a'_1 w_{ij\,1} \qquad \forall (i\,j) \epsilon A, \forall \xi \in \Xi \quad (16)$$

$$a'_{s-1} w_{ij\,s} \leq f_{ij\,s}^{\xi} \leq a'_s w_{ij\,s} \qquad \forall (i\,j) \epsilon A, \forall \xi \in \Xi;\ \forall s \epsilon \{2, 3, \ldots, S\} \quad (17)$$

$$w_{ij\,1} + w_{ij\,2} + \cdots + w_{ij\,s} = 1 \qquad \forall (i\,j) \epsilon A \quad (18)$$

$$w_{ij\,1}, w_{ij\,2}, \ldots, w_{ij\,s} \in \{0,\ 1\} \qquad \forall (i\,j) \epsilon A \quad (19)$$

Therefore, constraint sets (6b) and (6c) can be approximated as follows:

$$-MO_k^{\xi} \leq \sum_{(i\,j) \epsilon A_k} T_{ij} \left(\sum_{s=1}^{S} m_s f_{ij\,s}^{\xi} + c_s w_{ij\,s}\right) - \sum_{(i\,j) \epsilon A_k} T_{ij} \left(\sum_{s=1}^{S} m_s f_{ij\,s}^{\xi} + c_s w_{ij\,s}\right) \qquad \forall k \in K,\ \forall \xi \in \Xi \quad (6d)$$

$$\sum_{(i\,j) \epsilon A_k} T_{ij} \left(\sum_{s=1}^{S} m_s f_{ij\,s}^{\xi} + c_s w_{ij\,s}\right) - \sum_{(i\,j) \epsilon A_k} T_{ij} \left(\sum_{s=1}^{S} m_s f_{ji\,s}^{\xi} + c_s w_{ij\,s}\right) \leq MO_k^{\xi} \qquad \forall k \in K,\ \forall \xi \in \Xi \quad (6e)$$

One challenge is that the numbers of loops and constraints in (5) and (6), are exponentially large with respect to the number of pipes in the network.

The linearization method for constraint set (8) can be formulated as below [27]. We use a set of binary variables g_{ij}^{ξ}.

$$f_{ij}^{\xi} \leq g_{ij}^{\xi} U_{ij} \quad (8a)$$

$$f_{ji}^{\xi} \leq (1 - g_{ij}^{\xi}) U_{ij} \qquad \forall (i\,j) \epsilon A,\ \forall \xi \in \Xi \quad (8b)$$

$$g_{ij}^{\xi} \in \{0, 1\} \qquad \forall (i\,j) \epsilon A,\ \forall \xi \in \Xi \quad (8c)$$

2.3. Revised Two-Stage Stochastic Programming Formulation

For simplicity we relax constraint (8). Consequently, the final equivalent MILP consists of objective function (2), and constraints (3), (4), (5a), (5b), (6d), (6e), (7), (9), (10), (16), (17), (18), and (19). Moreover, the expected MILP second stage recourse function consists of objective function (2), and constraints (4), (5a), (5b), (6d), (6e), (7), (9), (10), (16), (17), (18), and (19).

2.4. Evaluation Procedure

The evaluation of the MILP and the MINLP has been done in a case when there is no break in the network. We use fmincon function in MATLAB to find an optimal solution for MINLP. Hence, we evaluate how well our MILP approximates the MINLP.

In addition, we introduce a sequential revised two-stage stochastic algorithm to find an optimality gap for the MILP optimal solution [28]. We used MATLAB and Gurobi.

3. Computational Study

We consider two networks, Networks 1 and 2. Network 1 consists of 117 pipes, 92 nodes, and 22 loops before the earthquake (Figure 2). The water pipe network length is 65749 meters. Figure 3 shows Networks 2, the Modena network. Networks 2 consists of 317 pipes, 272 nodes, and 46 loops before the earthquake. The water pipe network length is 71806.11 meters. For Network 2, we generated3000 random scenarios using Monte Carlo Simulation from a hypothetical earthquake [1]. 308 pipes out of the 317 pipes break at least once in the 3000 scenarios. However, many of the scenarios within the 3000 are repeated, and we note that there are in fact only 1505 unique scenarios.

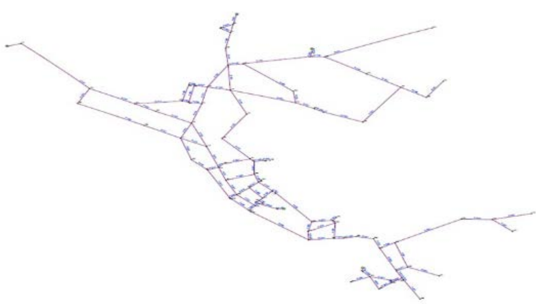

Figure 2: Network 1

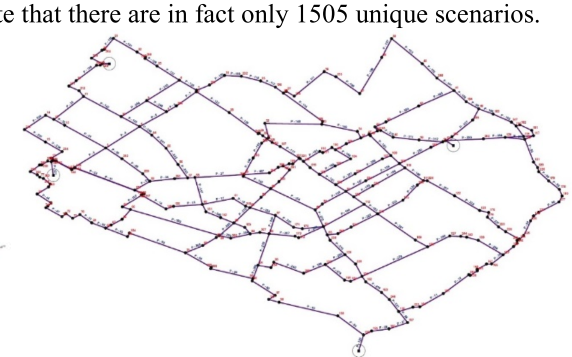

Figure 3: Network 2

3.1. Accuracy of MILP Recourse Function

First, we evaluate the accuracy of the MILP recourse function. For Network 1, we consider a single scenario case in which the earthquake does not break any pipes. We solve the MILP using Gurobi to determine the flows in Network 1. In this case model does not recommend any rehabilitation policy. Then, we use fmincon function in MATLAB to find an optimal solution for the nonlinear recourse function. The results show the linear approximated flow from the MILP recourse function is 210.5 liters/second while the nonlinear recourse function optimal solution from fmincon is 209.3 liters/second. Consequently, in this case the piecewise linear approximation is 99.4% accurate.

3.2. Sequential Revised Two-Stage Stochastic Programming

Since Network 2 is huge, we divide the 1505 unique scenarios into groups with five scenarios each, and we sort them by their descending probabilities. We assume rehabilitation cost is proportional to the pipe length, and the budget limit is 1500 meters. We consider the following sequential algorithm. First, we solve the MILP over the most probable five scenarios. Then, we reduce budget limit by the length of the pipes in the solution. We repeat this process until we have either no remaining scenario groups or budget limit. Table 1 shows the results. The final policy rehabilitates 4 pipes with a total length of 1498.97 meters and a serviceability of 373.25 liters/second.

Table 1: Sequential MILP

Iteration	Remaining Budget Limit (meters)	# of Rehabilitated Pipes
1 to 27	88.16	2
28 to 60	7.06	3
61	1.03	4

We use the method in [28] and determine the following optimality bounds for the solution:

$$373.25\left(\frac{\text{liters}}{\text{second}}\right) \leq z^* \leq 386.64\left(\frac{\text{liters}}{\text{second}}\right) \tag{20}$$

Consequently, the solution we find using the sequential algorithm is within 2% of optimality.

4. Conclusions

This study proposed the two-stage stochastic programming model for the water pipe rehabilitation problem with a recourse flow function to maximize the output flow right after an earthquake. We introduced the approximate mixed integer linear program (MILP) by manipulating the nonlinear constraints and using a piecewise linear approximation method. Therefore, the formulated mixed integer nonlinear program (MINLP) can be approximated by the MILP. Then we determined the rehabilitation policy over several randomly generated scenarios using the sequential algorithm. The evaluation of the concluded policy was done by using the method in [28], and the rehabilitation policy is within 2% of optimality.

In future research, the model can be used for rehabilitation plans of corroded pipes. In addition, we can consider leakage in the model, another objective function, using Benders Decomposition for solving the model, and developing software for helping municipalities to rehabilitate their water pipes, especially in cities where an earthquake may occur with higher probability.

Acknowledgment of Support

This material is based upon work supported by the National Science Foundation under Grant No. 1926792.

References

[1] Shahandashti, S. M., & Pudasaini, B. (2019). "Proactive Seismic Rehabilitation Decision-Making for Water Pipe Networks Using Simulated Annealing", Natural Hazards Review, 20(2), 04019003.

[2] Cross, H. (1936). "Analysis of flow in networks of conduits or conductors", University of Illinois at Urbana Champaign, College of Engineering. Engineering Experiment Station.

[3] Tayfur, G. (2017). Modern optimization methods in water resources planning, engineering and management. Water Resources Management, 31(10), 3205-3233.

[4] Djebedjian, B., Herrick, A., & Rayan, M. A. (2016, October). Modeling and optimization of potable water network. In 2000 3rd International Pipeline Conference. American Society of Mechanical Engineers Digital Collection.

[5] Caballero, J. A., & Ravagnani, M. A. (2019). Water distribution networks optimization considering unknown flow directions and pipe diameters. Computers & Chemical Engineering, 127, 41-48.

[6] Audu, H., & Ovuworie, G. (2010, July). MANAGEMENT OF WATER DISTRIBUTION INFRASTRUCTURE WITH GIS IN THE NIGER DELTA REGION OF NIGERIA. In WEST AFRICA BUILT ENVIRONMENT RESEARCH (WABER) CONFERENCE (p. 413).

[7] Abdel-Gawad, H. A. (2001, March). Optimal design of pipe networks by an improved genetic algorithm. In Proceedings of the Sixth International Water Technology Conference IWTC (pp. 23-25).

[8] Djebedjian, B., Yaseen, A., & Rayan, M. A. (2008, October). A new adaptive penalty method for constrained genetic algorithm and its application to water distribution systems. In *2006 International Pipeline Conference* (pp. 739-750). American Society of Mechanical Engineers Digital Collection.

[9] Yaseen, A. (2007). *Reliability-based optimization of potable water networks using genetic algorithms and Monte Carlo simulation* (Doctoral dissertation, M. Sc. Thesis, Mansoura University, Egypt).

[10] AbdelBary, A. G. M. (2008). *Optimization of Water Distribution Systems Subjected to Water Hammer Using Genetic Algorithms* (Doctoral dissertation, M. Sc. Thesis, Faculty of Engineering, Mansoura University).

[11] Bernhardt, K. L. S., & McNeil, S. (2004, March). An agent based approach to modeling the behavior of civil infrastructure systems. In *Engineering Systems Symposium, Tang Center, MIT*.

[12] Al-Khafaji, M. S., Mesheb, K. S., & Jabbar Abrahim, M. A. (2019). Fuzzy Multicriteria Decision-Making Model for Maintenance Management of Irrigation Projects. Journal of Irrigation and Drainage Engineering, 145(12), 04019026.

[13] O'Rourke, T. D., Jeon, S. S., Toprak, S., Cubrinovski, M., Hughes, M., van Ballegooy, S., & Bouziou, D. (2014). Earthquake response of underground pipeline networks in Christchurch, NZ. Earthquake Spectra, 30(1), 183-204

[14] Zouache, D., Moussaoui, A., & Abdelaziz, F. (2018). A cooperative swarm intelligence algorithm for multi-objective discrete optimization with application to the knapsack problem. *European Journal of Operational Research, 264*(1), 74-88.

[15] Manicassamy, J., Karunanidhi, D., Pothula, S., Thirumal, V., Ponnurangam, D., & Ramalingam, S. (2018). GPS: a constraint-based gene position procurement in chromosome for solving large-scale multiobjective multiple knapsack problems. Frontiers of Computer Science, 12(1), 101-121.

[16] Arin, A., & Rabadi, G. (2017). Integrating estimation of distribution algorithms versus Q-learning into Meta-RaPS for solving the 0-1 multidimensional knapsack problem. Computers & Industrial Engineering, 112(2017), 706-720.
[17] Chih, M. (2017). Three pseudo-utility ratio-inspired particle swarm optimization with local search for multidimensional knapsack problem. Swarm and Evolutionary Computation, In press(000), 1-18.
[18] Chen, Y., & Hao, J.-K. (2017). An iterated "hyperplane exploration" approach for the quadratic knapsack problem. Computers & Operations Research, 77(2017), 226-239.
[19] Song, B., Li, Y., Chen, Y., Yao, F., & Chen, Y. (2018, January). A Repair-based approach for stochastic quadratic multiple knapsack problem,. *Knowledge-Based Systems, 000*(2018), 1-11.
[20] Furini, F., Monaci, M., & Traversi, E. (2018). Exact approaches for the knapsack problem with setups. Computers & Operations Research, 90(2018), 208-220.
[21] Gendron, B., Hanafi, S., & Todosijević, R. (2018). Matheuristics based on iterative linear programming and slope scaling for multicommodity capacitated fixed charge network design. European Journal of Operational Research, 268(1), 70-81.
[22] Calvete, H., Galé, C., Iranzo, J., & Toth, P. (2018). A matheuristic for the two-stage fixed-charge transportation problem. Computers & Operations Research, 95(2018), 113-122.
[23] Nicholson, C., & Zhang, W. (2016). Optimal network flow: A predictive analytics perspective on the fixed-charge network flow problem. *Computers & Industrial Engineering*, 99(2016), 260-268.
[24] Zhang, W., & Nicholson, C. (2016). Prediction-based relaxation solution approach for the fixed charge network flow problem. *Computers & Industrial Engineering, 99*(2016), 106-111.
[25] McCormick, G. P. (1983). Nonlinear programming; theory, algorithms, and applications (Tech. Rep.).
[26] Ahuja, R. K., Magnanti, T. L., & Orlin, J. B. (1988). Network flows.
[27] https://www.leandro-coelho.com/linearization-product-variables/
[28] Mak, W.-K., Morton, D. P., & Wood, R. K. (1999). Monte carlo bounding techniques for determining solution quality in stochastic programs. *Operations research letters*, 24 (1-2), 47-56.