A Stochastic Simulation Scheme for the Estimation of Small Failure Probabilities in Wind Engineering Applications

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To ensure the feasibility of performance-based wind engineering (PBWE) frameworks, particularly when it involves computationally expensive nonlinear dynamic analyses and estimation of small failure probabilities, there is a need for efficient stochastic simulation schemes. To this end, an optimal stratified sampling-based Monte Carlo simulation (OSMCS) scheme is proposed to simultaneously estimate failure probabilities associated with multiple limit states including those which are implicitly defined. The scheme is based on the optimal allocation of Monte Carlo simulation (MCS) samples among the strata which are partitions of the uncertain parameter space, defined using one or more input random variables. The partitions enable simulation of rare events and the optimality guarantees minimum estimator variance for a target failure probability. The optimality criterion is derived and some theoretical aspects of the OSMCS estimator are discussed. To demonstrate the applicability and efficiency of the scheme a case study is presented and the implementation issues are also critically discussed.

Keywords: Structural safety, Monte Carlo methods, Wind Engineering, Stratified sampling, Variance reduction, Nonlinear modeling.

1. Introduction

In recent years, there has been rising interest in the adoption of nonlinear structural design methodologies for wind actions, which necessitates the development of efficient reliability assessment frameworks for various performance evaluations. One of the foremost challenges in such reliability assessment problems concerns the simultaneous estimation of small failure probabilities associated with rare events and multiple limit states. The problem is exacerbated when the limit state functions (LSFs) are not only nonlinear but cannot be explicitly expressed in terms of the response parameters. For example, the exceedance of the limit state of system collapse may need to be evaluated through a combination of indicators such as non-convergence of time history analysis, the deformed shape of the structure at the last converged time step, and the peak roof drift. The implicitness arises from the need to sufficiently validate the onset of a failure mechanism that cannot be simply expressed as a response measure exceeding a certain threshold. In the face of high-dimensional uncertainties and complex LSFs, traditional reliability methods, such as the second-order reliability method, are infeasible. With the use of simulation methods, it is important to note that each sample generated often involves the computationally expensive evaluation of a numerical model. Standard Monte Carlo Simulation (MCS) methods, with their simple random sampling strategy, require a large number of samples, roughly inversely proportional to a small target probability (e.g., $\leq 10^{-4}$), for achieving a specified accuracy. However, its robustness to the type of LSFs and dimensionality of the uncertain parameter space is desirable for the applications of this work.

Importance sampling techniques (Melchers (1989)) can explore a failure region efficiently by sampling from a distribution biased towards the failure region. However, it is infeasible to construct an effective importance sampling density function in the presence of a large number of uncertainties and complex failure regions. Subset simulation (Au and Beck (2001)) is extremely efficient in computing small probabilities associated with a LSF that can be expressed as a combination of response variables. The process of estimating a sequence of conditional probabilities is, however, unique to the LSF under consideration and hence, the samples cannot be used to evaluate the exceedance probabilities of other limit states. Within the context of performance-based wind engineering (PBWE) (Bernardini et al. (2015),Chuang and Spence (2017),Ouyang and Spence (2021)),

Proceedings of the 31th European Safety and Reliability Conference.
Edited by Bruno Castanier, Marko Cepin, David Bigaud and Christophe Berenguer

ISBN: 981-973-0000-00-0 :: doi: 10.3850/981-973-0000-00-0_output

a stratified sampling-based Monte Carlo simulation scheme was proposed by Ouyang and Spence (2020) wherein the sample space of wind speeds was partitioned to estimate various failure probabilities. However, the MCS sample allocation scheme was not discussed and the estimator variance was not computed.

In this work, an optimal stratified sampling-based Monte Carlo simulation (OSMCS) scheme is introduced to enable simultaneous estimation of failure probabilities using limited sample sets while minimizing the variance of a target failure probability estimate. Some theoretical properties of the OSMCS estimator as well as considerations on implementation are discussed. The efficiency of the scheme and its potential integration with nonlinear structural modeling environments are illustrated on a case study.

2. Stratified Sampling-based Monte Carlo Simulation

2.1. *Background*

Stratified sampling involves partitioning the probability space into mutually exclusive and collectively exhaustive subspaces called strata, to enable an explicit way of ensuring samples are collected from each of the user-defined strata in a preferred manner. With simple random sampling performed within each stratum, the estimator of the overall failure probability, $\hat{P}_f$, is given according to the total probability theorem as:

$$\hat{P}_f = \sum_{i=1}^{N_s} \hat{P}_{f|S_i} P(S_i) \tag{1}$$

where N_s = total number of strata; S_i $(i = 1, ..., N_s)$ are the strata partitioned based on a single or multiple input random variables; and $\hat{P}_{f|S_i}$ is the MCS estimator of the conditional failure probability, $P_{f|S_i}$. Each of the partition probabilities, $P(S_i)$, can be directly evaluated using the joint conditional distribution function (CDF) of the input random variables. In general, an analyst is free to select the number of MCS samples to be used in each of the strata and the division of the probability space into the strata S_i, ensuring only that the strata are mutually exclusive and collectively exhaustive partitions, so that Eq. (1) holds true.

2.2. *Optimal Sample Allocation*

Important insights about the distribution of MCS samples among the N_s strata are obtained when the variance of $\hat{P}_f$ is examined. From Eq. (1) and the expression for the variance of an MCS estimator, the variance of $\hat{P}_f$ can be computed. Subsequently, an optimization problem can be formulated to minimize the estimator variance and solved to obtain the optimal allocation of N MCS samples among the N_s strata. Accordingly, the i^{th} stratum is allocated $\tilde{n}_i$ MCS samples which is given by:

$$\tilde{n}_i = \frac{N\sqrt{P_{f|S_i}(1-P_{f|S_i})}P(S_i)}{\sum_{j=1}^{N_s}\sqrt{P_{f|S_j}(1-P_{f|S_j})}P(S_j)} \tag{2}$$

The tilde is used to indicate that the solution is optimal. The minimum $\mathrm{Var}(\hat{P}_f)$ that can be attained if the conditional failure probabilities, partitions S_i, and N are known in advance, is given by:

$$\mathrm{Var}(\hat{P}_f) = \frac{\left(\sum_{i=1}^{N_s}\sqrt{P_{f|S_i}(1-P_{f|S_i})}P(S_i)\right)^2}{N} \tag{3}$$

The stratified sampling-based simulation scheme based on the above-mentioned optimal sample allocation is referred to as the OSMCS scheme in this work. Other sample allocation strategies, such as equal allocation ($n_i = N/N_s$) and proportional allocation ($n_i = NP(S_i)$) are suboptimal and incur larger estimator variance. This optimal allocation problem is known in the theory of sample allocation as Neyman allocation (Dalenius (1950)).

2.3. *Statistical Properties*

It is important to characterize the estimator with its statistical properties. Firstly, it can be shown that OSMCS provides an unbiased estimate of the target failure probability. That is, $\mathbf{E}\left[\hat{P}_f\right] = P_f$, which follows from the unbiased estimation property of MCS estimators and the linearity of the expectation operator, $\mathbf{E}[\cdot]$. Secondly, similar to MCS, the OSMCS estimator is itself a random variable that converges to a normal distribution for large N. That is, $\hat{P}_f$ is governed by a normal distribution whose mean is P_f and variance is given by Eq. (3). This is because the MCS estimators are normal random variables and independent of one another, and $\hat{P}_f$ is a linear combination of these independent normal random variables according to Eq. (1).

3. Implementation Issues

The primary challenge in implementing the optimal allocation scheme is that the optimal sample sizes depend on the conditional failure probabilities which are unknown and to be estimated. However, approximate failure probabilities can be computed based on some test samples (e.g., $N/5$) to determine the optimal sample allocation for the remaining MCS samples. The preliminary estimation may be carried out using equal allocation of the test samples. It is useful to keep in mind that the optimal allocation scheme essentially suggests allocating a larger portion of the total samples to partitions with a larger probability of occurrence and those where the conditional failure probability is neither too small nor too large. Mathematically, this takes form $n_i \propto \sqrt{P_{f|S_i}(1-P_{f|S_i})}P(S_i)$, as seen in Eq. (2). The preliminary estimation followed by optimal sample allocation can be judiciously executed without affecting the overall efficiency significantly for real applications as discussed and demonstrated in the case study section. The choice of the stratification variable and the partitions should be motivated by the physics of the problem and qualitative considerations of the influence of the input random variables on the LSFs of interest.

In applications to wind engineering, the basic idea of stratification in stochastic simulation enables a direct means to investigate behavior that only occurs in a local range of wind speeds, for example, vortex shedding. Furthermore, the OSMCS scheme permits the evaluation of conditional probabilities involving multiple limit states, such as the estimation of the fragility curve for residual roof drift conditional on non-collapse in a specified time period.

4. Case Study

4.1. *Overview*

The OSMCS scheme is demonstrated using a probabilistic performance evaluation of a two-story two-bay steel frame assumed to be located in an urban region of Miami, USA, wherein failure probabilities associated with multiple limit states of interest are simultaneously estimated. The limit states considered in this example were system collapse, system first yield, and component fracture, which are referred to as LS1, LS2, and LS3, respectively. This problem is representative of performance-based reliability assessments in wind engineering. Of particular interest to this paper is the efficiency of the OSMCS scheme.

4.2. *Problem Setup*

4.2.1. *Building Model*

The building shown in Fig. 1 is a steel moment-resisting frame that was modeled and analyzed in the OpenSees simulation platform using fiber-based inelastic elements. The preliminary design for this building is based on satisfying the peak roof displacement limit of $H/300$, with $H = 10$ m, under a wind load corresponding to a 25-year return period and ensuring elastic response under a 700-year return period wind load. The section sizes following these design criteria were pro-

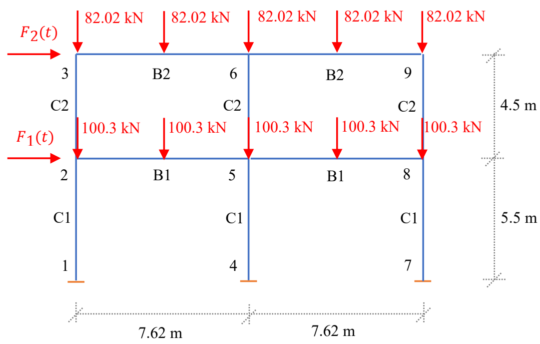


Fig. 1.: Two-story two-bay wind excited structure

vided in Chuang and Spence (2020) and used in this study. There are four unique structural section sizes and the gravity loads were applied as shown in Fig. 1. The floor diaphragms were assumed to be rigid. Each member was modeled using two force-based fiber elements, five integration points were considered per element and 14 fibers were used to discretize each section. Fiber damage due to low-cycle fatigue (LCF) and potential fiber fracture was modeled using the linear damage accumulation rule along with the modified rain flow cycle counting algorithm (Uriz (2005)). The Menegotto-Pinto material model was adopted to simulate the cyclic behavior of steel (Karamanci and Lignos (2014)). Large displacement effects were included through the use of corotational formulation. To represent the inherent structural damping, a Rayleigh damping model, as recommended for use in nonlinear analysis (Charney (2008)), was adopted.

The first two natural frequencies of the structure were approximately $f_1 = 0.51$ Hz and $f_2 = 1.22$ Hz, ensuring a certain degree of dynamic excitation by the wind forces. The structural uncertainties listed in Table 1 will, however, affect the modal frequencies, causing marginal deviations from the above-mentioned values.

4.2.2. *Wind Loads*

The stochastic wind load modeling used in this study follows the approach adopted in Chuang and Spence (2020). The wind speed is modeled using a local wind climate model to describe the mean wind speed profile and a spectral representation model to describe the fluctuating component. A power law wind speed profile is assumed and the target cross power spectral density matrix is based on Kaimal spectrum (Kaimal et al. (1972)) and coherence functions. The wind forces are assumed to act laterally in the plane of the frame. The spatio-temporally varying wind speeds are converted to wind load histories acting at the two floors using a quasi-steady model. The wind loads were generated for a duration of 10 minutes with a sampling time of 0.01 s. The first and last two minutes of the loads were ramped up and down, respectively, to account for the initial conditions when subjected to wind loads and mark the end of the wind event. The structural responses are recorded for the duration of the applied wind loads as well as for two minutes of free vibration at the end of 10 minutes.

4.2.3. *Stochastic Simulation Environment*

A wide range of uncertainties in the structural system and wind loads were considered in this study. The following structural random variables were considered: seven material model parameters, uniquely defined for each structural section in the frame; damping ratio, ζ, of the first two modes, which are considered to be equal; and the initial camber, uniquely defined for each column at mid-length. The material uncertainties include the Young's modulus E, strain hardening ratio b, yield strength F_y, fatigue material parameter ϵ_0, elastic-to-plastic transition parameter R_0, and two hardening parameters a_1 and a_3 (Karamanci and Lignos (2014)). The random initial camber is described by a random scale factor, δ_1, and a random sign, (± 1) based on the first buckling mode. The load uncertainties consist of the mean hourly wind speed at the building top $\bar{v}_H$, considered to follow a Type-1 extreme value distribution, and the independent and uniformly distributed phase angles used in the spectral representation model (Chen and Kareem (2005)). The parameters of the Type-1 distribution are estimated through calibration to the site-specific wind speeds provided in ASCE 7-16 (2016). The aforementioned uncertainties, except for the wind speed and those used in the spectral representation model, are presented in Table 1 along with their governing distributions.

Given the nature of the limit states of interest, the wind speed was identified as the dominant variable affecting them and hence selected as the stratification variable. Clearly, the exceedance of LS1 and LS3 are correlated and expected to occur only for extreme wind speeds. However, LS2 can be expected to be exceeded even under relatively lower wind speeds, but typically larger than the 700-year design wind speed. The partitioning approach was based on enforcing equal squared wind speed difference by recognizing that the load effect is approximately proportional to the square of the wind speed. A total of eight partitions and 1000 MCS samples were considered by taking note of the intended resolution of the wind speed intervals (WSI) and computational feasibility. To ensure the collectively exhaustive nature of the WSIs, the lower bound defining the first WSI is taken as zero while the last WSI is taken as unbounded from above. The lower bound of the

Table 1.: Summary of the basic random variables (CV = coefficient of variation).

Parameter	Mean	CV	Distribution
E	200 GPa	0.04	Lognormal
F_y	380 MPa	0.06	Lognormal
b	0.001	0.01	Lognormal
ϵ_0	0.077	0.161	Lognormal
R_0	20	0.166	Normal†
a_1	0.01	2	Lognormal
a_3	0.02	0.5	Lognormal
δ_1/L^*	0.0556%	0.77	Normal
ζ	0.015	0.4	Lognormal

†Truncated normal with lower and upper bound of 15 and 25, respectively.
$^*L$ = Column length.

last wind speed interval (WSI) was chosen to correspond to an annual exceedance probability (AEP) of 7×10^{-7}, a stipulated value in ASCE 7-16 (2016) corresponding to collapse for a risk Category II structure. The partition scheme is illustrated in Fig. 2. A preliminary estimation of the failure probabilities for the limit state of collapse (conditional and overall) was carried out using 20 MCS samples in each WSI, to implement the OSMCS scheme according to Eq. (2). The sample allocation was further adjusted manually to achieve a coefficient of variation (CV) of $10-15\%$ for the failure probabilities associated with both LS1 and LS2, by evaluating Eq. (3) using the test samples. The final sample allocation is shown in Table 2 along with the lower and upper bound wind speeds defining the partitions.

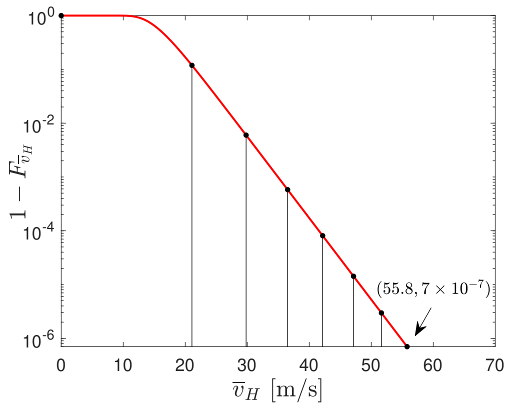


Fig. 2.: Partitioned non-directional hazard curve

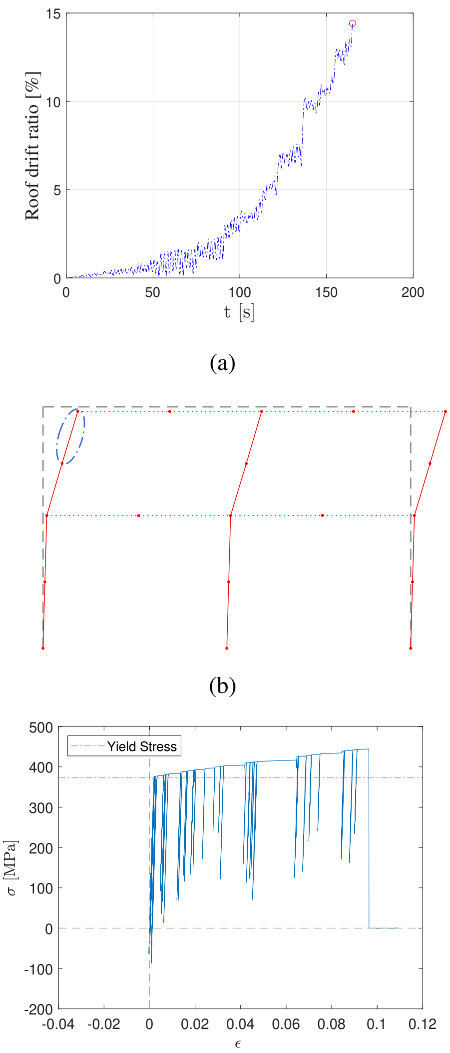
Table 2.: Sample allocation.

WSI	$\bar{v}_H^{\text{lower}}$ [m/s]	$\bar{v}_H^{\text{upper}}$ [m/s]	n_i
1	0.00	21.08	20
2	21.08	29.82	20
3	29.82	36.52	310
4	36.52	42.17	80
5	42.17	47.15	60
6	47.15	51.65	60
7	51.65	55.78	100
8	55.78	∞	350

4.3. *Results*

The results include the nonlinear responses required to describe the behavior of each random frame, as well as statistical information on these responses required to estimate the failure probabilities. Fig. 3 illustrates a collapse scenario with the use of roof drift ratio history, the deformed shape at collapse due to along-wind loading, and the stress-strain history of a fiber that witnessed significant accumulated damage due to LCF, leading to fracture. Similarly, Fig. 4 illustrates a non-collapse scenario highlighting the hysteretic stress-strain response and residual roof drift ratio when subjected to the reported wind loading. A comparison with the linear elastic response is also indicated.

The results of the probabilistic assessment are summarized in Table 3 for the three limit states of interest. In particular, the failure probabilities are expressed as AEP, and the 50-year reliability indices, β_{50} are also provided. The CV of the OSMCS estimators are provided and, as intended, they are approximately 15% for LS1 and LS2. The high CV for LS3 can be attributed to the deviation of the employed sample allocation from the optimal allocation for LS3 as well as LS3 having the smallest failure probability among the considered limit states. To emphasize the efficiency of OSMCS, N^{MCS} is presented, which equals the total number of samples if a standard MCS was used in lieu of OSMCS to achieve the same CV as the latter. Further, it is demonstrated that the stratified sampling-based Monte Carlo simulation scheme using equal sample allocation is indeed suboptimal, especially, when multiple small failure probabilities are to be estimated simultaneously. This is expressed using the quantity, $N^{SS\dagger}$, which equals the total number of samples if the suboptimal scheme was employed to achieve the same CV as OSMCS. The salient inferences concerning the OSMCS scheme are that: (1) the proposed scheme is several orders of magnitude



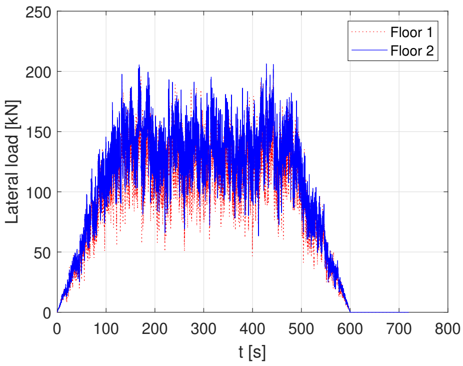
(a)

(b)

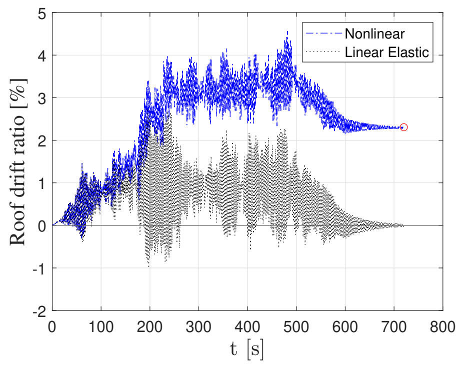
(c)

Fig. 3.: Sample illustrating collapse: (a) Roof drift ratio history; (b) Deformed shape at collapse; (c) Stress-strain history of a fractured fiber of the indicated portion of column.

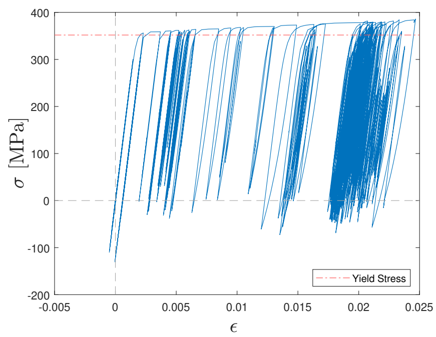
more efficient than a standard MCS scheme, especially so when the target probability is small; (2) the sample allocation can be tuned to work efficiently for the simultaneous estimation of multiple failure probabilities; (3) the scheme can be used for estimating probabilities associated with implicit LSFs, such as system collapse; (4) the sample allocation and efficiency of the estimator is dependent on the test sample set for a preliminary estimation of the failure probabilities. The preliminary assessment is partly subjective and needs to encapsulate the dependence structure of the LSF



(a)



(b)



(c)

Fig. 4.: Sample illustrating non-collapse: (a) Wind load history; (b) Comparison of roof drift ratio history with linear case; (c) Stress-strain history of a critical fiber.

and the input random variables.

5. Conclusion

An efficient stochastic simulation scheme, driven by stratified random sampling, was proposed in this work for wind engineering applications. Some theoretical properties and considerations on implementation were discussed. A practical illustration of the efficiency of the scheme was presented by integrating it with a fiber-based nonlinear modeling environment to solve a reliability assessment problem in the context of PBWE.

Table 3.: Failure probabilities and OSMCS efficiency.

LSF	LS1	LS2	LS3
AEP	1.61×10^{-7}	5.95×10^{-4}	3.06×10^{-8}
β_{50}	4.31	1.89	4.67
CV	16.8%	11.3%	74.7%
N^{MCS}	220,223,442	130,392	58,490,580
$N^{SS\dagger}$	1408	2713	1074

Acknowledgement

The research effort was supported in part by the National Science Foundation (NSF) under Grant No. CMMI-1750339.

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