Do They Need to See It to Learn It? Spatial Abilities, Representational Competence, and Conceptual Knowledge in Statics

Introduction

Students enter an engineering statics course with relevant knowledge and skills they have gained from prior academic work and life experiences that we could roughly place into three categories: math preparation, physics preparation, and a third category lumping together a collection of more general skills such as spatial visualization and problem solving. Each student brings a different mix of strengths and weaknesses in these three categories to their time focused on learning statics. Like most engineering fundamentals courses, statics does not introduce anything fundamentally new to this mix. Rather, the goal is to help students further develop their foundation and synthesize knowledge and skills toward more sophisticated applications. A particular student's experience of the learning activities we design to support this effort is likely highly dependent on the foundation they have brought to the course. More general academic skills and attributes such as motivation, self-regulation, self-efficacy, sense of belonging and mindset also influence how students engage with the course.

Existing research demonstrates the importance of math and physics preparation to student success in mechanics [1], [2], [3], [4], [5]. Problem solving skill is also clearly an important component to success [5]. The correlation of spatial abilities to broader measures of success and retention for engineering majors in general is well-established [6]. However, existing studies exploring the importance of spatial abilities to success in mechanics courses find mixed results. Many fundamental concepts such as free-body diagrams, moments, and vectors are inherently spatial, especially in three-dimensional applications [7]. Further evidence of the spatial nature lies in the finding that students' spatial abilities improve in statics at a higher rate than in other courses [8]. Research is less conclusive on the importance of spatial skills to student success in mechanics courses specifically. Helweg [9] finds a weak correlation between scores on the Pursue Spatial Visualization Test: Rotations (PSVT:R) [10] and final course grades. Anderson finds no significant correlation between students' spatial abilities and course performance, but does find that spatial abilities are a predictor of student gains in conceptual knowledge [1]. Higley also finds that spatial skills measures were not significant predictors of exam scores, but were a significant factor in identifying clusters of common mistake patterns [11]. In summary, existing research identifies spatial skills as a factor in student learning in statics and likely important to students' development of conceptual knowledge. The interaction with other measures of preparation is complex, however, and it is not clear how important spatial abilities are to eventual course success. It may be that students with lower spatial abilities are able to compensate by leveraging other strengths. It could also be that many of the students with lowest spatial abilities do not progress through prerequisite coursework to this point in the curriculum.

The cloudy picture on the importance of spatial skills to success in mechanics might become clearer if we consider more closely how this relationship is less direct than it may be in other foundational courses such as engineering graphics. In mechanics courses, students deploy their spatial skills to interpret the representations we use to communicate concepts. Understanding what each representation means and how to apply it effectively in problem solving is important to their development of both conceptual and procedural knowledge. Kozma and Russel [12] proposed the idea of representational competence (RC) in the context of chemistry education research to describe the ability to use multiple representations of a concept as appropriate for learning, problem solving, and communication. While there is still no consensus on RC as a unified theoretical framework [13], the construct is commonly used in the science education literature and is seen as a marker of domain expertise [14], [15], [16]. Existing engineering education research has applied the construct of RC to a lesser extent, but also makes connections between RC and students' conceptual understanding [17] [18]. In research more closely related to RC in mechanics, Johnson-Glauch explores how the representations chosen to communicate problem statements influence student problem solving in statics [19].

As stated above, it is clear that statics involves representations that are spatial in nature. It is less clear how spatial abilities influence student success in the course. The vector language of mechanics utilizes mathematical forms of representation as well as spatial forms of representation like diagrams. These two forms of representation convey overlapping but not identical information. This creates the potential for a scenario where a student is strong in one but weaker with the other. Although ultimately a student would need to achieve mastery of spatial representations to achieve true expertise, in the short term they may be able to use their strength with one form of representation to compensate for weakness with another. Figure 1 shows a diagram for how the student preparation categories described previously serve as foundation for their development of RC as they progress through a statics course. Note that we could apply the model envisioned here more broadly to think about RC in virtually any engineering fundamentals course.

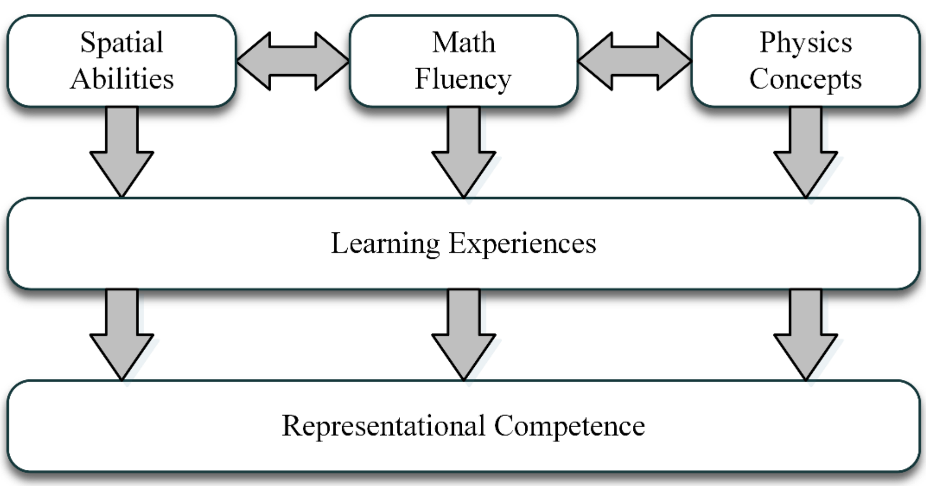


Figure 1. Conceptual model for how students might build on a foundation of prerequisite skills and knowledge to develop representational competence in a statics course.

In this paper, we explore the importance of spatial abilities to students' understanding of vector representations and the subsequent importance of students' RC with vectors to developing conceptual understanding in statics. Correlation of conceptual knowledge to overall course grades can vary widely across instructors and institutions for a variety of valid reasons [20]. Nonetheless, conceptual understanding is important to students' ability to transfer their learning to follow-on coursework as well as to eventual competency in engineering practice [21].

Two overarching research questions guide our study design and analysis:

RQ1: How important are spatial abilities to students' development of representational competence with vectors?

RQ2: How important is students' representational competence with vectors to their achievement of broader conceptual understanding in statics?

In order to address these questions, we administered multiple assessments at different times throughout statics courses at three community colleges. We also collected data on students' prerequisite course grades to serve as indicators of their math and physics preparation. Our analysis investigates how these various measures of student preparation and learning during the course predict conceptual knowledge measured at the end.

Study Design

The diagram in Figure 2 on the next page depicts a model for how conceptual learning might occur during a statics course when considered through the lens of the two research questions posed above. Students enter the course with a variety of preparation levels in relevant areas. This preparation influences how students experience learning activities in the first weeks that develop and review vector concepts and analytical procedures before and/or integrated with applications in the context of foundational statics concepts like forces, moments, and free-body diagrams. Students' development of RC with vectors in this context serves as preparation for learning experiences in the second half of the course that are focused on more advanced and integrated topics including rigid body equilibrium, structures, and internal forces.

We acknowledge this model is oversimplified in that conceptual learning is inherently nonlinear and complex [22], but still consider it useful for thinking about how RC with vectors might develop as a student progresses through the early emphasis of the course and how this progress might subsequently play into their learning in the second half. We hypothesize that students with lower RC might be less able to engage with more complex concepts later in the course that require more integration of multiple representations to develop. Examples include the process of integrating information across multiple free-body diagrams and sets of equilibrium equations in a frame analysis or integrating free-body diagrams, equations, calculus concepts and graphs of shear and moment diagrams. If a student is still devoting significant cognitive effort to coordinating and interpreting the basic vector representations, they may have less resources available to think about the more complex representations and underlying meaning.

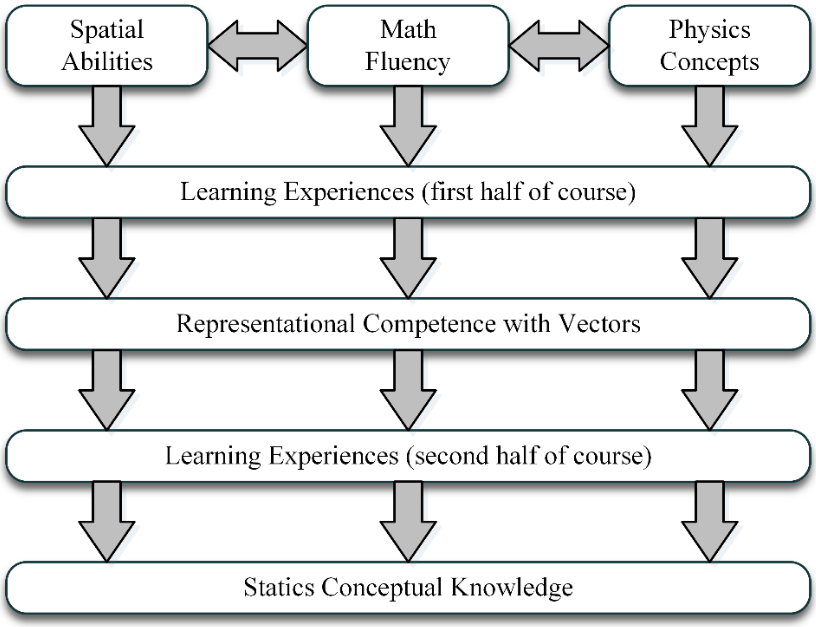


Figure 2. A simplified model for how representational competence with vectors might feed in to students' understanding of a broader set of statics concepts.

We employed three assessment instruments in the study. In all cases, students completed the assessments through online tools and received full credit on a nominal participation incentive regardless of their actual assessment score. Figure 3 shows the relative timing during the course.

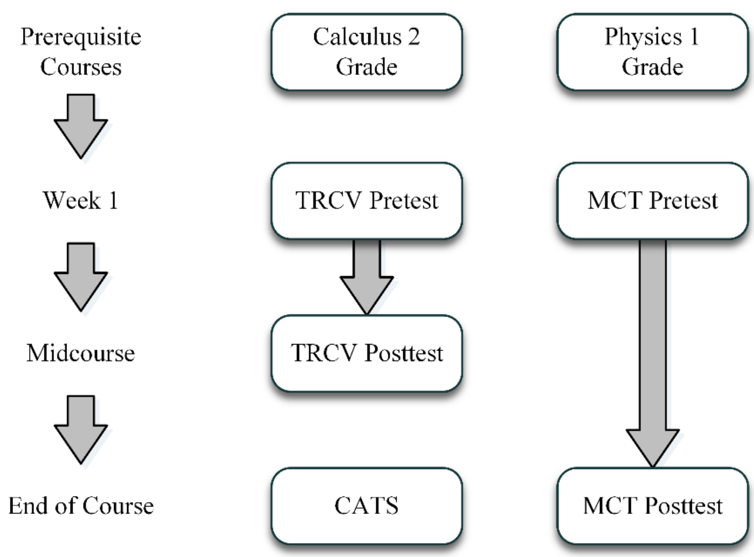


Figure 3. Study design illustrating the relative timing of assessments. TRCV = Test of Representational Competence with Vectors. MCT = Mental Cutting Test. CATS = Concept Assessment Test in Statics, formerly the Statics Concept Inventory (SCI).

The Mental Cutting Test (MCT) measures students' spatial skills [10]. We also considered the PSVT:R because of its wide use in the engineering education literature relating to spatial skills and student success. However, we ultimately chose the MCT for two reasons. The items on the MCT require integration of multiple spatial abilities including sectioning, rotation, and view translation from 3D to 2D. We hypothesize all three to be relevant to interpreting various statics representations such as 3D problem figures and free-body diagrams of multiple interacting bodies. Second, we were concerned about possible ceiling effects in administering the PSVT:R that might make this a less effective measure of variability in statics students' spatial abilities [8]. We administered the MCT during the first and last weeks of the term.

The Concept Assessment Test in Statics (CATS. formerly known as the Statics Concept Inventory) is widely used to measure conceptual statics knowledge [23]. We only administered the CATS at the end of the course based on suggestions that pretest scores differ little from random guessing [24] and to avoid assessment fatigue during the first week.

The Test of Representational Competence with Vectors (TRCV) measures fluency with vector representations and associated concepts [25]. The TRCV is a newer instrument that we further developed since initially proposing. We discuss the instrument and some of that development work in a little more detail below. We administered the TRCV in week one and at midcourse. The midcourse administration comes after coverage of moments and force system resultants, but before general rigid body equilibrium.

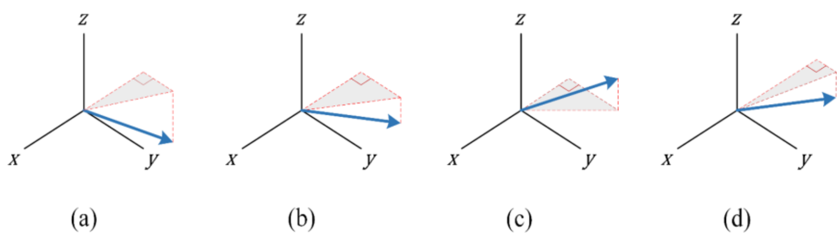
Test of Representational Competence with Vectors (TRCV)

The TRCV is a 16-item multiple-choice assessment of vector concepts and representations in a statics context. The items concern position vectors and force vectors in the context of vector concepts relevant to statics: vector addition, Cartesian components, dot products, and cross products. Assessment design focuses on testing each concept in both 2D and 3D with multiple questions integrating different combinations of representations classified as symbolic, numeric, diagram and narrative language. Figure 4 on the next page illustrates this approach with two items that require interpretation of 3D Cartesian component notation.

As previously reported, initial validation work included item analysis and correlation of assessment scores with students' productive use of multiple representations in problem solving on exams [25]. Subsequent development work led to a significant revision increasing the item count from 10 to 17 by disentangling some of the items that required synthesis of multiple vector concepts in addition to interpreting multiple representations. We further validated and refined the newer more focused items by selecting five items with the lowest discrimination scores (i.e. point biserial correlation less than 0.32) and comparing students' multiple choice answer selections with their written explanations for those choices. This analysis led to us discarding one item and making some minor revisions to question wording and/or answer choices on some of the other four.

This development work led to increased reliability of the assessment. The 10-item TRCV version 2.0 published previously had a Cronbach's alpha statistic of 0.60, indicating questionable internal consistency. The newer 16-item TRCV version 4.0 that we use in the present study had an alpha statistic of 0.80 for the pretest (considered good reliability) and 0.93 for the posttest (considered excellent reliability). The TRCV is available for instructors and researches to administer using the Concept Warehouse [26].

Question 7. A force vector is expressed in Cartesian components as $\vec{\boldsymbol{F}} = -150\hat{\boldsymbol{\imath}} + 100\hat{\boldsymbol{\jmath}} - 50\hat{\boldsymbol{k}}$ (Newtons). Which of the following figures best represents the direction of $\vec{\boldsymbol{F}}$**?** Note the shaded triangle in each figure lies in the xy plane.



Question 8. A force vector is expressed in Cartesian components as $\vec{\boldsymbol{F}} = -10\hat{\boldsymbol{\imath}} + 10\hat{\boldsymbol{\jmath}} + 10\hat{\boldsymbol{k}}$ (Newtons). The angle between $\vec{\boldsymbol{F}}$ and the positive z-direction is:

(a) Between $0°$ and $45°$

(b) Exactly $45°$

(c) Between $45°$ and $90°$

(d) Less than $0°$

Figure 4. Example items from the Test of Representational Competence with Vectors.

Study Population and Context

We administered the assessments in statics courses taught fall quarter 2020 at three community colleges, and then again at two of these colleges in winter quarter 2021. Table 1 summarizes the numbers in the study population. The first column gives course modality. All courses in the study were online and would otherwise be face-to-face if not for the COVID-19 pandemic. The modality column indicates the percentage of course credit hours taught using synchronous videoconference. For example, 40% synchronous would mean two hours per week live videoconference for a five credit course. Table 2 on the next page presents the population demographics. Students in the study self-reported this information as part of the IRB-approved informed consent process at the end of each course.

Table 1. Course modalities and sample sizes in the study population.

Institution and Term	Online Modality (% synchronous)	Enrollment (last week)	Number Consenting
CC1 F20	40%	21	17
CC2 F20	100%	31	17
CC3 F20	100%	25	22
CC1 W21	40%	13	11
CC2 W21	100%	24	13

Table 2. Demographic information of the study population along dimensions of gender, race/ethnicity, and age. All data is self-reported.

Category	Breakdown
Gender	
Female	11.4%
Male	82.3%
Other	1.3%
Prefer not to answer	5.1%
Race/Ethnicity	
Asian or Pacific Islander	25.3%
Black or African American	2.5%
Hispanic or Latinx	6.3%
Native American, Alaska Native or Indigenous	0%
White or Caucasian	50.6%
Multiracial or Biracial	8.9%
A race/ethnicity not listed	1.3%
Prefer not to answer	5.1%
Age	
17 or younger	1.3%
18-19	20.3%
20-22	31.6%
23-29	29.1%
30-39	11.4%
40 and above	5.1%
Prefer not to answer	1.3%

Results and Analysis

The overarching goal of this study is to use the frameworks posed above to gain insight into the two guiding research questions. We approach this with a series of bivariate correlation tests and multiple regression analysis to explore how the various assessment scores and course preparation measures interact with each other and/or predict later scores. Table 3 on the next page presents mean scores aggregated for the full study population for all of the factors included. Students self-report their grades in prerequisite calculus and physics along with their demographic information as described previously. See figure 3 above for the relative timing of the assessments. Note that the MCT pretest to posttest gain of 6.4% is slightly lower than the scores reported by Wood [8], but still significant with $p = .03$. End of course CATS scores are in the range reported by Steif [20].

We consider the MCT pretest score (MCTpre), calculus grade, and physics grade as three measures of student preparation that correspond to the spatial abilities, math fluency, and physics concepts we propose as three separate but related components to the foundation on which students build representational competence (see figure 1). Bivariate correlation tests between these three factors yield a significant correlation between calculus grade and physics grade with $R = 0.427$ ($p < .001$). We find no significant correlation between MCTpre scores and physics grade or math grade. The two course grades seem to be providing similar information about student preparation, and that information is different than what the MCT provides. Thus, although in principle we believe one could distinguish between relevant math skills and physics knowledge as this pertains to statics preparation, the course grades we use in this analysis do not cleanly disentangle those things, probably because grades are a "noisier" measure than a targeted and carefully constructed assessment like the MCT. So for purposes of our other analyses, we will assume the MCT scores capture spatial skills, and the two course grades capture some mix of partially overlapping "non-spatial skills," which could include math and science knowledge as well as other things.

Table 3. Assessment scores and course preparation measures. Calculus and physics course grades are self-reported.

Measure	N	Mean Score	St. Dev.
Calculus grade	78	3.49	0.56
Physics grade	75	3.27	0.69
MCTpre	79	54.9%	22.0%
MCTpost	75	62.4%	19.4%
TRCVpre	74	44.9%	22.6%
TRCVpost	60	60.0%	22.1%
CATS	67	45.7%	22.4%

Factors Predicting TRCV Scores

Next, we consider the relationship between these three predictors and TRCV scores, starting with TRCV pretest scores. It is important to account for the fact that we have data from three different institutions. The meaning of a particular course grade could vary by institution, especially considering that much of this prerequisite course work occurred during the onset of the COVID-19 pandemic in spring 2020 with the associated variability in institutional response and grading policy. Thus, in this analysis we statistically controlled for institution while seeing whether we can predict TRCV scores from the two grades and MCT scores. We used multiple regression to construct the model by sequentially adding predictors as Table 4 shows along with the relevant fit quality statistics.

Table 4. Model development to predict TRCV pretest scores using MCT, physics grade, and calculus grade while accounting for institutional effects.

Model	Predictors	R^2	R^2 Change	p-value
1	Institution	.008	.008	.768
2	Institution, Physics grade	.131	.123*	.036
3	Institution, Physics grade, Calculus grade	.229	.098	.061
4	Institution, Physics, Calculus, MCTpre	.410	.181**	<.001

*Statistically significant at $p<.05$, **Statistically significant at $p<.01$

Consider model 4 as an example of how we statistically control for institutional effects. The linear fit equation is:

$$(TRCV) = (a_0 + a_1 I_1 + a_2 I_2) + (b_0 P + b_1 I_1 P + b_2 I_2 P) + (c_0 C + c_1 I_1 C + c_2 I_2 C) + d_0 M, \quad (1)$$

where the variable I is a 2-element array representing institution that takes on the values [0 0], [1 0] and [0 1] for the three respective colleges in the study. The variables P and C represent grades in prerequisite calculus and physics respectively. Both of these predictors are multiplied by I to account for institutional differences in grading practices as discussed above. M represents MCT pretest score. We do not attach these scores to institution since this is a separately validated and targeted assessment, as is the TRCV. The coefficients a-d are computed using regression. In effect, the model computes three parallel regressions (one for each college) to predict TRCV scores from calculus and physics grades before aggregating the results and adding MCT as a third predictor.

As shown in table 4, both physics grade (R^2 change = 0.123, p = .036) and MCTpre score (R^2 change = .181, p < .001) are significant. Note that we added the different predictors to the model sequentially. If the predictors are uncorrelated with each other, the order in which we add them will not matter. If they correlate, like physics and calculus grades as discussed previously, then it may matter because correlated predictors carry overlapping information that would be redundant when used twice. So whichever predictor gets added to the model first gets credit for providing that information. Thus, it is important to test another variant of the model where we reverse the order in which we add physics and calculus grades. When we do so, we find that calculus grade now shows up as a significant predictor (R^2 change = 0.118, p = .043) while physics does not (R^2 change = 0.103, p = .052). We note that in both cases, the second predictor is near significance with a p-value only slightly higher than 0.05 and suspect that both factors would be significant with a larger sample size. This initial analysis supports the idea presented earlier that math fluency, physics knowledge and spatial abilities represent different components of foundational skills and knowledge students bring to a their statics course experience.

The analysis above gives some insight into the relationship between physics grade, calculus grade, MCT score, and TRCV score at the beginning of the course. We are more interested in what happens during the course. The TRCV posttest occurs roughly in the middle of the term, so our next analysis uses the same approach as above, but predicting TRCV posttest rather than pretest. Table 5 summarizes the results.

Table 5. Model development to predict TRCV posttest scores using MCT, physics grade, and calculus grade while accounting for institutional effects.

Model	Predictors	R^2	R^2 Change	p-value
1	Institution	.141	.141*	.017
2	Institution, Physics grade	.186	.046	.423
3	Institution, Physics grade, Calculus grade	.224	.038	.507
4	Institution, Physics, Calculus, MCTpre	.376	.252**	<.001

*Statistically significant at p<.05, **Statistically significant at p<.01

In this analysis, only MCT scores (among course preparation measures) add a statistically significant amount of predictive power (R^2 change = .252, p <.001). This result holds regardless of the order in which we add physics and math grades to the model. Interestingly, the effect of MCT scores is somewhat larger than the R^2 change we see in the analysis with TRCV pretest scores, and the R^2 change accounted for by the course grades is roughly half. One interpretation of this difference is that students may have learned how to apply more spatially oriented reasoning approaches to the TRCV items on the posttest, which led to MCT becoming a better predictor while the calculus and physics grades became worse predictors. Another possible explanation is that spatial skills may factor in to students' abilities to make gains on the TRCV in general during the first weeks of the course. We tested this hypothesis and found no significant correlation between MCT pretest scores and TRCV gains (measured as TRCVpost – TRCVpre). We also find some predictive power associated with institution in this model (R^2 change = .141, p =.017) which may reflect differences in instructional emphasis.

In summary, we conclude that spatial abilities (as measured by the MCT) appear to be important to students' development of RC with vectors and that this importance may increase as the course progresses. Physics and math preparation are likely also important, but the course grades we use in this study seem to be too broad a measure to disentangle their relative importance.

Factors Predicting CATS Scores

We next extend the analysis to consider how these assessment results might predict CATS scores. We will only consider spatial skills regarding initial course preparation because physics and calculus grades were not significant predictors of TRCV posttest scores. We conclude these two measures are too "noisy" to be particularly useful. This analysis focuses on the chain of assessments shown below in Figure 4. Here we hypothesize that the impact of spatial skills on statics conceptual knowledge is indirect and mediated [27] by representational competence, specifically RC with vectors as measured by TRCV.

Figure 4. Chain of assessments representing the learning model proposed in Figure 2.

We use a simpler model that no longer accounts for institutional effects because we are not considering course grades. Table 6 on the next page presents results from another series of bivariate correlation tests. MCT pretest scores correlate with TRCV posttest (r = .453, p < .001) and TRCV posttest results correlate with CATS (r = .577, p < .001). These are the two links in our hypothesized causal chain.

Table 6. Bivariate correlation results for MCTpre, TRCVpost and CATS.

Assessment	Statistic	MCTpre	TRCVpost	CATS
MCTpre	Pearson R	1	.453**	.383**
	p-value		<0.001	0.001
	N	79	60	67
TRCVpost	Pearson R	.453**	1	.577**
	p-value	<0.001		<.001
	N	60	60	56
CATS	Pearson R	.383**	.577**	1
	p-value	0.001	<.001	
	N	67	56	67

**Statistically significant at p<.01

We also find that MCT pretest scores correlate with CATS ($r = .383$, $p = .001$), not unexpected given the proposed chain. The question is whether that correlation occurs because MCT pretest directly impacts CATS, or whether it's only an indirect effect involving TRCV posttest. To test that possibility, we contrast two models:

1. MCT pretest predicting CATS after controlling for TRCV posttest
2. TRCV posttest predicting CATS after controlling for MCT pretest

Given the model proposed in Figure 4, we would expect a significant effect with (2) but not with (1). We again use multiple regression and consider the relative effects of adding predictors sequentially. Tables 7 and 8 present results for both models.

Table 7. Results using MCT pretest scores to predict CATS after controlling for TRCV posttest.

Model	Predictors	R^2	R^2 Change	p-value
1	TRCVpost	.333	.333**	<.001
2	TRCVpost, MCTpre	.340	.007	.445

**Statistically significant at p<.01

Table 8. Results using TRCV posttest scores to predict CATS after controlling for MCT pretest.

Model	Predictors	R^2	R^2 Change	p-value
1	MCTpre	.125	.125**	.008
2	MCTpre, TRCVpost	.340	.215**	<.001

**Statistically significant at p<.01

The results in Table 7 indicate that MCT pretest scores add no predictive power after controlling for TRCV posttest. In contrast, the results in Table 8 show that TRCV posttest results still add predictive power (R^2 change = .215, $p < .001$) after controlling for MCT pretest. These results suggest that the impact of spatial skills on statics conceptual knowledge is indeed mediated by representational competence, as reflected in the causal chain hypothesized above.

In a final test we consider whether the MCT posttest that we administered roughly concurrent with the CATS results in a stronger correlation. In doing so we find $R = .43$ ($p < .001$). This result is in line with findings by Anderson correlating scores on the Card Rotations Test and the Paper Folding Test with CATS scores, all administered concurrently [1]. But the correlation between MCT posttest and CATS scores in the current study is still noticeably lower than for TRCV posttest ($R = .58$, $p < .001$). Furthermore, when both TRCV posttest and MCT posttest are entered into a regression model to predict CATS scores, only TRCV posttest is a significant predictor ($p = .001$, versus $p = .127$ for MCT posttest). This result provides further evidence that the mediation model using representational competence better explains the relationship between spatial skills preparation and statics conceptual knowledge attainment.

Study Limitations

There are limitations to this study that we should acknowledge. First and foremost, the data was collected in the context of online learning during the COVID-19 pandemic with associated extraordinary stressors on many in the community college student population. The students involved would otherwise be engaged primarily in face-to-face instruction. It's possible that the patterns identified here may not generalize to more typical instructional contexts. We also acknowledge that some of the sample sizes are small when matching student results across multiple assessments. Lastly, this study pertains to patterns in how students approach learning that may vary across demographic groupings. Prior research concerning the importance of spatial skills in STEM education have identified significant differences across some of these categories [6]. Our study population is not large enough to disaggregate across demographics. Further research is necessary to test whether the model proposed in this study would hold across demographic categories and educational contexts.

Implications for Instruction

The key results of the study have implications to instruction. We find that (addressing RQ1) spatial abilities do indeed factor into students' ability to develop representational competence with vectors, and (addressing RQ2) RC with vectors mediates the importance of spatial abilities and correlates with development of conceptual knowledge in statics. The study also identifies the TRCV as a potentially useful instrument for assessing whether students are prepared to learn statics concepts as measured by the CATS. This last result may seem somewhat surprising because the TRCV focuses largely on 3D vector applications (e.g. 10 of 16 items are 3D), but the CATS uses exclusively 2D applications to assess conceptual knowledge with little direct connections to vector concepts such as 3D Cartesian components and cross products that are emphasized in the TRCV. We believe the relationship can be explained by applying the framework of RC. The vector language assessed by the TRCV is the vocabulary most statics courses use to develop the concepts assessed by the CATS. If students can understand this language, they are better prepared to learn the concepts.

Mechanics faculty, including those involved in this study, often lament how their students seem to resist our pleas to draw free-body diagrams as a first step in problem solving or to use vector notation and units appropriately in analysis. When viewed through the construct of RC, these seemingly disparate struggles start to align as examples of students not making effective use of these representations by integrating them into their problem solving and conceptual reasoning. Instead, they may view our insistence as just asking them to follow convention and prefer to use only the representations they are most comfortable with (e.g. for many this seems to be algebraic equations). This may relate to observations that students seem more likely to rely on memorized problem solving procedures (e.g. by referencing examples or homework solution manuals) than root their analysis in conceptual understanding [28], [29].

The results of this study suggest teaching strategies that scaffold students' development of RC may have promise in helping students learn concepts. Potential approaches include providing hands-on models and/or virtual simulations, as is commonly done in mechanics instruction, to provide students with concrete representations that may anchor their learning. Studies in chemistry education support this approach [14], [30]. We propose applying the framework of RC toward refining these learning activities with intent to foster RC development.

More generally, we propose that instructors could think more explicitly about how their students interpret and use the representations central to understanding mechanics concepts and problem solving. As stated by Moore [18] in the context of a heat transfer course: "thinking through multiple representations and translations within and among representations helps engineering students negotiate meaning when misconceptions or naıve conceptions are present." Mechanics instructors could consider providing more opportunities for students to practice translating and coordinating information across representations. Doing so may involve considering some representations that may not seem germane to an application or analysis at hand, but may serve useful more as basis for exploring the meaning and connections between representations. Additional instruments assessing RC in other areas of mechanics would also be helpful for use in formative assessment and evaluating the impact of interventions on student learning.

Conclusions

This paper presents a model and associated analysis that suggests an explanation for how spatial abilities factor in to students' development of statics conceptual knowledge by applying the construct of representational competence. Our analysis utilizes multiple regression to investigate interactions between assessment scores in multiple assessments administered throughout the term in statics courses at three different colleges. We find that spatial skills play a significant role in students' development of representational competence with vectors that we hypothesize is complementary to the role played by their preparation in prerequisite math and physics. That emerging representational competence also predicts their success in acquiring conceptual knowledge and seems to mediate the impact of spatial skills preparation. Ultimately we conclude that the framework of representational competence is useful for the design of educational interventions that seek to improve conceptual learning in mechanics as well as for the assessment of their impact on student learning.

Acknowledgement

This material is based upon work supported by the National Science Foundation under grant numbers DUE #1834425 and DUE #1834417. Any opinions, findings, and conclusions or

recommendations expressed are those of the authors and do not necessarily reflect the views of the NSF.

References

[1] E. Anderson, R. Taraban and D. Hudson, "A Study of th eImpact of Visuospatial Ability, Conceptual Understanding, and Prior Knowledge upon Student Performance in Engineering Statics Courses," in *ASEE Annual Conference and Exposition*, Austin, TX, 2009.

[2] G. Bischof and D. Rubesa, "Correlation between engineering students' performance in mathematics and," in *ASEE Annual Conference and Exposition*, Seattle, WA, 2015.

[3] K. T. Aung, R. Underdown and Q. Qian, "Vertical assessment of math competency among freshmen and sophomore engineering," in *120th ASEE Annual Conference and Exposition*, Atlanta, GA, 2013.

[4] K. J. Shryock, A. R. Srinivasa and J. E. Froyd, "Alignment of Preparation via First-Year Physics Mechanics and Calculus Courses with Expectations for a Sophomore Statics and Dynamics Course," in *ASEE Annual COnference and Exposition*, Vancouver, BC, 2011.

[5] R. Myose, S. Raze, K. Hoffman and A. Ghoddoussi, "Correlating Engineering Statics Student Performance with Scores of a Test over Pre-requisite Material Involving Problem Solving," in *Proceedings of the 2014 ASEE Midwest Section Conference*, 2014.

[6] S. A. Sorby, "Educational Research in Developing 3-D Spatial Skills for Engineering Students," *International Journal of Science Education,* vol. 31, no. 3, pp. 459-480, 2009.

[7] O. Ha and N. Fang, "Spatial Ability in Learning Engineering Mechanics: Critical Review," *Journal of Professional Issues in Engineering Education and Practice,* vol. 142, no. 2, p. 04015014, 2015.

[8] S. D. Wood, W. H. Goodridge, B. J. Call and T. L. Sweeten, "Preliminary Analysis of Spatial Ability Improvement within an Engineering Mechanics Course: Statics," in *2016 ASEE Annual Conference and Exposition*, New Orleans, Louisiana, 2016.

[9] O. Helweg, "Using The Purdue Spatial Visualization Test To Predict Success In Statics," in *ASEE Annual Conference and Exposition*, Albuquerque, 2001.

[10] R. Gorska and S. Sorby, "Testing Instruments for the Assessment of 3-D Spatial Skills," in *Proceedings of the 2008 ASEE National Conference and Exposition*, Pittsburg, PA, 2008.

[11] K. Higley, T. Litzinger, P. Van Meter, C. B. Masters and J. Kulikowich, "Effects of Conceptual Understanding, Math and Visualization Skills on Problem-Solving in Statics," in *ASEE Annual Conference and Exposition*, Honolulu, HI, 2007.

[12] R. B. Kozma and J. Russel, "Multimedia and Understanding: Expert and Novice Responses to Different Representations of Chemical Phenomena," *Journal of Research in Science Teaching,* vol. 34, no. 9, pp. 949-968, 1997.

[13] K. L. Daniel, C. J. Bucklin, E. A. Leone and J. Idema, "Towards a Definition of Representational Competence," in *Towards a Framework for Representational Competence in Science Education. Models and Modeling in Science Education*, vol. 11, K. Daniel, Ed., Springer, Cham, 2018, pp. 3-11.

[14] M. Steiff, S. Scopelitis, M. E. Lira and D. Desutter, "Improving Representational Competence with Concrete Models," *Science Education,* vol. 31, no. 3, pp. 344-363, 2016.

[15] P. Pande and S. Chandrasekharan, "Representational Competence: Towards a distributed and embodied cognition account," *Studies in Science Education,* vol. 107, no. 2, pp. 451-467, 2016.

[16] N. A. Rau, "Supporting Representational Competences Through Adaptive Educational Technologies," in *Towards a Framework for Representational Competence in Science Education. Models and Modeling in Science Education.*, vol. 11, K. Daniel, Ed., Springer, Cham, 2018, pp. 103-132.

[17] A. Johri, R. Wolff-Michael and B. Olds, "The Role of Representations in Engineering Practices: Taking a Turn towards Inscriptions," *Journal of Engineering Education,* vol. 102, no. 1, pp. 2-19, 2013.

[18] T. J. Moore, R. L. L. R. A. Miller, M. S. Stohlmann and R. K. Young, "Modeling in Engineering: The Role of Representational Fluency in Students' Conceptual Understanding," *Journal of Engineering Education,* vol. 102, no. 1, pp. 141-178, 2013.

[19] N. Johnson-Glauch and G. L. Herman, "Engineering representations guide student problem-solving in statics," *Journal of Engineering Education,* vol. 108, pp. 220-246, 2019.

[20] P. S. Steif and M. Hansen, "Comparisons Between Performances in a Statics Concept Inventory and Course Examinations," *International Journal of Engineering Education,* vol. 22, no. 5, pp. 1070-1076, 2006.

[21] R. A. Streveler, T. A. Litzinger, R. L. Miller and P. S. Steif, "Learning Conceptual Knowledge in the Engineering Sciences: Overview and Future Research Directions.," *Journal of Engineering Education,* vol. 97, p. 279–294, 2008.

[22] R. A. Streveler, T. A. Litzinger, R. L. Miller and P. S. Steif, "Learning Conceptual Knowledge in the Engineering Sciences: Overview and Future Research Directions. Journal of Engineering Education," vol. 97, p. 279–294, 2008.

[23] P. S. Steif and J. A. J. A. Dantzler, "A Statics Concept Inventory: Development and Psychometric Analysis," *Journal of Engineering Education,* vol. 94, p. 363–371, 2005.

[24] P. S. Steif and M. A. Hansen, "New Practices for Administering and Analyzing the Results of Concept Inventories," *Journal of Engineering Education,* vol. 96, p. 205–212, 2007.

[25] E. Davishahl, T. R. Haskell, J. Davishahl, L. Singleton and W. Goodridge, "Do They Understand Your Language? Assess Their Fluency with Vector Representations," in *126th Annual ASEE Annual Conference and Exposition*, Tampa, FL, 2019.

[26] M. Koretsky, J. Falconer, B. Brooks, D. Gilbuena, D. Silverstein, C. Smith and M. Miletic, "The AIChE concept warehouse: A web-based tool to promote concept-based instruction. Advances in Engineering Education," *Advances in Engineering Education,* vol. 4, no. 1, 2014.

[27] R. M. Baron and D. A. Kenny, "The moderator-mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations," *Journal of Personality and Social Psychology,* vol. 51, pp. 1173-1182, 1986.

[28] T. Litzinger, P. Meter, C. M. Firetto, L. J. Passmore, C. B. Masters, S. R. Turns, G. L. Gray, F. Costanzo and a. S. E. Zappe, "A Cognitive Study of Problem Solving in Statics," *Journal of Engineering Education,* vol. 99, pp. 337-353, 2010.

[29] P. S. Steif and A. Dollár, "Reinventing the Teaching of Statics," *International Journal of Engineering Education,* vol. 21, no. 4, pp. 723-729, 2005.

[30] A. T. Stull and M. Hegarty, "Model Manipulation and Learning: Fostering Representational Competence With Virtual and Concrete Models.," *Journal of Educational Psychology,* vol. 108, no. 4, pp. 509-527, 2016.