

# Learning & Teaching

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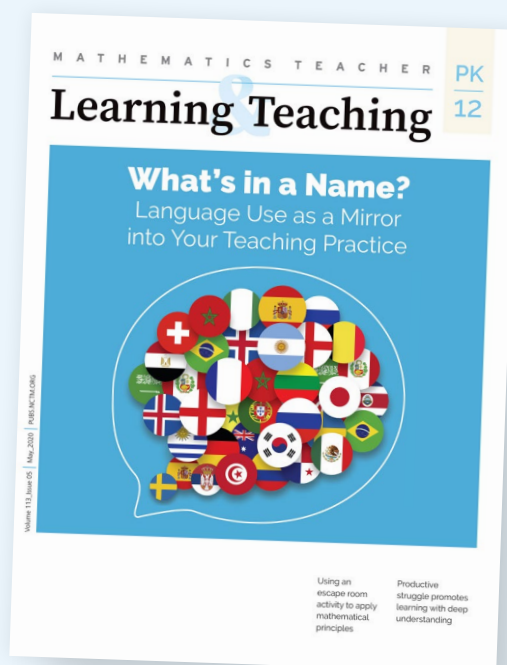
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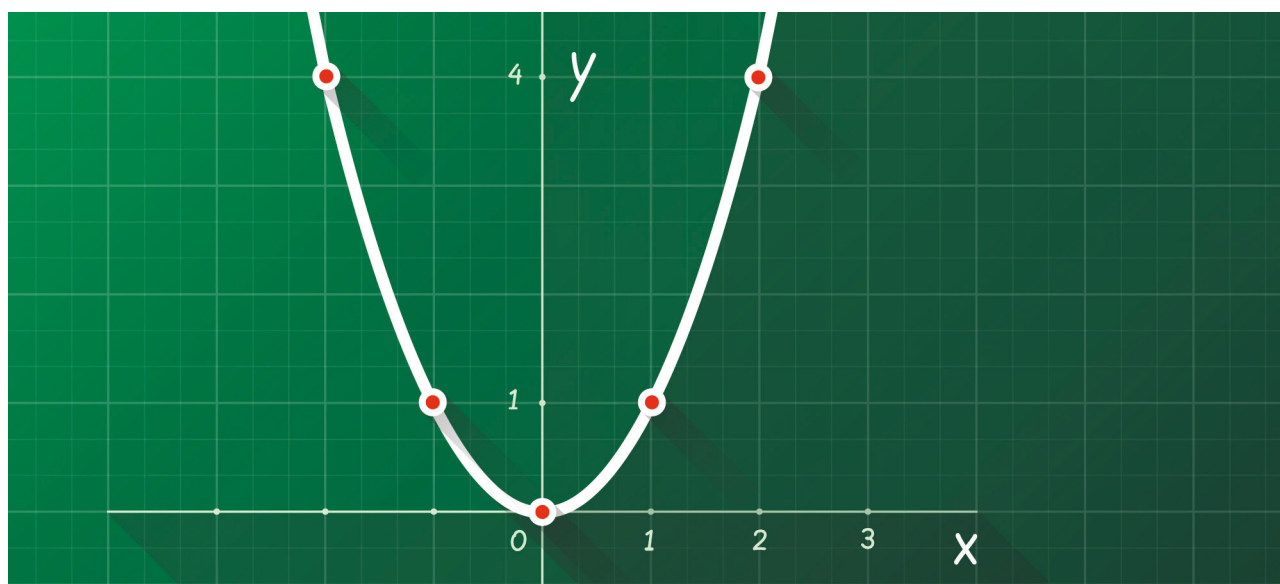


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# Parameters, Sliders, Marble Slides, Oh My!

Three different technological activities to explore parameters of quadratic functions each has its own pros and cons.

Nina G. Bailey, Demet Yalman Ozen, Jennifer N. Lovett, Allison W. McCulloch, and Charity Cayton

**Ms. Rose is preparing** to teach a lesson on quadratics in which her students explore the effects of parameters on quadratic functions. She knows that strategic use of technology is important and that it can allow students to investigate and conjecture the effect of the parameters on their own. However, she is uncertain about where to go from here. Ms. Rose asked her colleagues for recommendations, and they said that their students love Desmos Marbleslides. This conversation also reminded Ms. Rose about learning to use GeoGebra and Desmos in her teacher preparation program to have students directly change the value of the parameter.

Too many options exist, though, and Ms. Rose feels overwhelmed. She fears that selecting the wrong activity may be counterproductive and does not want to impede her students' learning in any way.

As can be seen by Ms. Rose's situation, selecting an appropriate technological tool for parameter exploration can be a daunting task given the numerous quality options, each with specific affordances and constraints. The goal of this article is to give examples of high school students exploring parameters of quadratic functions using different tools as a catalyst to discuss the pros and cons and demonstrate the mathematical

power of each. We illustrate how your learning goals should navigate you toward a productive option.

## THE POWER OF PARAMETER EXPLORATION

Parameters, like variables, are place holders for values. Specifically, parameters represent values for which a mathematical object (e.g., a function) is changed (i.e., transformed) while holding the main integrity of the object constant. Parameters are traditionally introduced during the study of particular function families (e.g., linear and quadratic), when students learn about the effect specific parameter values have on the graph of a function (NGA Center and CCSSO 2010).

Technology is explicitly recommended for parameter exploration because it enables students to try many values for each parameter very quickly (e.g., Edwards et al. 2009). When students are able to investigate and dynamically observe the effect of parameters on a function family, they are better able to visualize the transformation and to connect graphical and symbolic representations (e.g., Walker and Edwards 2017). This exploration may offer the needed motivation for students to want to understand why particular values of a parameter produce a predictable effect on the parent function. For example, why does a negative  $a$  value “flip” the parabola?

A variety of technological tools (e.g., GeoGebra and Desmos) and design styles (e.g., sliders and Marbleslides) can be used for parameter exploration. We will share examples from three different types of

activities using the technological tool Desmos: parameter explorations without sliders, parameter explorations with sliders, and an activity using a special component available in Desmos called Marbleslides. Each activity enables the exploration of parameters  $a$ ,  $h$ , and  $k$  of the vertex form of a quadratic function:  $y = a(x - h)^2 + k$ . The different features within each activity are summarized in table 1.

To understand the affordances and constraints of these activities, we video recorded pairs of high school students with varying experience (both with and without prior knowledge of quadratic functions) engaging with the activities. The video recorded sessions took place outside of class time and were not part of the class instruction on quadratic functions. After examining 26 students’ work across the different activities, we were able to identify typical behaviors of engagement. For each activity, you will see a video clip of a pair of students engaging with a portion of the activity that represents how students typically engage with that activity. Please note that every pair of students in the clips we have chosen was in a first-year mathematics course and had not been introduced to parameters within a quadratic function family unless we state otherwise. Each pair experienced only one of these activities so that we could examine the affordances and constraints of the design of the activity. We will discuss the students’ exploration and the affordances and constraints of the particular activity. In each example, the names indicated in the clips are those selected by the students’ themselves.

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Nina G. Bailey, [nbaile15@uncc.edu](mailto:nbaile15@uncc.edu), is working toward her PhD in curriculum and instruction with a concentration in mathematics education at the University of North Carolina at Charlotte.

Demet Yalman Ozen, [dyzi@mtmail.mtsu.edu](mailto:dyzi@mtmail.mtsu.edu), is working toward her PhD in mathematics education at Middle Tennessee State University in Murfreesboro.

Jennifer N. Lovett, [jennifer.lovett@mtsu.edu](mailto:jennifer.lovett@mtsu.edu), is an assistant professor of mathematics education at Middle Tennessee State University in Murfreesboro.

Allison W. McCulloch, [allison.mcculloch@uncc.edu](mailto:allison.mcculloch@uncc.edu), is an associate professor of mathematics education at the University of North Carolina at Charlotte.

Charity Cayton, [caytonc@ecu.edu](mailto:caytonc@ecu.edu), is an associate professor of mathematics education at East Carolina University in Greenville.

All five authors are interested in helping teachers implement tasks that incorporate technology effectively in their classrooms.

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### MULTIPLE GRAPHS ACTIVITY: DYNAMICALLY CHANGING THE PARAMETER VALUES WITHOUT SLIDERS

One way to explore the parameters  $a$ ,  $h$ , and  $k$  of the vertex form of a quadratic function,  $y = a(x - h)^2 + k$ , is to directly change the value of the parameter. We presented students with the parent function,  $y = x^2$ , graphed in blue. To explore the effect of changing the value of each parameter, students were asked to graph at least five more functions of each form (i.e.,  $y = (x - h)^2$ ,  $y = x^2 + k$ , then  $y = ax^2$  on individual pages of the activity). They were also instructed to consider how the new graphs compare to the parent function (see figure 1). We encourage you to try our Multiple Graphs activity before watching students engage with it.

In video 1, you will see Amya and Erica engaging with the Multiple Graphs activity. Amya and Erica's experience was very typical of the engagement we observed with this activity. As you watch, consider what Amya and Erica noticed about the effect of the parameters on the parent function.

Amya and Erica noticed that changing the value of  $h$  results in a horizontal translation as evidenced when Erica stated, "That's negative five right there;

it just went to the other side" when she noticed that parameter  $h$  translates in the opposite direction of its sign. Additionally, Amya noted that "it [the graph] just stays on top of the  $x$ -axis," indicating that all their graphs had vertices on the  $x$ -axis. Although they correctly expressed that changing the value of parameter  $h$  moves the parent function left or right, they did not explicitly determine why the value of  $h$  moves the parent function in the opposite direction along the  $x$ -axis compared to the sign of the value of  $h$ .

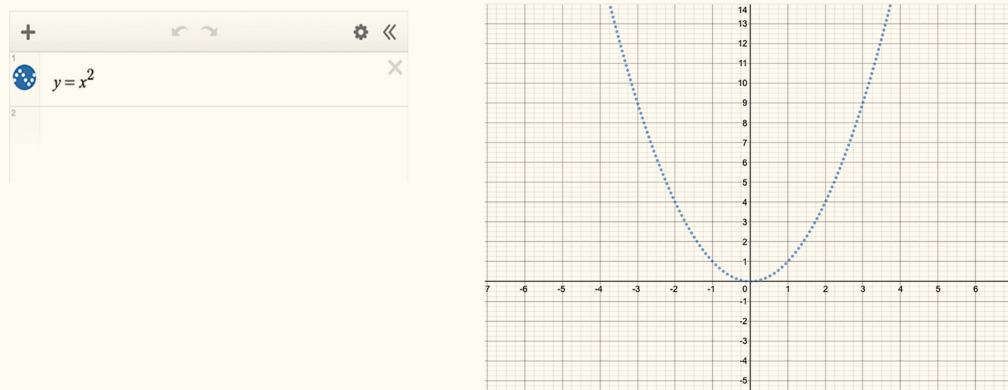
In this activity, Amya and Erica were able to test if functions are equivalent and what happens when the value of  $h$  is the same number with a different operation or sign. First, Amya and Erica graphed  $y = (x - 5)^2$  (shown in black in the video). Later, they graphed  $y = (x + 5)^2$  (shown in red) and noticed that those functions overlapped. They were able to see that those are equivalent functions that translate the parent function five units to the left. Immediately, they changed  $y = (x + 5)^2$  to  $y = (x - 5)^2$ . Erica stated that "if you do it negative, it's the opposite—gonna go to the other side." They were able to confirm their hunch that the graph "might go to the other side" and see the resulting translation of five units to the right. They also noted that changing the value of parameter  $a$  resulted in a

**Table 1** Summary of Activity Features

| Features                             | Activity                                                                  |                                                                                                                    |                                                        |
|--------------------------------------|---------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------|
|                                      | Multiple Graphs                                                           | Quadratic Sliders                                                                                                  | Quadratic Marble Slides                                |
| Number of graphs visible to students | Parent graph and at least five student-created graphs (6+)                | Parent and graph created on the basis of the sliders (2)                                                           | One (1)                                                |
| Style of parameter introduction      | Individual<br>$y = (x - h)^2$<br>$y = x^2 + k$<br>$y = ax^2$              | Cumulative<br>$y = (x - h)^2$<br>$y = (x - h)^2 + k$<br>$y = a(x - h)^2 + k$                                       | N/A                                                    |
| Style of changing parameter values   | Direct manipulation: typing the equation                                  | Dragging the slider                                                                                                | Direct manipulation: typing the value of the parameter |
| Affordances                          | Color coding                                                              | Students try integer/noninteger and positive/negative values for the parameter as they drag.                       | Goal oriented (i.e., positioned as a game)             |
| Constraints                          | Students might not try noninteger and negative values for the parameters. | A record of the graphs for comparison is not available as they try new values (only the current value is graphed). | Students need prior knowledge of quadratic parameters. |

**Fig. 1**

Graph at least five additional functions in the form  $y = (x - h)^2$ , changing the value of  $h$  each time. Keep track of what you notice about these graphs compared with the parent function. (Recall that the parent function is dotted.)

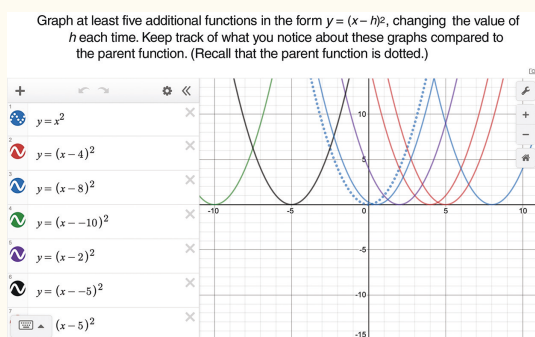


Try the Multiple Graphs activity before you watch students engage with it (in video 1).

vertical compression (i.e., wider parabola) or stretch (i.e., narrower parabola). Amya conjectured that a “smaller negative” value would produce a graph that is “bigger” than the original, referring to a wider parabola. They identified that negative values produce a graph that is “flipped over.” When they graphed  $y = -1x^2$ , Erica asked if it was “thinner than the original,” and the pair proceeded to discuss whether  $y = -1x^2$  is the same size as  $y = x^2$ . Although they did not indicate that no vertical compression or stretch existed, the pair was clearly grappling with the fact that changing parameter  $a$  seemed to produce two types of transformations.

The Multiple Graphs activity afforded Amya and Erica the opportunity to view multiple color-coded graphs that enabled continual comparison to the parent function. Additionally, the structure of the activity forced the students to try multiple values for each parameter. Although that is a benefit, it can also be a limitation because students will not necessarily try a variety of positive/negative and integer/noninteger numbers for the value of the parameter (e.g., Amya and Erica did not always try noninteger values). When the students did not try a wide variety of values, they encountered difficulty describing the effect of the parameter. This limitation could be overcome with prompting or questioning from the teacher asking students to consider what types of values they tried or to compare their observations with other groups or students.

### Video 1 Amya and Erica Engage with the Multiple Graphs Activity



[Watch the full video online.](#)

### QUADRATIC SLIDERS ACTIVITY: EXPLORING THE PARAMETERS WITH SLIDERS

Students can also explore the effects of parameters by dragging a slider to change the value of the parameter. In the Quadratic Sliders activity, the parent function  $y = x^2$  is graphed in blue and the translated function is graphed in red. Unlike the previous activity, only two graphs are visible in the Quadratic Sliders activity; however, students are able to drag the slider and dynamically observe the consequences of the change on the parent function (see video 2). Try the Quadratic Sliders activity.



Students typically slowly improved and refined their understanding of each parameter in the Quadratic Sliders activity. Video 3 shows such engagement. While watching Sara and Julian, consider what they were able to notice about the effect of the parameters on the parent function and how their experience with the tool differed from Amya and Erica's.

Sara and Julian noticed that altering the value of  $h$  “changes where the  $x$  is,” referring to the  $x$  value of the vertex. They attempted to probe why changing the value of  $h$  moves the parent function in the opposite direction along the  $x$ -axis. Julian explained that when the graph is translated to the right by two, then they need to subtract two from the  $x$  so that it will go back to the origin. He was grappling with how to explain that

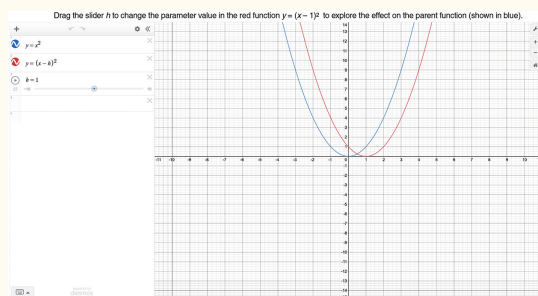
a negative value for parameter  $h$  results in a translation to the right. The students were able to compare the two graphs by moving the slider to determine the effect of  $h$ , but when asked to sketch the graph of a specific equation at the end, not featured in the clip, the pair translated in the wrong direction.

Recall that the Quadratic Sliders activity introduced the parameters cumulatively (i.e.,  $y = (x - h)^2$ ,  $y = (x - h)^2 + k$ , then  $y = a(x - h)^2 + k$  on individual pages of the activity). This style of parameter introduction enabled the students to better understand the role of the preceding parameter. Although this was true for the Multiple Graphs activity because one parameter is added each time, it was more evident within this activity with cumulative introduction. For example, after slider  $k$  was added, the students easily compared the effects of  $h$  and  $k$ . They both agreed that  $k$  “changes how high it [the graph] goes or low” referring to the  $y$ -value of the vertex of the parabola, and they noticed that the  $x$ -value changed as they dragged slider  $h$ .

After parameter  $a$  was added, Julian compared the effect of  $a$  to that of  $k$ : “So, the  $k$  changes where it is, up and down, but the  $a$  changes the variety,” and Sara added “changes how wide it goes.” Sara and Julian did this as each new parameter was added. For example, Sara summarized that “ $k$  goes up and down,  $h$  goes left or right,  $a$  determines how wide it is.” However, they were still unsure of how to determine how wide the graph is based on the value of  $a$ . While exploring how changing the value of  $a$  determines how wide the resulting function will be, Sara was surprised by the reflection of the graph over the  $x$ -axis when she tried negative values for  $a$ . As a result, Sara stated that  $a$  “changes the direction it goes,” indicating that the graph underwent a reflection over the  $x$ -axis. Consequently, the pair was able to compare the effects of each parameter by having the opportunity to drag each slider when needed and to dynamically observe the resulting function.

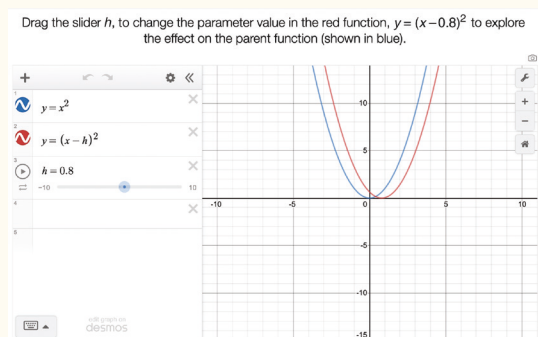
The structure of the Quadratic Sliders activity required them to try a variety of values (intentionally or not) for each parameter. Unlike the Multiple Graphs activity without sliders, Sara and Julian tried negative values and noninteger values while exploring and saw the immediate impact on the parent function. They focused on the transformation when moving the sliders and were not worried about picking values. Thus, they had more time to think about what is changing and consider which values of the parameter produced the change. Continuous exploration of the scaling and translating of the parent function seemed to allow the

### Video 2 Sample Directions for Parameter $h$ Slider



[Watch the full video online.](#)

### Video 3 Watch Sara and Julian Participating in the Quadratic Sliders Activity



[Watch the full video online.](#)

students to better understand the effect of each parameter. Although investigating how changing the value of each parameter affects the graph dynamically helps students understand the parameters' transformational effect on the parent function, this activity has one important limitation. The Quadratic Sliders activity uses sliders that produce a constantly changing graph that may make it more difficult to draw comparisons, which was not an issue in the Multiple Graphs activity. In the Multiple Graphs activity, Amya and Erica could refer to the color-coded graphs and corresponding values of the parameter. Remembering what the graph looks like for different values of each parameter using sliders may be cognitively draining for students. This may result in difficulties determining why the graph changes for particular values of a parameter.

### QUADRATIC MARBLESLIDES

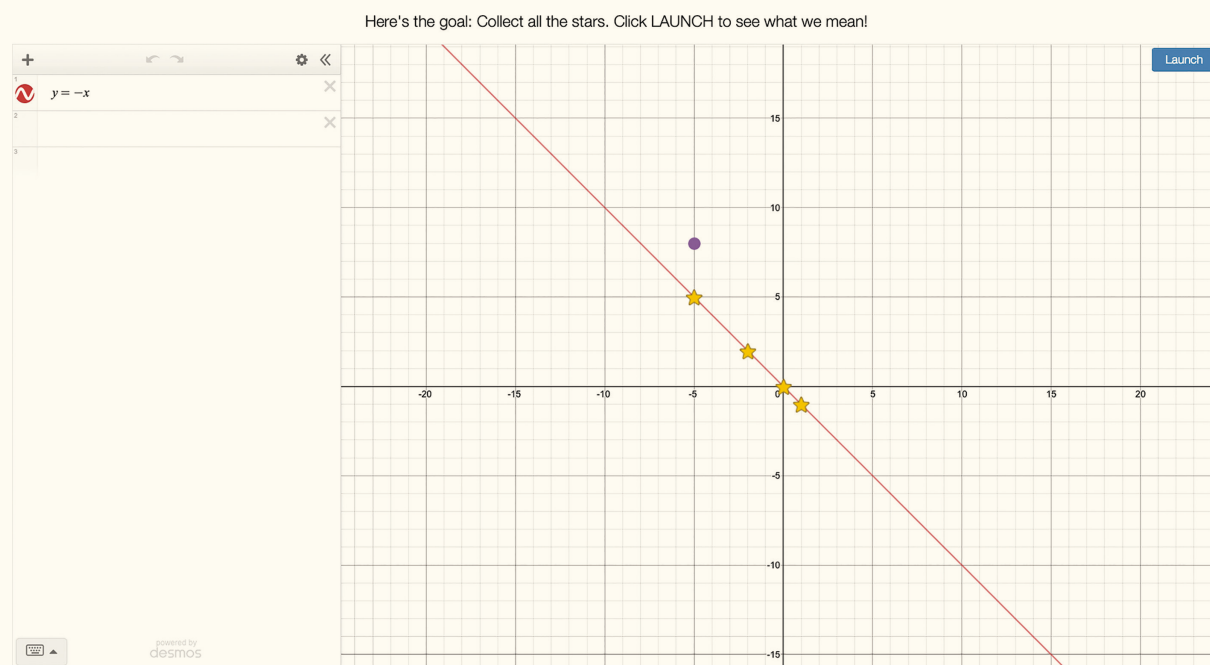
A third way to explore the parameters of quadratic functions is to use an activity that resembles a game. Our activity, Quadratic Marbleslides (created using the Marbleslide tool in Desmos Activity Builder), begins

with an example using a linear function (see figure 2), which models the goal for students after they click “Launch” and watch the purple ball (i.e., point) drop and slide down the line, collecting the stars (see below). Then students are asked to change the values of the parameters  $h$ ,  $k$ , and  $a$  of the quadratic function to position the graph of the function so that when the balls are launched, they will slide along the parabola and collect the stars (see video 4). Try our Quadratic Marbleslides activity.

Unlike the previous activities, many students with no prior knowledge of quadratic functions (including the following pair) struggled with the Quadratic Marbleslides activity. In video 5, you will watch Kaevel and Nasiyah engage with Quadratic Marbleslides. Pay attention to their strategy while watching the clip.

Kaevel and Nasiyah began this activity by employing a guess-and-check strategy. As the pair tries a stream of values for parameter  $k$  while not getting much closer to success, you can hear the frustration as Kaevel sighs, “Oh my gosh.” It appeared as though the pair had learned through their trial and error when they missed the launch and quickly changed  $k$

**Fig. 2**



*The Quadratic Marbleslides activity, which resembles a game, is a third way to explore the parameters of quadratic functions.*

to one to translate the graph up one unit and achieve success. Similarly, they changed  $k$  to negative four on the next graph to translate down four units. Although some learning was evident, it was not sustained. The pair reverted back to their guess-and-check tactic on the last graph in the clip. They began by plugging in a stream of values for  $h$  when the graph did not need to be translated horizontally. Certainly, there is value in guess and check if it leads to sustained learning. We anticipate that this pair would have had more success with either the Multiple Graphs or Quadratic Sliders activities that provided more structures for investigating the effect of each parameter.

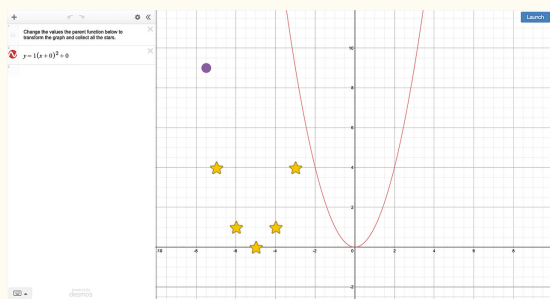
Beyond guess and check, students with no prior knowledge of parameters of quadratic functions also

did not typically consider the operators and order of operations as distinguishing factors among  $a$ ,  $h$ , and  $k$ . For example, another pair of students tried positive one-half for parameter  $k$ ; however, when they changed the value, they simply deleted the subtraction symbol. Although they thought that they had changed the value of  $k$  from negative one-half to positive one-half, they actually changed the value of  $k$  in such a manner that it became parameter  $a$ .

We have shared some drawbacks about using an activity like Quadratic Marbleslides as an introduction to parameters; let's consider some of the distinct advantages when it is used as a practice or review activity. Students who had prior knowledge of parameters of quadratic functions engaged with the activity as a means of refining their understanding. Many of these students enjoyed the game-like design of the activity and demonstrated excitement. Even students who did not immediately recall how to translate the graph as needed were able to test appropriate values with ease and confirm or adjust their conjectures. You will watch a short clip of Eden and Ethen (see video 6), who were in a second-year high school mathematics course. Pay attention to how their prior knowledge of quadratic parameters resulted in a very different experience than Kaebel and Nasayah had.

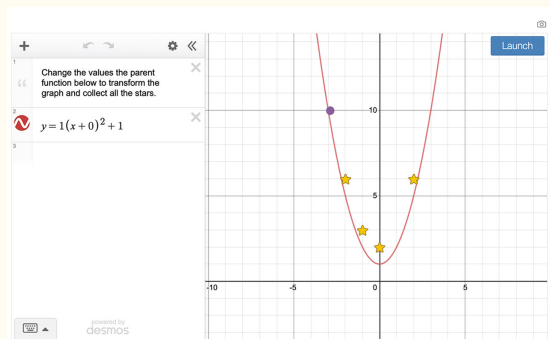
Ethen immediately asked Eden to change the value of  $a$  to 0.2 to make the graph narrower. When asked how they knew, Ethen quickly stated that “the bigger the number, the more narrow it is.” Eden then quickly tested this statement by changing the value of  $a$  to 10 to confirm their thinking. With some prior knowledge of

#### Video 4 Directions for the Quadratic Marbleslides Activity



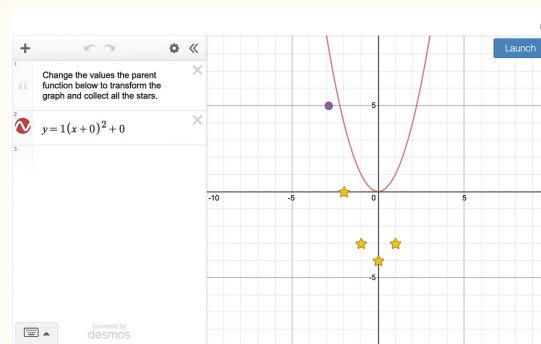
Watch the full video online.

#### Video 5 Kaebel and Nasiyah Engage with Quadratic Marbleslides Activity



Watch the full video online.

#### Video 6 Eden and Ethen Engage with Quadratic Marbleslides Activity



Watch the full video online.



quadratic parameters, students were able to more effectively employ guess-and-check strategies because they knew what types of values to test for which parameters and could quickly adjust when their guess did not produce the desired result.

The strength of this activity occurs when it is used as a review or skill refinement as opposed to an introductory lesson on quadratic parameters. Students with some experience were able to test appropriate values and make adjustments to translate or dilate the graph as desired, whereas students with little prior knowledge of parameters were forced to employ guess-and-check strategies. Although such strategies can be productive, there are better activity designs for introductory lessons. Furthermore, there is a danger of using an activity with direct manipulation of the equation when the students do not have sufficient prior knowledge to understand the relationships of the parameters, the order of operations, and the nuanced differences between operation and the sign of the value.

## AFFORDANCES OF THE DIFFERENT PARAMETER EXPLORATION ACTIVITIES

Let us return to Ms. Rose. With so many quality choices, how should she decide which is best for her students? Activity choice should be directly tied to the mathematical goal of the lesson. Consider the Quadratic Marbleslides activity; we saw that students with some prior experience were able to use this activity as a means of reinforcing their understanding of the effect of parameters  $a$ ,  $h$ , and  $k$  on the parent function. The students with no prior experience were able to use a guess-and-check strategy with some success; however, they were unable to focus on connections. If the goal is for students to make such connections, then their focus could be more effectively placed on the effect of each parameter in an activity that allows them to explore and make conjectures.

If Ms. Rose is introducing the effect of parameters for the quadratic function family for the first time, activities that promote exploration and conjecture are a better choice. Both Multiple Graphs and Quadratic Sliders drew students' attention to the variant and invariant features of the graph to investigate the effect of changing the value of the parameters. Specifically, the Multiple Graphs activity enabled students to compare many graphs at once and match the color coding back to the

parameter values to carefully consider the effect. One downfall is that students may not try enough of a variety of value types (e.g., positive/negative and integer/noninteger) to gain a complete understanding of the parameter's effect. On the contrary, the Quadratic Sliders activity forced students to try a variety of value types. However, the trade-off is that students are able to see the graph for only the current slider value. No record remains for what the graph looked like for each value students tried while dragging the slider. Both activities nurtured exploration and aligned well with goals that are focused on describing the effect of each parameter.

Given these differences, thinking about whether you want students to observe (e.g., take note of invariances), explore, review, or test conjectures when selecting a technological tool is important; this will inform the tool you select, how you design student interactions with the tools, and the questions you ask. You should be aware of the importance of how technology tools can be used strategically to support meaningful learning; unproductive use restricts students' opportunity to engage and reason with mathematics. When deciding between activities that promote exploration, such as Multiple Graphs and Quadratic Sliders, you should consider the ways that students will interact with the technology. For example, Quadratic Sliders encourages students to try non-integer values without prompting. However, Multiple Graphs provides students with recorded evidence of the effect of several different parameter values for ease of comparison (i.e., color-coded graphs), yet Quadratic Sliders afforded better comparison across parameters (e.g., after parameter  $h$  was introduced, parameter  $k$  was added to the equation and not introduced separately). The focus of this article was to explore how students with different prior experience engage with these specific activities, but you may use a combination of activities as well as intentional sequencing to offer students experiences that build on one another. In addition, if your goal is to have students make connections across not only symbolic and graphical representations but also tabular representations, you may add a dynamically linked table as well.

Given that it can be overwhelming to select an appropriate activity, we encourage you to try these activities with your students. Remember that you should use the mathematical goal for your students as a guide to weigh the affordances and constraints of the design style. —

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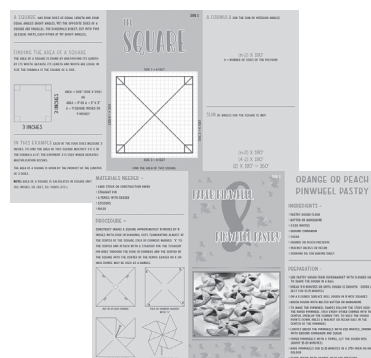
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