AMTE's 2021 NTLI Fellowship: Using a Framework to Teach Preservice Mathematics Teachers How to Professionally Notice within Technology-Mediated Learning Environments

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Abstract: Professional noticing of students' mathematical thinking in a technology-mediated learning environment is a complex, but incredibly important skill for preservice secondary mathematics teachers to develop. The purpose of this study is to examine how explicitly sharing a framework for professional noticing of students' mathematical thinking in technology-mediated learning environments and providing opportunities for practice influence pre-service secondary mathematics teachers' engagement with professional noticing in technology-mediated environments. A pre- and post- video-based assessment was used to examine changes in PSMTs' professional noticing as a result of engaging with the framework. The results of this study suggest that using this framework to support pre-service teachers' development of this skill has promise, especially related to coordinating students written and spoken mathematical thinking with their technology engagement.

Effective mathematics teaching positions students' thinking as the primary resource for making informed pedagogical decisions. The importance of professional noticing of students' mathematical thinking is emphasized in AMTE's *Standards for Preparing Teachers of Mathematics* (Bezuk et al., 2017): "well-prepared beginners commit themselves to noticing, eliciting, and using student thinking to assess student progress in understanding the mathematics and to adjust instruction in ways that further support and advance learning toward the intended learning goals" (p. 16). Professional noticing is a complex practice that becomes more challenging with the introduction of a technology tool. Research has shown that the ways students interact with a technology tool can provide insight to their mathematical thinking and learning (e.g., Dick & Hollebrands, 2011; Trouche & Drijvers, 2010). Thus, noticing student thinking in such a context requires paying attention to not only what students say and write but also the ways they engage with the technology.

As one-to-one device classrooms and open-source technological tools became more prevalent, it is vital to carefully consider the preparation of teachers to notice in such environments. This is particularly important, as research indicates that pre-service secondary mathematics teachers (PSMTs) have difficulty describing how technology tools inform students' mathematical understanding (e.g., Chandler, 2017; Wilson et al., 2011). Thomas et al.'s (2015) work provides evidence that providing teachers a framework to guide their professional noticing is a worthwhile strategy in non-technological contexts. Thus, we conjecture that a similar strategy would be helpful to support PSMTs' noticing in technology-mediated learning environments. The focus of this study is to investigate the ways in which PSMTs engage in professional noticing when using a framework specific to noticing students' thinking in technology-mediated learning environments.

Background Literature

Professional noticing (Jacobs et al., 2010) is defined as a set of often hidden pedagogical skills that includes attending to students' strategies, interpreting their understanding, and deciding how to respond on the basis of those understandings. The practice of professional noticing is a challenging practice for all teachers, particularly preservice mathematics teachers (e.g., Jacobs et al., 2010; Krupa et al., 2017; Monson et al., 2020). This practice is further complicated when students use technology as it requires the dual and interconnected attention to students' mathematical thinking and the ways they engage with the technology as they are learning. Our focus here is specifically the technological tools used in the teaching of mathematics.

Research has shown that student engagement with a technological tool is essential because technological tools mediate students' sense-making (e.g., Baccaglini-Frank & Mariotti, 2010; Dick & Hollebrands, 2011). Furthermore, the ways students interact with a technology tool can provide insight to their mathematical thinking and learning (e.g., Arzarello et al., 2002; Baccaglini-Frank & Mariotti, 2010; Trouche & Drijvers, 2010). Research has demonstrated that PSMTs struggle to explicitly coordinate students' mathematical thinking with their engagement with the technology. Lovett et al. (2019) studied how PSMTs notice students' mathematical thinking in a technology-mediated learning environment. They indicated that the majority of PSMTs either placed more emphasis on students' spoken and written responses than their technology engagement or they attended to students' spoken and written responses separately from technology engagement. These findings suggest that PSMTs have trouble coordinating attending to both students' spoken and written response, and technology engagement. A similar trend emerged for interpreting; PSMTs more frequently interpreted students' spoken and written responses separately from technology engagement. Additionally, they found some evidence of coordination between students' spoken and written responses and technology engagement. Such findings illuminate the potential benefit PSMTs may draw from explicit instruction on professional noticing within technology-mediated learning environments. Thomas et al. (2015) indicated that using a framework supported teachers' development of noticing; thus, it is our aim to use a framework that supports PSMTs' professional noticing of students' mathematical thinking within a technology-mediated environment.

A Framework for Professional Noticing in a Technology-Mediated Learning Environment

Dick et al. (2020) suggested a framework for Professional Noticing of Students' Mathematical Thinking in Technology-Mediated Learning Environments [NITE]. Grounded in research indicating that PSMTs struggle to coordinate students' work with technology with their spoken and written mathematical thinking (Lovett et al., 2019), NITE focuses on technology use when attending and interpreting students' mathematical thinking. Specifically, they suggest fine-grained layers of attention and interpretation building on the noticing framework proposed by Jacobs et al. (2010) to include technology engagement while acknowledging the interrelated nature of the three noticing components. The framework (Fig. 1) separates the attention to and interpretation of students' written or spoken responses from the attention to and interpretation of students' technology engagement to draw attention to the importance of considering and coordinating this aspect of students' strategies, and emphasizes the necessity of vertical coordination between the components as skills of attending and interpreting are interwoven (Superfine et al., 2017).

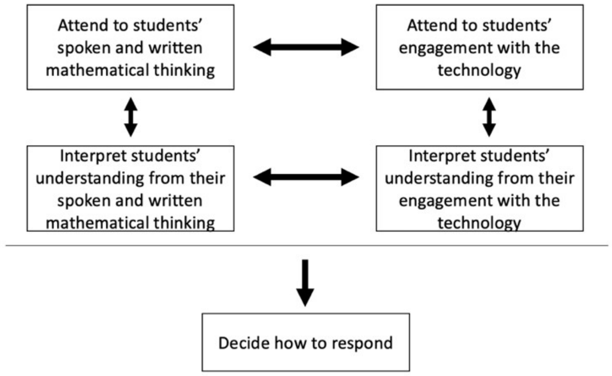

Figure 1. Professional Noticing of Students' Work in a Technology-Mediated Learning Environment

Context

To support PSMTs in their development of the skill of professional noticing in technology-mediated learning environments, we are in the process of designing curriculum materials that use the NITE framework and video cases of secondary students. Guided by the tradition of design research in curriculum development (e.g., Clements, 2007; Cobb et al., 2003), we are engaging in cycles of refinements based on feasibility and pilot testing of the materials as they are being developed. The first lesson introduced the NITE framework. PSMTs were provided with a 3-minute video clip of a pair of students engaged in a Desmos activity in which they used sliders to explore the parameters of a quadratic function in vertex form (i.e.,). This particular clip was focused on the students' exploration of . PSMTs watched the video and then were asked to attend to and interpret the students' thinking. Next, each of the components of the NITE and the importance of coordinating among them were discussed. PSMTs then watched the same clip again and refined their attend and interpret responses. Next, as a whole class we built upon the work of the small groups and created what we agreed were robust attend and interpret responses -fully coordinating what the students wrote and said along with the ways they engaged with the sliders to describe and make sense of their understanding of the effect the parameter has on the graph of the function. Finally, we wrapped up the lesson by sharing tips for effective attending and interpreting, and discussed how the practice of professional noticing is foundational to the 5 Practices for Orchestrating Productive Mathematics Discussions (Thomas et al., 2015) providing the bigger picture of what we were building toward across this course and others in the program.

At the time of this study, tasks for four modules had been developed and were being piloted (a total of seven modules will eventually be created). The design of these tasks was guided by our design principles for examining student practices in a technology-mediated learning environments (Dick et al., 2020). Mathematical topics included the concept of function, rate of change, and function families. In each case, PSMTs examined carefully selected video clips of pairs of students working together on technology-based tasks (e.g., Desmos, GeoGebra). The question prompts that accompanied the videos were guided by the NITE framework, often beginning with a focus on attending and interpreting students' thinking and later adding other practices that build on noticing (e.g., questioning, predicting, selecting, sequencing, and connecting). For example, one of the tasks was around a Desmos activity named *Function Carnival*. PSMTs had completed the Function Carnival task in a prior lesson. Here they watched a video of a pair of students working on a portion of the activity in which they were to draw the time vs. distance traveled graph for a car that was traveling along a curvy road. PSMTs collaborated in an interactive platform (i.e., GoReact) to tag moments in the video that they thought were mathematically important given the learning goals (Fig.2). Next, PSMTs were asked to write up their interpretations of the students' current understandings related to the learning goals (i.e., describing qualitatively the functional relationship between two quantities by analyzing a graph, describing qualitatively the functional relationship between quantities by analyzing a simulation of their interaction, and sketch a graph that exhibits the features of a function described through a simulation of the resulting action). While only a subset of the modules were ready to be piloted at this time, additional video-based tasks were used with the PSMTs throughout the semester in similar ways. These included small groups of students working on technology-based geometry and statistics tasks.

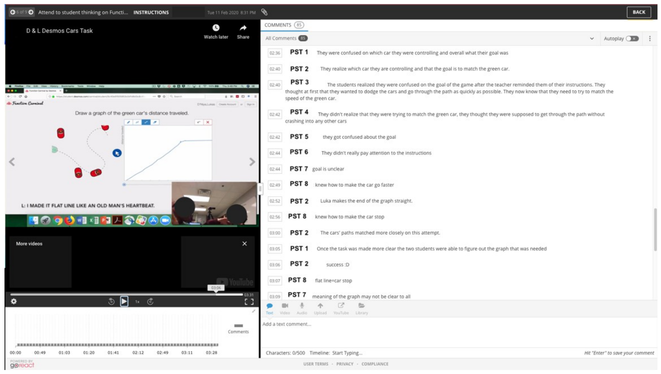

Figure 2. PSMTs Collaborate to Attend to Students' Mathematical Thinking on a Desmos Activity

The Study

As noted above, this study is situated within the context of a larger project that is creating a series of modules for mathematics teacher educators to use with PSMTs to examine secondary students' mathematical practices. These materials are being created using an iterative design and refinement process guided by the design principles for examining student practices in a technology-mediated environment (Dick et al., 2020). The data presented in this study was from the first iteration of the implementation of project materials. Specifically, this study focuses on the introductory module aimed at providing PSMTs a framework that connects research to practice and provides them a lens through which they can examine student work in technology-mediated environments.

Research has illustrated that PSMTs and practicing teachers often struggle with the skill of deciding how to respond (e.g. Dick, 2017; Jacobs et al., 2010), and the skill of deciding is even more complex for a task situated in a tool-mediated learning environment. Given this complexity, we believe it is first important to understand the ways in which PSMTs attend and interpret. We aim to answer the following research question: How does explicitly sharing the NITE framework and providing opportunities for practice during a single course influence PSMTs' engagement with professional noticing skills of attending to and interpreting students' mathematical thinking in technology-mediated environments?

The context for this study is a course focused on teaching secondary mathematics with technology at a university in the southeast United States that occurred in Spring 2020. The earlier portion of the course was carried out in person, and the latter was carried out remotely (a mixture of synchronous and asynchronous online settings) due to COVID-19. While the transition to an online setting in the middle of the semester did change the structure of some of the course materials, it did not change the content of the course materials or plans for data collection.

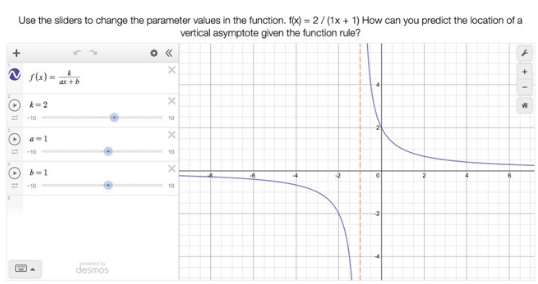

Figure 3. Snapshot of the Desmos Activity

PSMTs completed a written pre- and post- video case noticing assignment during the first and last weeks (respectively) of the course. The assessment included a video clip of students working on a task and written prompts for the PSMTs. The three minute video clip showed a pair of students engaged with a Desmos activity that enabled the dynamic exploration of the relationship between rational functions and vertical asymptotes (Fig. 3). Prior to completing the noticing assessment and in line with the design principles, PSMTs engaged with the Desmos activity as a learner so they would have context for the video clip they would later examine. After PSMTs watched the video clip, they responded to two noticing prompts (Fig. 4). One prompt focused on attending to the students' spoken and written mathematical thinking and engagement with the technology. The second prompt focused on interpreting the students' understanding of vertical asymptotes. The written responses to these prompts are the data sources for this study.

1. Describe how the students determined the location of the vertical asymptote for a rational function of the form $f(x) = \frac{k}{ax+b}$.
2. Interpret the students' current understanding of vertical asymptotes. Provide evidence from the video to support your claims.

Figure 4. Noticing Prompts

Participants

Eight of nine PSMTs enrolled in the course agreed to participate in the study. Unlike the typically female dominated population of US teachers (National Center for Education Statistics, 2020), our group of PSMTs was an even split (50% female, 50% male, 0% other). All of the eight participants, had completed at least through Calculus 2, are math majors and secondary mathematics education minors, and are preparing to be high school math teachers. The participants will be referred to using pseudonyms.

Data Sources and Analysis

Data included the PSMTs written responses to the prompts in the noticing assessment. We collected eight responses on the pre-noticing assessment and seven on the post-noticing assessment. The one PSMT who did not complete the post-assessment was excluded from analysis. Data was analyzed using a coding rubric (Fig. 5 and Fig. 6) designed based on the NITE framework and included four components: attending and interpreting students' spoken and written mathematical thinking and technology engagement. To develop the coding rubric, the research team identified the mathematically significant details of the students' exploration in the video clip according to the four components. Similar to the coding scheme used by Jacobs et al. (2010), the rubric included three levels (i.e., lacking, limited, and robust) on each of the four components in the framework. Since research indicates that the skills of attending and interpreting are interwoven (e.g., Superfine et al., 2017), we coded PSMT responses across both prompts (e.g., if a student attended in the interpret prompt, it was coded as attending). The coding rubric was refined using a broader cross-institutional sample of responses from the larger project. We used an iterative process of refinement to achieve consistent application of the codes by the entire research team (DeCuir-Gunby et al., 2011). Using the final coding rubric, the research team assigned two researchers to individually code the data and all discrepancies were discussed by the entire team until reconciled.

Verbal Spoken & Written Mathematical Thinking	Robust	• Students recognize that only *b* and *a* affect the location of the vertical asymptote • Students recognize that rather than $\frac{b}{a}$ it is the opposite ($-\frac{b}{a}$) **and** explain it by saying it is one of those "weird flippy thingys that graphs do" or some paraphrase of this student language • Students write x = *-b/a* (or *-x = b/a)* as the location for a vertical asymptote
	Limited	• Two of the bullets above
	Lacking	• One or none of the bullets above or incorrect
Technology Engagement	Robust	• The students change each of the sliders and watch / discuss how they do (or do not) affect the location of the vertical asymptote • The students set ***K***=0 so that the function is no longer rational **OR** they state that k has no effect • The students **change** ***b*** and then **move** **a** and notice both have something to do with the asymptote. (e.g. "I think whatever ***b*** is is your vertical asymptote, but it has something to do with **a** too.") • They test the conjecture with multiple values of ***a*** and ***b*** on the sliders • Once they conjecture that the vertical asymptote is located at ***x = -b/a*** **AND** then test the conjecture with **additional** values of ***a*** and ***b*** on the sliders
	Limited	• Three or four of the bullets above • Exception: If one of the 3-4 includes noting that k=0 and they are no longer looking at a rational function, it should be scored as robust.
	Lacking	• Two or fewer of the bullets above or incorrect

Figure 5. Rubric for Attending to Students' Spoken and Written Mathematical Thinking and Technology Engagement

Verbal Spoken & Written Mathematical Thinking	Robust	• The students understand that the location of a vertical asymptote can be determined by ***x* = -*b*/*a*** (**-*x* = *b*/*a***) (i.e., they have a procedure for locating the vertical asymptote) • The students have not yet connected their rule why it makes sense in the context of a rational function (they have not connected ***a*** and ***b*** to the denominator of the rational function and the fact that it can't be 0 **OR** to continuity). • The students have not yet connected their rule to setting the denominator of the function equal to 0 to solve to explain why the vertical asymptote is located at ***x* = -*b*/*a*** rather than ***x* = *b*/*a***
	Limited	• Two of the bullets above
	Lacking	• One or none of the bullets above or incorrect interpretation
Technology Engagement	Robust	• The way the technology allowed them to just see the asymptote and not tie it to rational function (the function is not rational when they are determining and testing their rule) - connected to rational functions being undefined • The way the technology allowed them to test and make sense of the "weird flippy thing", specifically related to the negative value • The way that they used the sliders to change and make sense of the location of the asymptote, but have not explained what a vertical asymptote is (connecting to continuity or why the denominator matters)
	Limited	• Two of the bullets above
	Lacking	• One or none of the bullets above

Figure 6. Rubric for Interpreting Students' Spoken and Written Mathematical Thinking and Technology Engagement

Findings

Through comparison of the rubric levels (i.e., lacking, limited, robust) of the pre- and post-assessment, we documented level changes of PSMTs' engagement with professional noticing of students' mathematical thinking in technology-mediated environments. We discuss the findings (Tab. 1 for summary of results) according to the four components of the NITE framework in the rubric: attending and interpreting students' spoken and written mathematical thinking and technology engagement.

	Attend		Interpret	
	Spoken and Written Mathematical Thinking	Technology Engagement	Spoken and Written Mathematical Thinking	Technology Engagement
Remained robust	1	0	0	0
Improvement in level	3	3	0	1
Same level	2	4	3	6
Decline in level	1	0	4	0

Table 1. Summary of PSMTs' change in coding level from pre- to post-noticing assessment.

Attention to Students' Spoken and Written Mathematical Thinking

In regard to PSMTs' attention to students' spoken and written mathematical thinking, three of the seven PSMTs improved from either lacking or limited to robust, one maintained a score of robust, and two PSMTs maintained a score of limited. As an example of the improvement from lacking to robust, we share Naomi's responses on the pre- and post-assessment. On the pre-assessment Naomi (coded lacking) stated:

> They started by setting a to 0 and moved b and then set b to 0 and moved a. By doing that they finally figured out that a and b are closely related when figuring out the vertical asymptote. … Eventually they realized that to find the vertical asymptote they got the equation of a/b and switch the sign.

Naomi did identify that the students arrived at a formula, but incorrectly identified the formula as divided by instead of negative divided by . Furthermore, Naomi did not explicitly discuss how the students determined that only and affect the location of the asymptote, nor did she discuss the language used by the students. This starkly contrasted with Naomi's post-assessment (coded robust):

> They set 'k' to 0 and moved both 'a' and 'b' to determine that the asymptote moves when sliding the slider negatively and positively. By moving the sliders 'a' and 'b' they came to realize that 'a' is half of 'b' and that determines the location of the vertical asymptote. They determined that the vertical asymptote is the variable 'b' divided by the variable 'a'. … Then they came to the conclusion that the sign in front of the

> vertical asymptote is opposite sign of the division when they mention that they think "it's one of those flippy thingys". The students current understanding of vertical asymptotes is that it is variable 'b' divided by variable 'a' and the sign flipped.

On the post-assessment, Naomi correctly attended to all three details of the students' mathematical thinking and identified the formula that the students produced, took note of the students' language, and addressed that only and affect the location of the asymptote. PSMTs who showed improvement consistently demonstrated more detailed responses on the post-assessment; however, there was no consistent pattern of which details PSMTs included on the post-assessment but did not include on the pre-assessment.

Attending to Students' Engagement with the Technology

When attending to technology engagement, there were five details PSMTs had to include for their response to be considered robust. None of the PSMTs were able to capture all five details in their pre- or post-assessment. However, six of the seven PSMTs showed improvement from lacking to limited or maintained a score of limited in terms of attention to students' engagement with the technology. There were three PSMTs that improved their scores from lacking to limited. Consider Taylor as an example of a PSMT who improved from lacking on the pre-assessment to limited on the post. On the pre-assessment she did not take note of when or how the students used the technology to refine their understanding of which variables affect the location of the asymptote, nor did she comment on how the students tested their conjectures using the technology:

> They knew from the graph that both a and b changed the vertical asymptote, it was just determining how they were connected. They figured out that it was b/a and then changing the sign which is the equivalent to setting it equal to zero and solving the equation, they just didn't know to solve the equation that way.

On the post assessment, Taylor explicitly noted how the students manipulated the sliders:

> At first the students didn't think you could determine the asymptote with those numbers alone, and also thought a and b did the same thing, while moving all the sliders. When moving the sliders, they said that k didn't move the vertical asymptote, but a and b did. Then moving only a and b sliders, one student said she thought it could be b/a because it wasn't exactly b or a. Then, the other student noticed that 2/-4 which was b/a was -1/2 and the vertical asymptote was at ½. So, they determined it was b/a by trying it multiple times with different numbers but giving it a negative which they referred to as "one of those weird flippy thingy's" "I would tell my friend to divide b/a=-x".

Notice how Taylor articulated how the students tested multiple values to refine their understanding. Even though Taylor did not attend to how the students changed and then moved to figure out that the vertical asymptote is determined by both and (bullet 3 on the rubric), she was the only PSMT to attend to the fact that the students continued to test their conjecture after they formulated a rule (bullet 5 on the rubric). Even though no PSMTs attended at a robust level, since Taylor was able to capture this detail, it is reasonable to conclude that PSMTs are capable of attending to technology engagement at the robust level.

Interpretations of Students' Spoken and Written Mathematical Thinking

We did not see parallel growth in PSMTs' interpretations of students' spoken and written mathematical thinking as we did in their attending. Instead, three PSMTs maintained a score of limited while the remaining PSMTs regressed. PSMTs did interpret students' understanding of the location of the vertical asymptote (i.e.,); however, continued to struggle interpreting the students' understanding in terms of rational functions and/or setting the denominator equal to zero (Fig. 6). While it is disheartening that four students regressed in their interpretations, three of the PSMTs' responses were considered robust on the pre-assessment. Olivia's response typified these PSMTs' responses that included all three details:

> The students seem to have a good understanding of how to calculate the location of a vertical asymptote. … However, there are two concepts regarding vertical asymptotes that they still need to understand. They knew from experience that the answer was the opposite sign of what you would think initially, but they were unsure of exactly why... Another concept that may be absent from the students' understanding of vertical asymptotes is why the location is only dependent on a and b. Based off the video, you cannot say for sure that they know when the denominator in the function is zero, the value is undefined. Mia and Evelyn understand how to find vertical asymptotes, but not why they exist.

Although we do not know why these PSMTs regressed in their interpretations, we know that it is possible for PSMTs to interpret the students' mathematical thinking at a robust level.

Interpreting Students' Engagement with the Technology

In terms of interpreting students' technology engagement, there was little evidence of growth. Five PSMTs maintained a score of lacking and one a score of limited. Logan was the only PSMT to demonstrate improvement from a score of lacking to limited. On the post-assessment Logan was able to connect the language to how the technology helped the students to their understanding of vertical asymptotes:

> The students point out that they "think [the vertical asymptote] is b divided by a because -2 (b) divided by 4 (a) is – ½." Although the arithmetic is correct, the numbers do not quite match what is shown on the graph. The vertical asymptote is ½ rather than – ½ but the students point out that they "think it's one of those flippy thingies"

Common among the PSMTs whose interpretations were coded as lacking was that they either included only an interpretation of the students' use of the technology to make sense of the "weird flippy thing" or did not mention the students' engagement with the technology at all.

Discussion and Implications for Teaching

Findings from this study suggest that using the NITE framework did support PSMTs' professional noticing of students' mathematical thinking within a technology-mediated environment; this is consistent with Thomas et al.'s (2015) findings in non-technological environments. However, it did not seem to support PSMTs on all components of the framework in the same way. For example, results show that PSMTs can develop the skill of attending to technology engagement in coordination with students' spoken and written mathematical thinking—and the NITE framework appears to support them in their engagement with this work. Despite this encouraging finding, there was not a parallel growth in the PSMTs' skill of interpreting technology engagement in coordination with the students' spoken and written mathematical thinking. Our findings are consistent with the literature indicating that interpreting students' mathematical thinking is a difficult practice (e.g., Jacobs et al., 2010) as PSMTs' interpretations are dependent upon attending as the skills are interwoven (e.g., Superfine et al., 2017). In fact, when compared to other studies that focused on developing PSMTs' noticing, these findings suggest the skill of interpreting in a technology-mediated environment is even more challenging than when technology is not involved.

While we cannot fully explain the lack of growth related to PSMTs' interpretations, we share a few potential reasons for this result. It is possible that the PSMTs' content knowledge of rational functions influenced their interpretations, which would be consistent with prior research connecting noticing to content knowledge (e.g., Dick, 2017; Dreher & Kuntze, 2015; Sanchez-Metamoros et al., 2015). While the PSMTs all had extensive prior experiences with rational functions, they were not explicitly discussed in this course. As a result, it is possible that the PSMTs' interpretations of this particular video example is not representative of their skills as a whole. Another possible interpretation is related to PSMTs' visions of high quality mathematics instruction, which also influences the practice of noticing (e.g., Sherin et al., 2008; Sherin, 2014). If PSMTs hold the belief that teaching should focus on imparting procedural knowledge, it follows that those PSMTs would focus on the students finding a rule for locating the vertical asymptote instead of focusing on how the students are or are not grappling with the conceptual meaning. Lastly and importantly, the COVID-19 pandemic undoubtedly influenced the PSMTs. There were multiple PSMTs that had robust interpretations of the students' written and spoken work in the pre-assessment that regressed in the post-assessment. As the skill of interpreting demands a heavier cognitive load than attending, it is possible that the PSMTs' emotional loads from the pandemic (e.g., stress and anxiety) interfered with their available cognitive load to interpret to the best of their ability. Specifically, we noted that the PSMTs' responses on the post-assessment were less detailed than the pre-assessment.

Implications for the Design of the Project Modules

Based on the findings from this study, several important refinements to the design of the assessment and modules have been made. Given that noticing is a complex skill that develops over time (Jacobs et al., 2010) the fact that we only piloted a subset of the module materials, yet we saw growth in PSMTs' skill of attending is actually quite encouraging. Findings here suggest that PSMTs may need more opportunities to engage with examples of students working in technology-mediated learning environments and that PSMTs' early experiences may benefit from careful scaffolding using the language of the NITE framework. As a result, we have refined our introductory module to include an explicit example of what robust attention to and interpretation of students' written or spoken responses and of students' technology engagement entails. We also revised the activities to more carefully scaffold PSMTs' noticing development throughout the modules. For example, early activities asked the PSMTs to attend to and interpret students' spoken and written mathematical thinking and attend to technology engagement separately. Activities later in the module combine students' spoken and written mathematical thinking and technology engagement for the skill of attending and for the skill of interpreting to emphasize the horizontal coordination that should take place (Fig. 1). In addition, we have included questions that require PSMTs to consider how interpreting students' mathematical thinking is dependent upon the ability to attend and how these skills are interwoven (e.g., Superfine et al., 2017). We anticipate that the explicit language in the prompts, the inclusion of scaffolds, and additional experiences will benefit PSMT growth.

Finally, the pre- and post- assessments have been updated to explicitly reflect the language of the NITE framework. The original attending prompt in the pre- and post-noticing assessment stated, "Describe how the students determined the location of the vertical asymptote for a rational function of the form ?". We originally used the term "describe" because PSMTs had not been introduced to the term "attend" prior to being introduced to the framework. Findings suggest PSMTs might not have read "describe" as equivalent to being asked to "attend", so the prompt has been changed to read: "Attend to (i.e., describe in detail) how the students determined the location of the vertical asymptote for a rational function of the form ". We hope that this will not only clarify expectations but also support PSMTs' recognition that any time they are asked to describe students' thinking (regardless of the terms used in the prompt) it should be at the level of detail described in the attend components of the NITE framework.

Conclusion

Professional noticing of students' mathematical thinking in a technology-mediated environment is a complex, but incredibly important skill for preservice teachers to develop. The results of this study suggest that using the NITE framework to support PSMTs' development of this skill has promise, especially related to coordinating students' attention to written and spoken mathematical thinking with their technology engagement. We hope that with further refinements to the project modules based on the findings of this study, we see similar results related to PSMTs' interpretations.

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