Optimized SVM constellations for SDM fibers

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***Abstract*—Stokes vector modulation (SVM) can be used together with multimode and mulitcore fibers. We study various algorithms for optimal geometric constellation shaping and bit-to-symbol mapping in the generalized Stokes space. We evaluate the performance of these constellations for optically-preamplified direct-detection receivers.**

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I. Introduction

A recently-proposed, promising modulation format for inter-data-center networks (IDCNs) is Stokes vector modulation (SVM) [1]. SVM allows to increase spectral efficiency compared to conventional binary intensity modulation while still using relatively inexpensive direct-detection (DD) receivers [1].

It is possible to generalize SVM for use with multimode or multicore fibers (space division multiplexed (SDM) fibers) [2]. We refer to this modulation technique here as *mode vector modulation* (MVM) to distinguish it from the conventional SVM over single-mode fibers (SMFs). MVM uses the spatial degrees of freedom of SDM fibers jointly, rather than independently, to transmit information from the transmitter to the receiver.

MVM constellations can be represented as points in a generalized Stokes space with $N^2 - 1$ dimensions, where N is the total number of polarization and modal degrees of freedom of a multimode or multicore fiber [3]. The optimization of the positions of the constellation points in the generalized Stokes space and the allocation of bits to symbols to minimize the error probability of MVM links are non-trivial tasks that were not tackled before in the literature.

In this paper, we design M-ary MVM equienergetic constellations with points on the unit Poincaré hypersphere. Geometrically-shaped constellations are obtained by numerical optimization of mathematical/physical analogs [4], [5] using the method of gradient descent [6]. Finally, bit-to-symbol mapping is optimized by using simulated annealing. Both optimizations yield significant improvement over naively generated constellations and encodings.

II. Theoretical Model

Given a constellation $(|s_i\rangle)_{i=1}^{M}$ of N-dimensional unit Jones vectors (which we refer to as a (N, M)-MVM constellation), we take d_{ij} as the corresponding Euclidean distance in the generalized (N^2-1)-dimensional Stokes space. Generalizing the *Thomson problem* [5], we define the potential function $U = U\left((|s_i\rangle)_{i=1}^{M}\right) := \sum_{i=1}^{M-1}\sum_{j>i}^{M}\frac{1}{d_{ij}}$, which is proportional to the Coulombic potential energy between identical point charges. This gives us a differentiable function, which we can minimize by means of *gradient descent* [6], noting that we must renormalize each $|s_i\rangle$ to unit length after each step.

Starting from a pseudo-randomly-generated (or a given geometrically-constructed) collection $(|s_i\rangle)_{i=1}^{M}$, we iterate gradient descent as above to obtain numerically optimized constellations. While this is a non-convex optimization problem, we observed good convergence performance across a wide range of initial conditions.

Having geometrically optimized a constellation $(|s_i\rangle)_{i=1}^{M}$, we turn to optimizing the assignment of bits to each symbol in order to minimize the bit error probability. For cubic lattices in any dimension (e.g., pulse amplitude modulation (PAM) and quadrature amplitude modulation (QAM)), this can be achieved by using *Gray coding* [7], which has the property that adjacent symbols in the constellation differ by a single bit (Figure 1a). More complex constellations (e.g., the spherical 16 point constellation of Figure 1b or MVM constellations as in Figures 1c-1d) do not admit Gray coding.

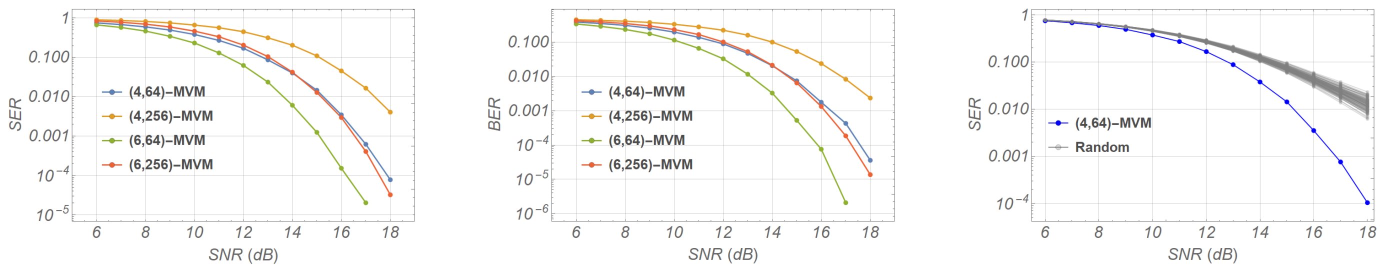
We assume that, in the exclusive presence of additive Gaussian noise with zero mean and standard deviation σ introduced by the direct-detection receiver, the conditional probability $P(|s_i\rangle \,|\, |s_j\rangle)$ of a symbol error whereby i is detected when j was sent depends asymptotically on d_{ij} as $P(|s_i\rangle \,|\, |s_j\rangle) \propto d_{ij}^{-1}\exp\left(-\frac{d_{ij}^2}{8\sigma^2}\right)$. Further letting b_i denote the bit sequence assigned to $|s_i\rangle$ and using $h(b_i, b_j)$ for the *Hamming distance*, we define the objective function

$$\xi = \xi\left((|s_i\rangle)_{i=1}^{M}, (b_i)_{i=1}^{M}, \sigma\right) := \sum_{i=1}^{M}\sum_{j\neq i}^{M}\frac{h(b_i, b_j)}{d_{ij}}\exp\left(-\frac{d_{ij}^2}{8\sigma^2}\right). \tag{1}$$

Finding a coding $(b_i)_{i=1}^{M}$ that minimizes ξ for a noise level σ thus serves as a proxy for minimizing the bit error probability for a given constellation, even when receiver noise is not the dominant noise type.

(a) (b) (c) (d)

Fig. 1: (a) An example Gray coding. (b) A spherical constellation with 16 points does not admit a Gray coding. (c–d) Two-dimensional projections of the $(4, 256)$-MVM constellation in Stokes space.



(a) Symbol Error Rate (b) Bit Error Rate (c)

Fig. 2: (a–b) Monte Carlo simulation of Symbol Error Rate (SER) and Bit Error Rate (BER) vs. Signal-to-Noise Ratio (SNR) for various (N, M)-MVM constellations. (c) Monte Carlo SER performance of the optimized $(4, 64)$-MVM constellation against randomly generated constellations.

Given the rapidly scaling combinatorics inherent in finding a coding that minimizes ξ, we turn to numerical minimization. *Simulated annealing* is a class of algorithms suitable for such combinatorial minimization problems [8]. By stochastically accepting or rejecting new candidate solutions, such algorithms are able to more fully explore the search space and avoid becoming trapped near local extrema of the non-convex objective function.

III. Results and Discussion

In order to validate our constellation designs and bit encodings, we use a Monte Carlo method to evaluate the performance of direct-detection MVM links dominated by *amplified spontaneous emission* (ASE) noise. Figures 2a–2b show the *symbol error rate* (SER) and *bit error rate* (BER) as a function of *signal-to-noise ratio* (SNR). Across all (N, M) combinations tested, our optimized constellations displayed significant SER performance gains over randomly-generated constellations. Figure 2c shows one such comparison for the $(4, 64)$-MVM constellation. Similarly, our optimized bit encodings show significant BER performance gains when compared against randomly-generated codings. We further observed a strong correlation between the number of simulated annealing iterations and the encoding performance.

For a desired SER or BER, we observe a 1–2 dB SNR penalty incurred by quadrupling the number of points and a 2–4 dB SNR gain by increasing the number of modes. Our optimized MVM constellations achieve a threshold 1% BER performance for SNRs in the 13–17 dB range.

IV. Summary

We generated optimized MVM constellation families with approximately equidistant points on the unit Poincaré hypersphere and evaluated the performance of the optically-preamplified MVM DD receivers using these constellations. Simulation results show the superiority of MVM compared to conventional SVM over SMFs.

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