Examining the Distribution of Authority in an Inquiry-Oriented Abstract Algebra Environment

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Recent large-scale research points to evidence of inequitable outcomes between women and men in inquiry-based mathematics education (IBME) courses. One explanation for differing outcomes may be that women are having different experiences in these courses than men. Specifically, the ways in which students garner mathematical authority and leverage their authority in both whole class and small group contexts may differ between students. Framing authority as a relation between people determined by their mathematical activity, we present an exploratory analysis of the authority relations between students as they engage with tasks developed for an IBME abstract algebra course. Findings suggest there are indeed discrepancies in the amount of time students have mathematical authority. We present examples of situations in which discrepancies are visible to begin examining the underlying nature of these discrepancies.

Keywords: Abstract algebra, inquiry, mathematical authority

Many researchers have positioned inquiry-based mathematics education (IBME) (Laursen & Rasmussen, 2019) as means to support student-centered undergraduate classrooms. These paradigms share commonalities: “student engagement in meaningful mathematics, student collaboration for sensemaking, instructor inquiry into student thinking, and equitable instructional practice to include all in rigorous mathematical learning and mathematical identity-building" (p. 129). Until recently, the first three pillars have dominated research in this paradigm with equity largely underexplored at the undergraduate level (Adiredja & Andrews-Larson, 2017). Consider the context of abstract algebra—there are several research-based curricula (e.g., Larsen, et al., 2013) and studies exploring student and teacher activity in inquiry classes (e.g., Fukawa-Connelly, 2016). Such work reflects a positive reality: students are engaging in meaningful mathematics. However, recent large-scale research points to differing outcomes for students such as Laursen, et al. (2014) and Johnson et al.’s (2020) contradictory results when looking at the outcomes for women and men in these types of courses.

It is possible that students are having different experiences in these courses in terms of their ability to leverage their authority in both whole class and small group contexts. Possessing mathematical authority can aid students in developing deeper conceptual understanding of concepts and developing more productive identities and dispositions toward mathematics (Langer-Osuna, 2017). In addition, mathematical authority can result in improved motivation and engagement because of increased choice and responsibility. However, typical mathematics classrooms can possess structures that remove authority from students (Wagner & Herbel-Eisenmann, 2009). Although IBME courses may differentiate in several ways from these typical classrooms, IBME courses may still possess structures that serve to restrict or remove mathematical authority from students. While the restriction of authority is typically framed as teachers (or other institutional figures) removing authority from students by limiting their opportunities to engage with mathematics, it is possible that students themselves may restrict authority from others. Given that IBME courses provide more opportunity for students to engage with mathematics and with each other, a related question then is whether and how student-to-student authority relations might work to restrict authority from some students.

While authority relations have been analyzed to some degree in the K-12 setting (e.g., Amit & Fried, 2005; Cobb, Gresalfi & Hodge, 2009), they remain largely implicit in the undergraduate setting. As we conjecture that authority relations may provide an explanatory mechanism for differing student experiences in IBME classrooms, developing a theory of authority relations in the advanced undergraduate setting is important. Thus, in this paper, we focus on a group of four students engaged in task-based interviews designed to mimic an IBME classroom in order to carefully analyze their authority relations. We focus on a lab setting because (1) some of the highly studied curricula at this level have been developed based on student thinking/activity in similar settings (e.g., Larsen, et al., 2013) and (2) the lab setting provides opportunity to explore in depth the ways that students are interacting in a group without extraneous classroom influences. This exploratory analysis focuses on the following research question:

How do mathematical authority relations manifest among participants as they engage with abstract algebra tasks?

Theoretical Framing

A key distinction of authority within a mathematics classroom is between pedagogical and mathematical authority (Wilson & Lloyd, 2000). In this paper, we primarily focus on the latter. In other words, we focus on describing authority that is derived from engaging with mathematics rather than sources of authority that are perhaps institutional in nature (e.g., the pedagogical power of a teacher to determine a discussion topic). We assert that authority is visible by observing people's actions. Symbolic interactionism describes relationships between individuals and asserts that communication is how people make sense of and act upon their world. We adopt the theoretical lens of symbolic interactionism and use Goffman's (1981) notion of authorship and animation to describe authority relations as interactions between people and the activities in which they are allowed to take part. In this paper, mathematical authority is defined as a dynamic and negotiated relationship between people (or groups of people, or organizations, etc.) where one party defers to another in a mathematical situation (Lambert, Hicks, Koehne & Bishop, 2019). This definition stands in contrast to definitions of authority that rely on characteristics or immutable traits of an individual, such as perceived status due to age or possessing a PhD, and definitions that consider authority to be an attribute of an individual that may evolve over time (e.g., Engle et al., 2014). Instead, we conceptualize authority as a dynamic, moment-to-moment feature of interactions that is not carried beyond the situation in which the authority is manifested (i.e., a student with a "dominant" personality does not necessarily maintain mathematical authority from one class session to another.)

Table 1. The AAA Authority Framework for Authority Relations

Authorship	Refers to the significant contribution to the mathematical ideas under consideration.	Asks: Who is the primary source, or author, of the mathematical ideas?
Animation	Refers to who is publicly communicating or extending mathematical ideas.	Asks: Who is communicating mathematical ideas in the classroom?
Assessment	Refers to judgements/evaluations made about an idea.	Asks: Who is assessing mathematical ideas in the classroom and who is being assessed?

To operationalize authority relations, we utilize the AAA (read "triple A") framework (Lambert et al., 2019). AAA consists of three components of activity: authorship, animation, and assessment of ideas. Table 1 broadly outlines the AAA framework; we discuss specifics of the components of AAA in the methods below. Aligned with the AAA framework, we draw upon two concepts from Goffman's (1981) Forms of Talk: participation frameworks and footing. Participation frameworks offer an assignment of participation types to everyone involved in an event. In the context of AAA, the possible types of participation are the author, the animator, or the assessor of mathematical ideas. Footing refers to a participant's alignment with others within a participation framework as well as the obvious or subtle shifts in that alignment from moment-to-moment. Communication is then accomplished by attending to everyone's participatory status during an event. In the context of AAA, this amounts to identifying who holds the status of authorship, animator, or assessor from one event to another. In the next section, we describe how these concepts assist in coding our data.

Methodology

The data in this study were collected as part of a larger project focused on adapting student-centered teaching practices from the K-12 setting to the undergraduate setting. The data consists of two 2-hour long task-based interview sessions with a group of four undergraduate students (one woman and three men) at a large, public university in the United States. To begin analyzing our data, the first author segmented each interview by attending to footing: (a) shifts in participation structure, (b) shifts in the mathematical content focus, or (c) the introduction of a new task or activity. Three types of segments were identified: segments that emulated whole-class (WC) activity (i.e., everyone engaged in focused discussion with the interviewer facilitating), segments that emulated small group (SG) activity (i.e., participants worked in pairs; one or more participants worked without the interviewer present), and segments that were either not mathematical or involved independent work with no student interaction.

The AAA framework was originally developed to attend to the collective authority of all students (contrasted with the teacher) rather than the authority of individuals in a given moment. In other words, if one student authors an idea in a class discussion, then students collectively receive credit for authorship. In this paper, we adapt AAA to track authority for individuals. Each segment that was labelled as WC or SG was assigned codes of which individuals were the holders of authority for each of authorship, animate-speak, animate-scribe, and assessment. Authorship is based on opportunities to contribute which are constrained by the overall focus of a segment. Authorship involves establishing the focus or making a significant mathematical contribution to the overall focus of a segment. It is assumed that the interviewer will direct classroom activities and functions, such as assigning a worksheet or posing a question to the class, and thus, these activities do not count toward authorship.

The component of animation is divided among speaking and scribing activity. Speaking is the public oral communication of mathematical ideas. Speaking may include (a) revoicing another's idea, (b) expressing disagreement/confusion with another's ideas, (c) sharing new ideas/content, and (d) asking probing questions to elicit another's ideas. Scribing includes the communication of ideas which occurs publicly through (a) inscribing mathematical content on a board/document camera, (b) and gesturing to written work. A key aspect of scribing is board control, which often involves placing oneself at the front of the classroom.

Finally, an assessment is defined as an explicit statement that indicate the correctness, validity, or sensibility of a given mathematical contribution (such as a statement, argument, or strategy). Any disagreement or argumentation between students is considered as explicit assessment. Acts of assessment may or may not be followed up with a justification. Who makes a justification is not necessarily the same individual who makes the initial assessment and justification can be presented with varying degrees of quality.

For each segment, we determined which students held authority for the activities of authorship, animate-speak, animate-scribe, and assessment by assigning a code to each student that participated in each activity. Segments that were particularly difficult to code were brought to the group to resolve coding and interpretations. Once each segment was coded, the length of time for which each student was involved in a segment in which they held authority for each activity was determined for both of whole class and small group time. In the next section, we present the results of this analysis.

Results

There was a total of 69 coded segments across the interviews: 55 WC and 14 SG. Figure 1 below represents the lengths of time (in minutes) for which each of the participants successfully took the opportunity to participate across both interviews in both WC and SG contexts.

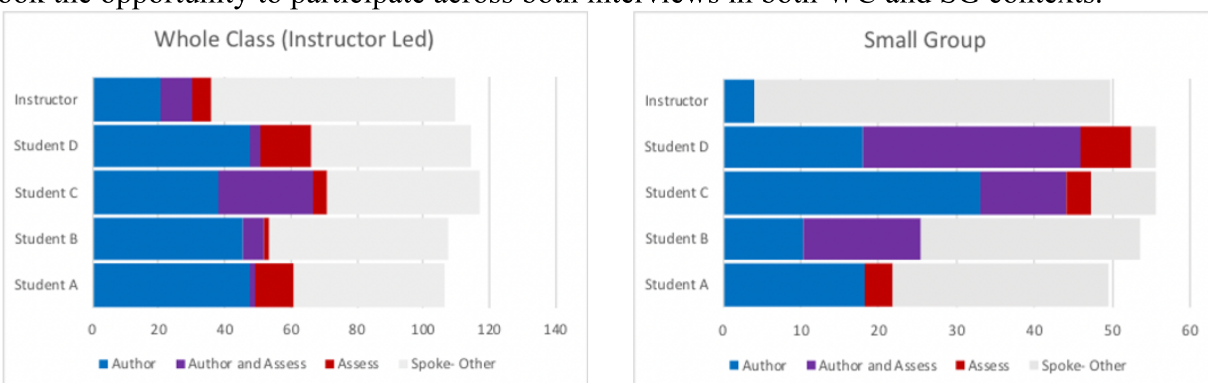

Figure 1. Lengths of time (in minutes) participants held authority.

There is clear unbalanced participation across all AAA activities. Across WC segments, Student C was moderately dominant in both of authorship and assessment. More striking, however, is the discrepancy in time spent as a holder of authority when shifting the participation structure to SG settings. In particular, Student A was a holder of authority for each of author, speak, and assess the least amount of time across all SG segments. In contrast, Students C and D spent the greatest amount of time as holder of authority for each of author, speak, and scribe, while Student D's participation in assessing increased dramatically from WC to SG. Thus, the dominant holder of authority for an activity shifted depending on the participation structure. In the sections that follow, we seek to begin disentangling the underlying reasons for such difference and explore possibilities for these disparities by examining a variety of excerpts.

Authoring Public Ideas

Within the two interviews, Students C and D were found to be the author of mathematical ideas more than A and B, especially during SG time. In contrast, Student A was found to be an author for the least amount of time compared to the other students across the total time. The following segment illustrates a particular example of Student A failing to achieve authorship despite making utterances that contribute to the mathematical conversation. In this segment, the teacher-researcher (TR) is prompting students to share out their thoughts on how they would begin proving a theorem:

TR:	So what are the types of things that we think about when we're going to prove something?
Student B:	What you're given.
Student A:	<concurrently with Interviewer> …assuming.
TR:	What you're given. All right, so let's go ahead and keep track of these things. Let's say ... "what you're given." <writes phrase on board> All right, and what are the things that we're given?
Student B:	G and H are isomorphic.
Student A:	< concurrently with Student B > …are isomorphic.
TR:	G and H are isomorphic. <*writes phrase on board*>. Okay.
Student B:	G is abelian.
TR:	G is abelian. <*writes phrase on board*>. Anything else?
Student B:	They're groups.
TR:	G and H are groups. <*writes phrase on board*> Anything else?
Student C:	That's about it.
Student D:	That's all that I see.

Although Student A does in fact contribute independent mathematical ideas in response to the interviewer's questions, Student A's contributions are either: (a) made concurrently with Student B who speaks up over Student A, or (b) spoken while the instructor is already acknowledging and making public Student B's contribution. Further, the instructor only animates Student B's contributions (through re-voicing and scribing on the board.) Because Student A never successfully puts forth an idea that is clearly recognized or acknowledged by the group, she does not receive authorship credit in this segment. It is also noteworthy that Students C and D's turns of talk at the end effectively halt further opportunity for authorship from others.

Refusing the Opportunity to Speak

The length of time in which students took opportunities to speak were seen to be relatively high for all students across all situations. This is to be expected due to the usual ease of which individuals can contribute by simply making a statement relative to the mathematics. However, we found it worth investigating small discrepancies to make sense of why a student may not have spoken in a segment. In the following segment, Students B and D have just finished giving a presentation at the board while Students A and C are asked to respond to the presentation with a comment and a question:

TR:	So, something that makes sense, something that you have a question about?
Student C:	Yeah, something that made sense, I really thought you did a great job explaining the definition well in tandem using the diagram. I understood everything you said, as you were showing the mapping from *a* ... is phi of *a* equal to phi *b*, *a* equal to *b* <*referring to 1-1 property*> you did everything very well in sync, with the instruction and explanation-wise. **Looks to Student A who silently nods her head in agreement**
TR:	Question?
Student C:	**Looks back at Student A who shakes her head** Just when we were talking about the operations, I thought it was really well-done, writing down the operations, what you were using, although this equation at the

bottom ... you all did switch back and forth of operation, or dot, or star, your saying of it.

Examples of students not speaking can appear in two ways: passively and actively. Passive non-speaking may occur when a student does not contribute verbally, but without acknowledgment or invitation by others. In contrast, this segment illustrates active non-speaking. Although, Student C immediately begins answering the first question posed to the pair, Student A refuses Student C's invitation (nonverbal cue by looking back at her) to both elaborate on Student C's initial comment, and to initiate a verbal response to the second prompt – indicating through gesture her choice to turn down the invitation.

Examining Who Assesses and How

Assessment can be tied to other authoritative activities, chiefly by taking up authority by speaking, but also through authoring and scribing. Who assesses has implications for who the main actors are within a mathematical discussion. However, the form of assessment may also influence who is taking up authority among the other activities. In this excerpt, the students are engaged in a debate following a prompt by the interviewer about making a modification to a diagram on the board that was scribed as part of a presentation by Students A and C (authors.) In particular, Student B has pointed out a flaw of the diagram not explicating the mapping of an element *a* in a group *G* to its image: φ(*a)*.

Student C:	We don't know for... in G, what *a* itself maps to. We know what *a* operation *b* maps to, but we don't know what *a* maps to.
Student B:	I don't know why not. You wrote "phi of *a*"
Student C:	This <*points at board*> is in G, though. Or, I'm sorry, this is in H, this is not in G. I only think ... because we only work in terms of elements in G when we're using them in tandem with another operation ... if that makes sense.
Student A:	But the diagram is confusing.
Student C:	Yeah. Well, I think ... I'm just saying I think the operation is necessary, because, unless we're working with a kernel. If we find a kernel, then we could say… *a* is in kernel, and it maps to *e*. We could say that for sure, but we don't know any single element that maps to another element in H.
Student B:	Even though we know that it's onto and one-to-one.
Student C:	Well, yeah, but we don't know the exact element. We know there is a kernel.

Note the variety of assessment that occur in this excerpt and who makes it: (a) assessments of an idea intended to significantly alter another authored idea, thereby establishing new authorship; (b) detailed assessments either for (or against) an authored idea put forth by oneself (or another); and finally, (c) assessments that either confirm or deny the contribution of another without making new contribution. The first type of assessment is primarily made by Student B who has begun the argument with Student C. The second of these types is done only by Student C as he is working to defend a diagram drawn during his presentation. His assessment of Student B's critique allows for him to maintain authorship, as well as give him an opportunity to scribe (by gesturing at the board.) Finally, Student A contributes a brief assessment that serves only to affirm up Student B's argument, thereby limiting her opportunity to receive authorship credit.

Discussion

Inquiry approaches to mathematics instruction often emphasize “supporting students in maintaining authority and ownership of the mathematics” (Kuster et al., 2018). However, the quantifier (all students) is often left implicit in these stated principles and the ways we analyze student activity in such settings. Through our analysis of just four students in a relatively open setting, we were still able to document discrepancies in authority relations. The AAA authority framework allowed for observation of which activities students had the opportunity take part in and provided a lens to further parse individual segments. In contrast with prior work using AAA, we examined authority at the individual student level illustrating important differences across our students including: differences in authority within small groups versus whole class segments, substantial differences in who assesses in small group, and differences in participation at a base level as reflected by speaking. The chosen examples of authority relations allow for insight into potential explanations for the differences in participation in authoritative activity: student interactions may prevent others from holding authority (either inadvertent or not), students may simply choose to not participate, and finally, authority may be limited or extended dependent on the type of assessment one partakes in. If a primary aim of IBME is for students to have authority and ownership of mathematics, then we suggest that it is vital to explore authority relations in these settings. If students leave these courses with different experiences (as suggested in Johnson, et al. (2020), and we argue further evidenced by this analysis), we may need to revisit the implicit “all students” quantifier to better understand the experience of “some students” in terms of authority relations.

The current form of AAA provided an insightful global view of authority. Based on our current analysis, we have several conjectured routes of adaptation to the framework that can further support its use in relation to students individually rather than collectively. One limitation of our current analysis is that authorship (and other activities) for a segment may vary in degree and contribution. Future research can move away from a binary approach to analyze both nature and degree (such as primary author for idea #1). Further, we conjecture that tracking particular mathematical ideas that are the focus of authorship, animation, and assessment can provide richer illustrations. This is particularly salient when looking at small groups where conversation often ebbs and flows in less directed ways than in whole class instruction. Finally, we note the important role invitations and barriers played in shaping authority relations. Future analysis can expand on the nature of these contributions.

The results of this work point to suggestions for improving equitable teaching practice. Gerson and Bateman (2010) argue that the limiting of teachers’ mathematical authority is one way of helping to ensure shared authority in the classroom between students and the teacher. However, teachers may also consider ways of utilizing their pedagogical authority to assist students in having more opportunity to author, speak, scribe, and assess. For example, the shutdown of further responses at the end of the first example presented above could have been mitigated by an intervention from the teacher-researcher through extending another invitation to respond and further authored ideas may have become public. Thus, striking a delicate balance between limiting and leveraging the types of authoritative power held by a teacher can assist in creating a more equitable distribution of mathematical authority.

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