Using Bézier Curves to Quantify Embryonic Curvatures: A Cross-Species Study

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Introduction: The body flexures that occur during embryonic development are vital for proper organ formation; they are necessary for proper heart and brain development. Body flexure is often only referred to qualitatively. It is important for a *quantitative* system to be developed for studying curvature. This paper will discuss the effectiveness of using Bézier curves to model the cervical and thoracic flexure of embryos. We present comparative results from three species: White leghorn chicken, Japanese quail, and Zebra finch.

Materials and Methods: The system of Hamburger and Hamilton [1] divides the incubation period of the chick into 46 stages and these are referred to as HH stages in this paper. We analyzed cervical (top arrow in Figure 1A) and thoracic flexure (bottom arrow in Figure 1A). Points along the dorsal curvature were selected using ImageJ and processed through MATLAB where third order cubic Bézier curves were fit to the data and control points of the Bézier curves were recorded (Figure 1A). The first and last control points, P0 and P3, are simply the first and last points in the data set. The second and third control points, P2 and P3, can be used to quantify the cervical and thoracic flexures respectively. Images of other species were acquired from previous studies [2,3]. These were at similar stages of development and analyzed in the same manner to compare body flexures across species (Figure 1C and D). Affine transformations were required to plot the curves in the same grid; Bézier curves are invariant under this process. The origin for all the transformations was the point of maximum curvature in the cervical region.

Results and Discussion: The Bézier curve was able to successfully capture the cervical and thoracic flexures at HH Stage 14. However, at Stage 15 the sharpness of the thoracic flexure was not appropriately fit and required a composite Bézier curve (results not shown). The increase in curvature in the cervical and thoracic regions between HH 14 to HH 15 is observable through the change in appearance of the Bézier curves over time (Figure 1B). The trajectory of the second and third Bézier control points reflect the increase in the curvature as they move farther away from the embryo (Figure 1B).

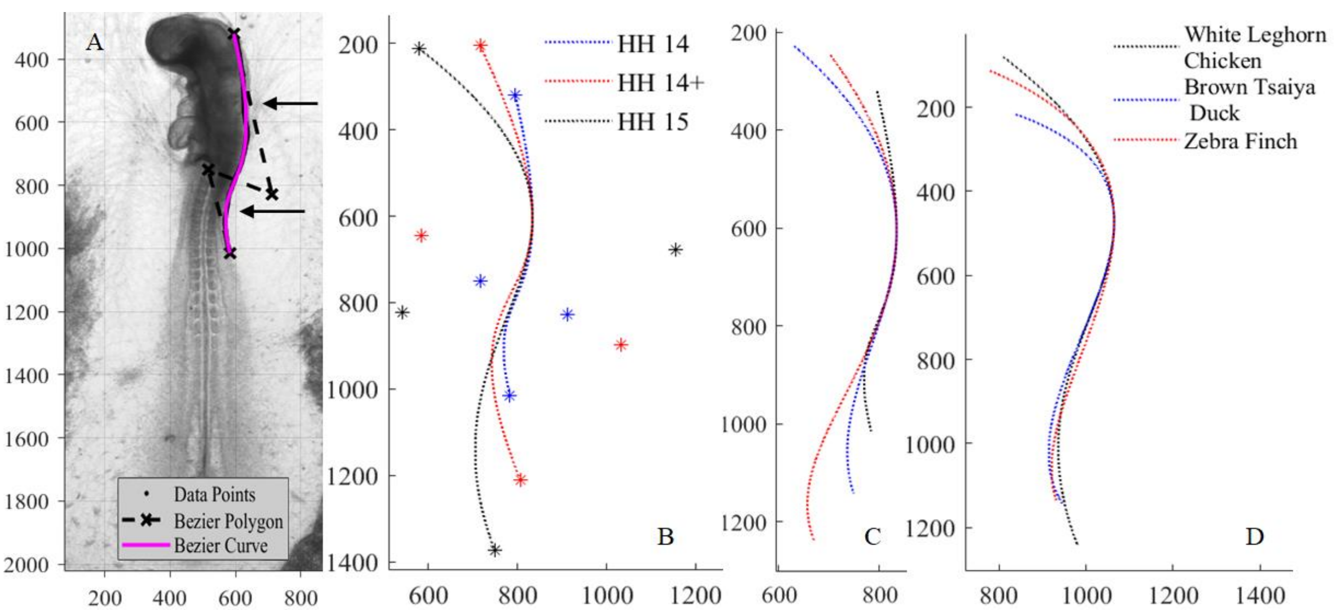

Figure 1: (A) Data points, Bézier curve, control points, and Bézier polygon for the chick at HH 14. (B) Progression of Bézier curves and control points from HH14 to HH 15 in the chick embryo. (C) Bézier curve for chick, duck, and finch at HH 14 and HH15 (D). The total length of the embryo is about 7mm and the axes represent pixel dimensions.

Conclusions: Our cross-species analysis shows the flexures are remarkably similar in character in avian embryos at the same developmental stage. Quantification of the body flexures enables a more complete understanding of embryonic development and the opportunity to quantitatively study flexure, a crucial morphogenetic event.

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