

SECONDARY MATHEMATICS TEACHERS' EFFORTS TO ENGAGE STUDENTS THROUGH ACADEMIC AND SOCIAL SUPPORT

Amanda Jansen
University of Delaware
jansen@udel.edu

James A. Middleton
Arizona State University
jimbo@asu.edu

Adi Wiegel
Arizona State University
awiegel1@asu.edu

Catherine Cullicott
Arizona State University
ccullico@asu.edu

Xiaoxue Zhang
University of Delaware
veraz@udel.edu

Gabriel Tarr
Arizona State University
Gabriel.Tarr@asu.edu

Kelly Curtis
University of Delaware
kbcurtis@udel.edu

Teachers' efforts to support students, both academically and socially, can play a role in how high school students productively engage with mathematics in the moment. To examine the connection between teacher support and student engagement, we conducted an exploratory mixed-methods study combining data from 20 high school classroom observations with student self-reports taken during the observed activity. Our findings indicate that when teachers provide academic support to their students during a lesson, they are also likely to provide social support. Higher teacher support of both kinds correlates with higher student self-efficacy, as well as social and cognitive engagement. Investigating relationships between observations of teaching and students' self-reports of engagement in-the-moment is a potentially revealing approach for uncovering engaging instructional strategies in secondary mathematics classrooms.

Keywords: High School Education; Affect, Emotion, Beliefs, and Attitudes; Classroom Discourse

Supporting secondary students' engagement with mathematics is a problem that can be addressed through instructional design. Typically, students' self-efficacy, enjoyment, and sense of the utility of mathematics decreases as they move through middle school and into high school, and the longer students are in high school, the more disengaged they tend to become (e.g., Chouinard & Roy, 2008). In this study, we explore the ways in which teachers' attempts to support students, both academically and socially, can play a role in how high school students productively engage with mathematics in the moment.

Mathematics Engagement

Engagement in school is students' expression of affect, beliefs about themselves, sense of belonging, and observable behaviors in the school setting (Jimerson, Campos, & Greif, 2003). Engagement is thus a complex meta-construct that simultaneously accounts for cognitive, affective, and socio-behavioral dimensions (Fredricks, Blumenfeld, & Paris, 2004). More specifically, mathematics engagement is the interactive relationship between students and the subject matter. It is manifested in the moment through expressions of behavior and experiences

of emotion and cognitive activity, and it is constructed through opportunities to do mathematics, as situated in both current and past experiences (c.f., Middleton, Jansen, & Goldin, 2017).

For students to learn mathematics, they must be engaged. For example, in a study of almost 4,000 middle school and high school students in Western Pennsylvania, researchers found that higher levels of cognitive, behavioral, emotional, and social engagement predicted students' course grades in mathematics (Wang, Fredricks, Yea, Hofkens, & Linn, 2016). According to Greene (2015), it is well-established in prior research that motivation constructs such as students' self-efficacy support students' engagement in ways that lead to learning. To understand how instruction can support learning, it is important to understand how instruction supports students' motivation and engagement. To do so, we must account how students experience engagement and how teachers' practice creates a context for students to engage.

Students' engagement is malleable and situated. Engagement is influenced by teachers' instructional practices in the moment and by the classroom climate (Anderson, Hamilton, & Hattie, 2004). Mathematical work, social interactions, and identity enactments differ from classroom to classroom. Understanding how teaching influences engagement requires attending to students' experiences, both collectively and subjectively, as they attend to important social and psychological features of their learning environment (Fraser, 1989).

Potentially Engaging Mathematics Teaching Practices

Mathematics instruction is considered to be stronger when teachers provide students with both *social support* for working together on content and *academic support* for accessing rigorous mathematical content (Shernoff et. al., 2016). Academic and social support can take a variety of forms. Academic support may include opportunities for sense-making and reasoning (Stein, Grover, & Henningsen, 1996), opportunities to make conceptual connections (Hiebert & Lefevre, 1986), pressing students to explain their thinking (Engle & Conant, 2002; Kazemi & Stipek, 2001), providing students with specific and detailed feedback (Stipek, Salmon, Givvin, Kazemi, Saxe, & MacGyvers, 1998), and opportunities to solve mathematics tasks in context (Koedinger & Nathan, 2004), or some combination of these. Social support may include motivational discourse with a focus on learning, positive affect, and encouragement of collaboration with peers (Turner, Midgley, Meyer, Gheen, Anderman, Kang, & Patrick, 2002), positioning students as competent (Cohen & Lotan, 1995; Gresalfi, Martin, Hand, & Greeno, 2009), accountability practices in the classroom (Horn, 2017), or some combination of these.

It is worth investigating whether and how these productive classroom practices support students' engagement at the high school level. Many of these prior studies were conducted in middle grades or upper elementary grades, and not all of these prior analyses of strong mathematics instruction were empirically linked to students' engagement. Thus, the research question that guides this study is: *What ways of providing academic and social support during instruction lead to productive engagement in-the-moment in secondary mathematics classrooms?*

Methods

This exploratory, mixed-methods study was conducted using the first year of data (pilot data) from a three-year NSF-funded study. In Spring 2018, we observed lessons and surveyed students in on-grade level ninth-grade mathematics classes in two states (one in the Southwestern region of the United States and one in the Mid-Atlantic region). We chose these locations because schools in these areas take different curricular approaches: integrated mathematics (Mid-Atlantic) and topics-based courses (Southwest). Mid-Atlantic courses were titled Integrated Math 1 or Integrated Math 2. Southwest courses were Algebra I or Geometry. The three Mid-Atlantic

schools implemented a block schedule with approximately 90-minute class periods. In the Southwest, the class periods were approximately 50 minutes long.

We gathered data from six schools – three from each state. In the Mid-Atlantic, the schools' demographics ranged from 12-34% low income, 25-60% white, 27-47% Black, and 6-21% Latinx. In the Southwest, the schools' demographics ranged from 76-94% low income, 1-6% white, 1-16% Black, and 77-96% Latinx.

Teachers were recruited by soliciting nominations of teachers from district curriculum supervisors and the mathematics coaches. We invited nominated teachers to participate in the study. The nine participating teachers averaged 10.6 years of teaching (min = 1 year, max = 26 years). All had completed a Master's degree or were in progress of doing so. One of the teachers in the Mid-Atlantic identified as Black, two Southwest teachers identified as Latinx, and one Southwest teacher identified as Asian. The rest of the participating teachers identified as White.

Teacher-selected, Potentially Engaging Activities

We observed 20 lessons. Lessons were selected by asking teachers to identify a lesson with an activity that they conjectured would be engaging for their students. The teachers provided written rationales for their conjectures about why this activity would engage their students. We video recorded the entire lesson, but we focused our analysis on these teacher-selected episodes. In the Mid-Atlantic, we analyzed 12 lesson episodes (four lessons from each of three teachers) and in the Southwest, we analyzed eight lesson episodes (two lessons from two teachers, one lesson from four other teachers).

These observations were paired with Experience Sampling Method (ESM) surveys (c.f., Shernoff, Ruzek, & Sinha, 2017) for the teacher-selected episode. Administering the ESM immediately after the teacher-selected activity allowed us to capture students' in-the-moment interpretations of their experiences during that activity. Each item was ranged on a Likert Scale from 1 to 5 points.

- *Interest* – 3-items: “I enjoyed the activity I was just working on,” “The activity I was just working on was personally relevant to me,” and “I think the topic covered in the activity I was just working on is interesting” ($\alpha = .783$).
- *Cognitive Engagement* – 3-items: “How hard were you trying during the activity you were just working on?”, “I was on task during the activity I was just working on,” and “How hard were you concentrating on the activity you were just working on?” ($\alpha = .782$).
- *Perceived Instrumentality* – 3-items: “I will use what I learned from the activity I was just working on when I grow up,” “I will use what I learned from the activity I was just working on in future courses,” “How I performed on the activity I was just working on will affect my future success” ($\alpha = .758$).
- *Self-Efficacy* – 3-items: “Rate how much you understand the math covered in the activity you were working on,” “I felt successful in the activity I was just working on,” and “I felt challenged by the activity I was just working on” (reverse-coded) ($\alpha = .548$).
- *Social Engagement* – 4-items: “I built on others' ideas during the activity I was just working on,” “I felt like my contribution was respected during this activity I was just working on,” “I felt supported by my teacher in the activity I was just working on,” and “I had the opportunity to ask questions during the activity I was just working on” ($\alpha = .709$).

Data Analysis

Our unit of analysis for coding observations was an episode consisting of a teacher-selected, potentially engaging activity. We analyzed episodes surrounding this activity using rubrics

(ranging on a 0 to 3-point scale) developed to document instructional practices which have the potential to support secondary students' engagement, as well as the prevalence and quality of those practices. Seven specific rubrics assessed the *academic support* that teachers provided and seven specific rubrics assessed *social support* that teachers provided:

Academic Support – Seven specific rubrics: (1) Sense-making and reasoning, (2) Connections and coherence, (3) Pressing students to explain, (4) Contexts of tasks, (5) Mathematical correctness, (6) Mathematical language precision, (7) Feedback.

Social Support – Seven specific rubrics: (1) Whole-class discussion, (2) Small group work, (3) Status-raising, (4) Motivational discourse, (5) Enthusiasm about mathematics, (6) Attention to students' lives, (7) Accountability and high expectations.

When coding, we applied the rubrics to 10-minute increments for each episode. At least two coders were assigned to each episode, and coders met to reconcile and resolve all coding disagreements. To assign an overall score for each rubric to an episode, we calculated the resolved rubric means across the 10-minute increments. To determine an overall academic support score for an episode, we took the mean of the mean resolved scores across the seven relevant rubrics across the 10-minute increments. We used this same procedure to create an overall social support score. To examine relationships between teaching practices and students' engagement in the moment, we began by computing Pearson correlations between the overall academic and social support rubric scores and the ESM scale scores across the 20 classes (316 students had complete data). Next, we examined the correlations between the specific observation rubrics, and the mean ESM scale scores for each class. Using these exploratory analyses, we selected a lesson that showed strong support practices, both academic and social, and high student engagement, as indicated by ESM responses. We then conducted an in-depth qualitative analysis of instruction during this lesson to explore what instruction looked like during a lesson when students reported high levels of productive engagement.

Results

Our analyses revealed that teachers who strongly enacted opportunities to provide academic support also strongly enacted opportunities to provide social support (see Table 1 for correlations), and both types of support correlated with student self-efficacy, though less so with other engagement variables. Although one might assume that a secondary teacher might be better at providing either academic support or social support rather than both, we found that most of these teachers tended to be either strong or weak at both. When these teachers provided strong opportunities for students to be supported academically, students reported experiencing higher self-efficacy and more productive social engagement. Similarly, when these teachers provided opportunities for students to be supported socially, students also reported higher self-efficacy.

Among the engagement scales, students' reports of their social engagement correlated with their cognitive engagement, their interest, and self-efficacy in the observed activity. Overall, these results suggest that when teachers supported their students along academic and social dimensions, students felt more confident, which related to their cognitive engagement and social engagement during the lesson.

Table 1: Relationships Between Teaching and Student Engagement (n=20 classes)

Observation Scores (Teaching)		Experience Sampling Method Survey (Student Self-Reports: In-the-moment Engagement)				
	Academic Support	Social Support	Interest	Cognitive Engagement	Perceived Instrumentality	Social Engagement
Social Support	.766**	---				
Interest	0.096	0.112	---			
Cognitive Engagement	0.385	0.356	.579**	---		
Perceived Instrumentality	0.028	0.011	.602**	0.184	---	
Social Engagement	.453*	0.44	.620**	.895**	0.199	---
Self-Efficacy	.591**	.559*	0.074	.505*	-0.02	.566**

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

Relationships Between Specific Observation Rubrics and Student Engagement

Table 2 displays relationships between the specific observation rubrics applied to each lesson and self-reports from students in those class periods on ESM scales. For academic support, efforts to support students' use and understanding of mathematical language significantly correlated with self-efficacy and social engagement, while feedback significantly correlated with self-efficacy. For social support, status raising significantly correlated with self-efficacy. Students appeared to feel more confident when teachers positioned students as competent in specific ways (status raising), helped students understand mathematical language, and gave students more detailed feedback focused on concepts. Students also participated more by sharing their thinking during class (social engagement), often by negotiating meaning, when the teacher developed mathematical terminology (mathematical language).

Table 2: Significant Relationships Between Specific Observation Rubrics and ESM Scales

Specific Observation Rubrics			
	Academic Support: Mathematical Language	Academic Support: Feedback	Social Support: Status Raising
Self-Efficacy	0.647**	0.559*	0.784**
Social Engagement	0.495*	0.164	0.376

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

When these teachers' instruction was observed to have higher mean scores for academic and social support appeared, students appeared to have opportunities to engage productively and feel confident. However, results from the specific observation rubrics (7 academic support rubrics and 7 social support rubrics) did not indicate many significant correlations with students' motivation and engagement on the ESM. These results suggest supporting engagement involves a more complex interaction of observed support variables than a single rubric can detect for a few observations. Academic and social support behaviors may interact in ways that are not easily categorized by our specific rubrics for academic and social support. Moreover, students' reports of their engagement may not be highly related to specific teaching support behaviors, but they may be related to more general patterns (hence the strong correlations of mean academic support and social support to our ESM scales). In other words, our ESM scales may detect general feelings about the class sessions and not make fine distinctions that relate to specific observed support behaviors.

Teaching that Supports Productive Engagement

It is unsurprising that secondary students have better opportunities to engage productively when teachers provide both academic and social support, but it is worth exploring what such support looks like in practice. Correlational analyses indicated that specific forms of academic support productively engaged students: opportunities for students to learn about precise mathematical language and receive specific feedback about concepts. A specific form of social support appeared to productively support engagement: teachers' efforts to explicitly raise students' statuses.

However, it is essential to investigate these instructional practices qualitatively to seek insights about enacting them. Our analyses suggest that when these teachers enacted specific academic supports that correlated with engagement (mathematical language and feedback), teachers also supported students' engagement by engaging them in sense-making and reasoning, making connections, and pressing students to explain, as these forms of academic support appeared to also take place when discussing mathematical language and when the teacher gave feedback. Additional social supports that qualitatively appeared to co-occur with status raising were having whole-class discussions that were collaborative and using motivational discourse with students.

We illustrate these features of instruction that appear to support productive engagement among secondary students by describing an episode from the third observation of Kathy's teaching in the Mid-Atlantic region. This episode of teaching was rated as having some of the highest academic and social support scores in the sample (2.07 and 2.00, respectively, out of 3) and Kathy's students reported some of the highest self-efficacy and cognitive engagement scores on the ESM in the sample (4.33 and 4.11, respectively, out of 5).

Kathy selected a card sorting task as a potentially engaging activity. Each card had a type of representation of systems of linear *equations* or systems of linear *inequalities*. The representations were symbolic, graphical, or in story problem form. The activity began with students working in groups for about nine minutes to sort the cards and compare how they were similar and different. They were expected to identify distinguishing features of systems of linear equations and systems of linear inequalities and record these ideas on sticky notes. After working in groups, the teacher led the students in a four-minute whole class discussion about the cards. Below, we share some examples of this first round of whole-class discussion during this activity.

Kathy: (Rings bell.) Bring in back in 5, 4, 3... Here's where we're going next: You have to make some decisions about which ones [cards] were inequalities and which ones were

equations. Those decisions lead to this question: How are systems of equations and inequalities the same or different? There's something you had to think about and you used that information to make your decisions in your groups. Now before I have you go and summarize that on your post-its, if you haven't done so yet, let's go ahead and make sure that we understand all eight of these correctly.

On a SmartBoard, Kathy wrote an E for equations and an I for inequalities on the projections of the cards as students identified them using “shout it out” (choral class responses). Kathy and her students had interactions that centered around their ideas. For example:

Student 1: Inequations.

Kathy: Did you say inequations? Inequations. It's one or the other. It's supposed to be an inequality. [Student 2] tell us why this is an inequality [symbolic representation].

Student 2: It doesn't have y>equals.

Kathy: Doesn't have that equals, y>equals. Which makes this one what, [Student 2]? [points at a different set of two equations] Equation or inequality?

Student 2: Equations.

This example illustrates that the purpose of the lesson was focused on making sense of mathematical language (inequalities and equations) and concepts (similarities and differences between systems of equations and inequalities). Students were expected to make connections between multiple representations, as they encountered each type of system as symbols, graphs, and story problems. The teacher then gave feedback using revoicing and redirecting. Students were pressed to explain (“tell us why this is an inequality”). This discussion included other similar moments.

Teacher: [reading the problem on the SmartBoard] Two CDs and 4 DVDs cost \$40.

Student 3: Equation.

Teacher: Whoa, it's like jeopardy! You didn't let me finish. 3 CDs and 5 DVDs cost \$55.

Why an equation here?

Student 4: Why a what?

Student 3: Because it's giving \$40, right?

Student 4: Because there's no budget.

Student 5: And the other, \$55.

Teacher: It's giving an exact amount for a certain number of items. It's not a budget, it's not a limit or restriction. It's exactly equal.

In this interaction, Kathy's feedback revoiced the students (“not a budget”) as an example of a constraint that could be a signal for an inequality rather than an equation, which helps position that contribution as valuable. She also gave more detailed conceptual feedback in response to the multiple student contributions about how the representation aligned with being an inequality, in contrast to a shorter evaluative statement or a statement about procedures.

Then, students had another short amount time to work in groups (four minutes) and reflect upon what they identified as important features in systems of linear equations versus linear inequalities. They wrote these reflections on post-its. The teacher brought the students back

together as a whole class for two minutes to debrief and highlight the features on the post-its that groups put on a blackboard. Below is an excerpt of how Kathy debriefed the group work.

Kathy: All right stop what you are doing and look at me. You are looking, I appreciate that. [Student 6], open your eyes, pick your head up. These are ideas from your class. I will condense them so there are no repeating ideas. Here are some ideas that your class came up with. Inequalities have the symbols less than or greater than, less than and equal to, greater than and equal to. A system of equations equals something. How many solutions are there to a system of equations? [Student 7] was holding up one finger. If we're talking a linear system with straight lines, how many times can they cross in a graph?

Students: Once

Kathy: Just once. That may change when we talk about things that are not linear, maybe graphs that curve, but for what we're seeing, one solution. This same idea, one has equal signs and the other doesn't. I'll put it in the middle. Oh, here's a new idea! You only shade on the graph when you have an inequality. this one is about shading. An inequality has a feasible region. Does anyone know what a feasible region is? [Student 8], say that again?

Student 8: The shaded part.

Kathy: The shaded part on your graph. It shows all of your solutions.

Motivational discourse was present when the teacher expressed excitement about students' new ideas ("Oh, here's a new idea!") and when she expressed appreciation for students when they paid attention. There was an explicit opportunity to understand mathematical language, at least to some degree, around terms such as "feasible region" and systems of equations and inequalities. There were also attempts to credit students with productive thinking anonymously (reading off what groups wrote on post-its) and by calling attention to students who were calling out productive ideas [students 7 and 8], which could potentially serve to raise students' statuses. Students were also pressed to explain features of systems of inequalities and systems of equations in during the previous classroom discussion and in groups in writing on the post-its. Finally, Kathy gave students feedback on their thinking by reflecting back what she noticed as similarities and differences in groups' responses. (Analysts observed a limitation on mathematical accuracy: linear systems can also have infinite solutions or no solutions.)

This description of a classroom episode illustrates some of the ways that Kathy enacted teaching that aligned with students' self-reports of productive engagement during this activity. Kathy supported students academically through opportunities to make sense of concepts and make connections among representations, and through opportunities for students to negotiate meaning about mathematical language. She pressed students to explain their reasoning and gave them feedback to specifically support understanding of mathematical concepts. Kathy also supported students socially through motivational discourse and status-raising efforts, particularly in terms of how she appreciated students' thinking when they shared ideas and elevated their ideas through revoicing them. Students were asked to explain multiple times throughout the activity, which supported the development of meanings of language and concepts and provided opportunities to position more students' ideas as valuable, to raise their statuses. Thus, for Kathy, both of these two aspects together – the academic and the social support she provided – appear to bolster the mathematical engagement of her students.

Conclusions

Secondary mathematics students in this study reported higher levels of engagement, particularly higher self-efficacy and social engagement, when they had opportunities to negotiate meaning about mathematical language, received specific feedback targeted toward making sense of concepts, and when teachers made the effort to raise students' status. This is an exploratory study; our future analyses will include more observations and ESM measures across an academic year, allowing for more sophisticated statistical analyses. Nevertheless, our preliminary results suggest that investigating the relationships between observations of teaching and students' self-reports of engagement in-the-moment is a potentially revealing approach for uncovering engaging instructional strategies in secondary mathematics classrooms.

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