Introduction

Many students know procedures for working with equivalent fractions. They may be able to generate sets of equivalent fractions, list fractions that are equivalent to a given fraction, or verify that two fractions are equivalent. Although students may develop procedural fluency with equivalent fractions, difficulties with developing conceptual understandings of fraction concepts are well-documented (Mack, 1990). In particular, understanding why fraction-equivalence procedures work and developing deep understandings of what it means for two fractions to be equivalent can be difficult.

Children may find certain strategies helpful for navigating equivalent fractions. The "splitting" strategy involves constructing an equivalent fraction by splitting each of the parts of a given fraction into the same number of equal-size pieces (Empson, 2001). For example, ¾ can be transformed into 6/8 by splitting each fourth in half. In a "chunking" strategy, equal numbers of parts are grouped or "chunked" together to construct an equivalent fraction (Empson, 2001). 5/10 becomes ½ when tenths are "chunked" together in pairs. Since fraction equivalence is such an important topic, the goal of our study was to investigate students' thinking about and understanding of fraction equivalence and design an instructional sequence to help their understandings develop.

Our Study

Our study took place in the context of a 10-week summer program during which lessons were designed week-by-week according to the cycle shown in Figure 1. Four students, who we will call Dalton, Will, Carol, and Camilla, participated in the fraction-equivalence lessons. The students had just finished third grade prior to the summer program.

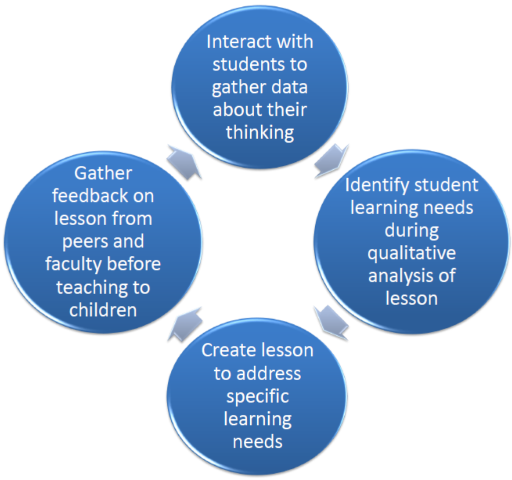

Figure 1. The Instructional Cycle

Lessons 1 and 2

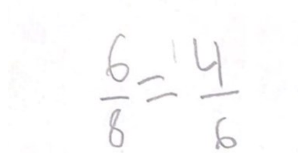

Figure 2. Example of an incorrect equivalent fraction given by a student.

During our first lesson we wanted to gauge what the children already understood about fractions. We had students try to recognize fractions from a visual representation, then create their own fractions and draw corresponding representations. In order to see how deep our students' understandings of fraction equivalence were, we asked them to list pairs of fractions that they thought were equivalent. Student's utilized paper and pencils and simply wrote out examples of fractions they deemed to be equivalent. We did not provide manipulatives for this activity because we wanted to see student's baseline understanding of fraction equivalence without the reliance or aid that manipulatives offer. We found that the children generated both correct and incorrect examples. This data showed us that our students did not necessarily understand what fraction equivalence meant or how to tell if fractions were equivalent. We knew in order to develop a conceptual understanding of fraction equivalence; our students must first have a correct understanding of what is means for fractions to be equivalent.

During lesson two, students were given some correct and incorrect examples of equivalent fraction pairs. We included the incorrect examples students generated during lesson 1 in order to reintroduce the students to their prior misconceptions and give them the opportunity to correct them. Students were asked to determine whether two given fractions were equivalent using fraction bars. Offering our students fraction bars as a visual representation of different fractions helped our students to develop a basic conceptual understanding of what equivalent fractions had to look like. Upon utilization of these manipulatives, we found three of our four students were able to successfully recognize equivalent fractions. We found that the use of visuals was very helpful for our students and encouraged them to visually compare fractions in order to determine if they were equivalent or not. This allowed them to internalize and develop the notion that for two fractions to be equivalent they must look the same.

Lessons 3 and 4

For these two lessons we shifted our focus to using fraction strips to further students' understanding of fraction equivalence. Students cut out and labeled the fraction strips themselves, then used them to find pairs of equivalent fractions. For example, students were able to use a "lining up" strategy to see that two of the 1/6 pieces created the same size piece as the 1/3 piece. Students were able to ask questions about what fraction strips were equivalent and find the answers themselves, promoting curiosity and ownership of their mathematical learning. For lesson four we wanted students to develop a deeper understanding of why fractions are equivalent, and to realize that multiple fractions

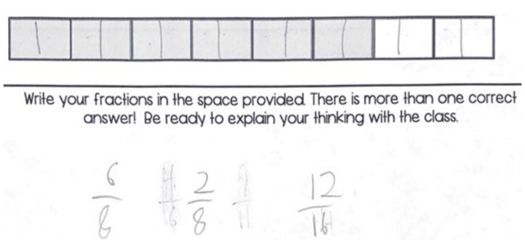

Figure 4. Student work showing multiple equivalent fractions found from one visual representation.

can be seen within a single visual representation. We had noticed our students tended to rely on unit fractions, so worked with fractions that do not simplify into unit fractions. When we took out the possibility of finding unit fractions, the students naturally began to use “splitting” and “chunking” strategies. One student showed both splitting (by drawing vertical line segments) and chunking (by grouping the eighths into pairs) in the same diagram.

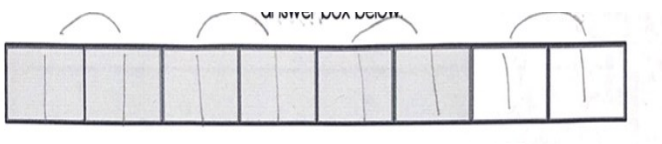

Figure 5. Student work showing beginning understanding of splitting and chunking ideas.

Lessons 5,6 and 7

For the last three lessons we focused on utilizing different types of concrete manipulatives in order to reinforce the ideas of “splitting” and “chunking” as concrete strategies for our students to understand and use when creating equivalent fractions. Manipulatives helped develop students’ reasoning patterns and explanations, increasing students’ use of mathematical language. Students were much more likely to use the terms “splitting” and “chunking” in their explanations when they could split or chunk physical objects into groups. In lesson five we used buttons to help with this; students created fractions using different button attributes, such as grouping them together according to color, number of holes, and size. Figure 6 shows one student’s work using the chunking strategy. The student had 12 buttons and chunked them into different groups based on as many characteristics as they could find. For example, when looking at how many tiny buttons they had,

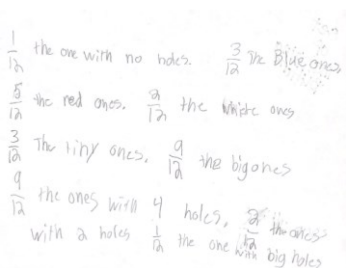

Figure 6. Example of all of the fractions a student was able to come up with utilizing the button manipulatives.

the student chunked the manipulates in three tiny buttons and then 9 larger buttons. This helped the student record both 3/12 for tiny buttons and 9/12 for bigger buttons.

In lesson six, when we did similar activities with snap cubes, we noticed that even though students used splitting and chunking strategies to build pairs of equivalent fractions, they now struggled to use vocabulary to explicitly describe what they were doing. In lesson seven we used the snap cubes again in a different situation to prompt students to use splitting and chunking vocabulary more explicitly. Students worked through an activity sheet and then taught their own strategies to each other.

In these lessons, we noticed that students at first struggled with creating equivalent fractions that could not be simplified by dividing numerator and denominator by 2. For example, we found that our students had difficulty recognizing that 3/12 and 1/4 are equivalent because 1 is not half of 3 and 4 is not half of 12. When students worked with a snap-cube representation of a fraction with an odd denominator, they realized they were unable to “double” the denominator by cutting the cubes in half. This forced students out of their comfort zones and directed them toward exploring splitting in other ways.

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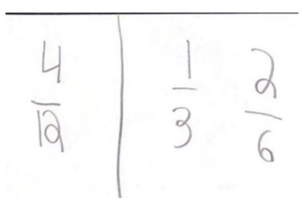

Figure 7. Shows the original fraction a set of snap cubes represented and then two equivalent fractions a student was able to create by breaking apart her snap cubes.

Summary/Discussion

Our students helped us a gain a deeper understanding of how their conceptual understanding of equivalent fractions developed. We found that the use of splitting and chunking resulted in students relying on the strategy of doubling or halving the numerator and denominator of given fractions. Learning progression 3.NF.3.b, “Recognize and generate simple equivalent fractions and explain using models” seemed to be the most difficult for our students, but the use of discrete manipulatives seemed to help our students move away from their reliance on doubling to make an equivalent fraction. Overall, our students demonstrated improved conceptual understandings of creating equivalent fractions. We hope that our findings allow other teachers to critically think about how to develop students’ conceptual understanding of equivalent fractions.

References

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