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Conversion between ^{3}He Melting Curve Scales below 100 mK

Yefan Tian · Eric Smith · Jeevak Parpia

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Abstract We provide the conversion parameters to allow a ^{3}He melting curve thermometer to be used to calibrate secondary thermometers to the PLTS2000 temperature scale [1]. Additional fits to the phase diagram of superfluid ^{3}He are also provided using the melting curve P, T measurements and of the phase diagram of superfluid ^{3}He [2] as a bridge. Further the melting curve measurements of Osheroff and Yu [3] are also used to extend the scale below 0.9 mK.

Keywords Thermometry at mK temperatures · PLTS scale · Greywall ^{3}He scale · Melting Curve Thermometer

1 Introduction

The melting curve of ^{3}He offers the possibility of a readily transferable practical temperature scale below 100 mK. Measurements by Greywall [2] (T, P values) that relied on a combination of Fermi liquid behavior of ^{3}He and the susceptibility of paramgnetic salts differ from those of the more modern PLTS2000 scale [1] that relied on a number of thermometers including platinum NMR thermometry and ultimately on noise thermometry. The publication detailing the PLTS2000 scale [1] does not provide a direct conversion from $P(T) - P_A$ to T (where $P(T)$ is the measured pressure along the melting curve at some temperature T, and P_A is the pressure of the superfluid transition to the A phase at melting pressure) [4].

In both the publications [1, 2], the melting curve pressure P is expressed as a polynomial in terms of T, leaving it up to researchers to effect the inversion. It is common usage to calibrate a secondary thermometer against the superfluid transition at various pressures, since these represent fixed points ranging from ∼2.5 mK at melting pressure to ∼0.9 mK at 0 bar. In the past, the phase diagram of superfluid ^{3}He was thoroughly investigated by Greywall [2]. Thus, the purpose of

Jeevak Parpia
E-mail: jmp9@cornell.edu

Yefan Tian, Eric Smith, and Jeevak Parpia
Department of Physics, Cornell University, Ithaca, NY, 14853 USA

this paper is to provide useful interpolation relationships and generate the phase diagram of superfluid ^{3}He in terms of the PLTS2000 scale. The observation of the superfluid transition in ^{3}He, together with the transition from the B phase to the A phase at various pressures below the melting pressure can allow a secondary thermometer to be calibrated even if a melting curve thermometer is not available.

However, there are small (and not insignificant) differences in the pressures of the "fixed points" (the pressure of the minimum in the melting curve, P_{min}, the pressure at the superfluid transition along the melting curve, P_{A}, the pressure of the equilibrium B to A transition, $P_{\mathrm{A-B}}$ and the pressure of the Neel transition, P_{N}) in the data published by Greywall [2] and in the publication disseminating the PLTS scale [1]. These differences potentially complicate the conversion from the Greywall to the PLTS scale, especially in the region between P_{A} and P_{min}. Adoption of the PLTS scale with the relationships provided here should allow a calibration of a secondary thermometer against the phase diagram of ^{3}He. Conversion of data taken using the Greywall scale to the PLTS scale can be done over limited P, T range to a certain stated accuracy, inherent because of the nonlinearity of the differences between the assigned pressures of the fixed points on each scale. We address this in the section immediately before the concluding paragraphs.

2 Temperatures for the PLTS2000 and Greywall scales along the melting curve.

Many investigations of the properties of normal and superfluid ^{3}He rely on the ubiquitous melting curve thermometer (MCT) [5] in which a sealed sample of ^{3}He is cooled via a heat exchanger along the melting curve. Providing the ^{3}He is pure ($\leq$10-20 ppm ^{4}He content) and calibrated with a precision pressure gauge, the measurement of pressure can be related to the temperature. We assume here that the superfluid transition temperature along the melting curve can be accessed yielding P_A, the pressure at which the A transition (T_c) occurs. The most recent temperature scale in the mK regime utilizes the melting curve [1], and is designated as T_{PLTS}. Unfortunately, T_{PLTS} does not provide a map onto the phase diagram of ^{3}He as measured by Greywall [2]. Fortunately, since both the measurements reference pressure relative to the pressure of the superfluid transition at the melting curve P_{A}, the temperature scale provided by Greywall, (T_{G}) and T_{PLTS} can be mapped onto one another.

At first glance the two temperature scales are simply linearly related. However, as we shall see, the fits differ by more than a linear factor especially in the important region below 3 mK. To effect the conversion, we first had to relate the two scales to one another. This is best done through the original parameterization where the pressure along the melting curve ($P - P_A$), is expressed in terms of a polynomial in temperature in each scale. In fact these expressions are inconvenient, since the pressure is measured using the melting curve and should be the input to determine the temperature of the thermometer. Thus, we first generate fits for the temperatures T_{PLTS} and T_G in terms of $P - P_A$, the pressure along the melting curve (rather than the reverse as provided in References [2,1]). We then provide three further fits, a conversion from T_G to T_{PLTS}, an expression (and tables) using this conversion for the superfluid transition temperatures expressed in terms of the PLTS200 scale for ^{3}He as a function of pressure, and we also obtain an expression for the equilibrium transition temperatures for the A to B transition as a function of pressure for the PLTS scale in zero magnetic field.

By subdividing the temperature between 5.6 and 100 mK (accessible by dilution refrigerators), and below 5.6 mK (accessible with nuclear demagnetization apparatus) we obtain two expressions written as ninth order polynomials, yielding fits that show minimal residual differences (≤ 1 μK). The relation between $P-P_{\mathrm{A}}$ and T^h_{PLTS} in the range of 5.6-100 mK is given by the polynomial (with T in mK, P and P_A in mbar) in Equation 1,

$$
\begin{aligned}
&T^h_{\mathrm{PLTS}} = \sum_{i=0}^{9} b^h_i (P-P_{\mathrm{A}})^i, \\
&\text{with} \\
&b^h_0 = 2.5257068036689, && b^h_1 = -0.024555787070591, \\
&b^h_2 = -1.7994349872600 \times 10^{-6}, && b^h_3 = -6.0072773787192 \times 10^{-9}, \\
&b^h_4 = -5.8885557756054 \times 10^{-12}, && b^h_5 = -3.9102041149206 \times 10^{-15}, \\
&b^h_6 = -1.6605359484626 \times 10^{-18}, && b^h_7 = -4.3792852104458 \times 10^{-22}, \\
&b^h_8 = -6.5042399327047 \times 10^{-26}, && b^h_9 = -4.1677548758514 \times 10^{-30}.
\end{aligned}
\tag{1}
$$

In a similar manner, we write the relation between $P-P_{\mathrm{A}}$ and T^l_{PLTS} between 0.9-5.6 mK in Equation 2,

$$
\begin{aligned}
&T^l_{\mathrm{PLTS}} = \sum_{i=0}^{9} b^l_i (P-P_{\mathrm{A}})^i, \\
&\text{with} \\
&b^l_0 = 2.4442188707375, && b^l_1 = -0.026522783446809, \\
&b^l_2 = -2.7665556176467 \times 10^{-5}, && b^l_3 = -2.2800036357244 \times 10^{-7}, \\
&b^l_4 = 3.9890194953355 \times 10^{-10}, && b^l_5 = 1.2845430171276 \times 10^{-11}, \\
&b^l_6 = -6.9521369379387 \times 10^{-13}, && b^l_7 = -1.5658128424388 \times 10^{-14}, \\
&b^l_8 = -1.1687750824147 \times 10^{-16}, && b^l_9 = -3.0194721850282 \times 10^{-19}.
\end{aligned}
\tag{2}
$$

The Greywall temperature scale [2] was also inverted to provide the relationship between $P-P_{\mathrm{A}}$ and T^l_{G} between 0.9-5.6 mK. However, the residuals in this case are not so well controlled. Nevertheless, we were able to obtain suitable fits with only a marginally worse residual near the joining point at 5.6 mK. The results are shown in Figures 3, 4. The fits are also given below in Equations 3 and 4

$$
\begin{aligned}
&T_{\mathrm{G}}^{h} = \sum_{i=0}^{9} b_i'^h (P - P_{\mathrm{A}})^i,\\
&\text{with}\\
&b_0'^h = 2.5716301676528, && b_1'^h = -0.025282877529959,\\
&b_2'^h = -3.1520828852106 \times 10^{-6}, && b_3'^h = -8.3143192317460 \times 10^{-9},\\
&b_4'^h = -8.5254298085516 \times 10^{-12}, && b_5'^h = -5.8213178708483 \times 10^{-15},\\
&b_6'^h = -2.5330094160263 \times 10^{-18}, && b_7'^h = -6.8138014359791 \times 10^{-22},\\
&b_8'^h = -1.0289746107350 \times 10^{-25}, && b_9'^h = -6.6784487609959 \times 10^{-30}.
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
&T_{\mathrm{G}}^{l} = \sum_{i=0}^{9} b_i'^l (P - P_{\mathrm{A}})^i,\\
&\text{with}\\
&b_0'^l = 2.4917569885793, && b_1'^l = -0.027314027643057,\\
&b_2'^l = -3.0515894619175 \times 10^{-5}, && b_3'^l = -2.4516567321204 \times 10^{-7},\\
&b_4'^l = 1.5835099524554 \times 10^{-9}, && b_5'^l = 5.3200305749153 \times 10^{-11},\\
&b_6'^l = -3.9737644865275 \times 10^{-13}, && b_7'^l = -1.8507486467695 \times 10^{-14},\\
&b_8'^l = -1.6373166353048 \times 10^{-16}, && b_9'^l = -4.6689405796748 \times 10^{-19}.
\end{aligned} \tag{4}
$$

Having obtained the values for the temperatures on the two scales from a common value of $P - P_A$, we can readily plot the two temperatures against each other and obtain fits over the temperature ranges from 0.9 mK and 5.6 mK and between 5.6 mK and 100 mK. These results are plotted in Figure 6 and the expression to convert from T_G to T_{PLTS} is given below in Equations 5 and 6

$$
\begin{aligned}
&T_{\mathrm{PLTS}} = \sum_{i=0}^{6} a_i T_{\mathrm{G}}^i,\\
&\text{with}\\
&a_0 = -0.14265343150487, && a_1 = 1.2810635032153,\\
&a_2 = -0.22689947807354, && a_3 = 0.084337673002034,\\
&a_4 = -0.016928990685839, && a_5 = 0.0017611612884063,\\
&a_6 = -7.4461876859237 \times 10^{-5}.
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
&T_{\text{PLTS}} = \sum_{i=0}^{9} a_i T_{\text{G}}^i, \\
&\text{with} \\
&a_0 = 0.020353327019475, && a_1 = 0.96670033496024, \\
&a_2 = 0.0019559314169033, && a_3 = -9.5551084662924 \times 10^{-5}, \\
&a_4 = 3.2167457655106 \times 10^{-6}, && a_5 = -7.0097586342143 \times 10^{-8}, \\
&a_6 = 9.6909878738352 \times 10^{-10}, && a_7 = -8.2126513949290 \times 10^{-12}, \\
&a_8 = 3.8886762300964 \times 10^{-14}, && a_9 = -7.8713540127550 \times 10^{-17}.
\end{aligned} \tag{6}
$$

These plots (Figures 1-5) and fits Equations (1-5) enable us to obtain the phase diagram in P, T coordinates for the second order normal to superfluid transition temperatures together with the line of equilibrium (first order) B phase to A phase transition temperatures in the next section. For reference we list a few representative values of $P - P_A$ and the fitted values of T_{PLTS} , T_G in Table 1.

$P - P_{\text{A}}$ (mbar)	T_{G} (mK)	T_{PLTS} (mK)	$P - P_A$ (mbar)	T_{G} (mK)	T_{PLTS} (mK)
52.7 (Néel) [1]	-	0.90181	-20	3.02805	2.96544
52.5 (Néel) [2]	0.93038	-	-40	3.55027	3.47422
52	0.94925	0.92816	-60	4.06314	3.97500
50	1.01862	0.99938	-80	4.56979	4.47062
48	1.08492	1.06691	-100	5.07223	4.96287
46	1.14949	1.13219	-140	6.06953	5.94170
44	1.21295	1.1959	-180	7.06190	6.91750
42	1.2756	1.25843	-220	8.05309	7.89352
40	1.33761	1.31999	-260	9.04534	8.87169
38	1.39907	1.38073	-300	10.0401	9.85327
36	1.46002	1.44074	-340	11.0384	10.8391
34	1.5205	1.5001	-380	12.0409	11.8299
32	1.58053	1.55886	-420	13.0482	12.8260
30	1.64013	1.61708	-460	14.0607	13.8279
28	1.69931	1.67479	-500	15.0787	14.8358
26	1.75808	1.73202	-540	16.1026	15.8500
24	1.81646	1.78881	-620	18.1689	17.8983
22	1.87446	1.84518	-700	20.2613	19.9742
20.2 (A-B) [1]	-	1.89558	-800	22.9158	22.6101
20 (A-B) [2]	1.93209	-	-1000	28.3644	28.0273
18	1.98938	1.95677	-1200	34.0160	33.6538
16	2.04633	2.01204	-1400	39.8912	39.5090
14	2.10295	2.06699	-1600	46.0126	45.6138
12	2.15927	2.12163	-1800	52.4059	51.9919
10	2.2153	2.17598	-2000	59.1003	58.6708
8	2.27104	2.23006	-2200	66.1298	65.6818
6	2.32651	2.28388	-2400	73.5342	73.0623
4	2.38173	2.33746	-2600	81.3603	80.8563
2	2.4367	2.3908	-2800	89.6645	89.1174
0 (A)	2.49143	2.44393	-3000	98.5146	97.9109

Table 1 $P - P_{\text{A}}$ and T_{G}, T_{PLTS} from the polynomial functions provided in [1,2].

3 The phase diagram for superfluid ^{3}He in T_{PLTS}

This section should be especially useful for researchers who are studying the properties of superfluid ^{3}He below the melting curve, where features such as T_c, P and

T_{AB}, P, can be used to calibrate secondary thermometers. These features were measured by Greywall [2] but not part of the PLTS scale [6].

Having established the conversion between two temperature scales, it is possible to calculate the T, P coordinates of the superfluid transition temperature $T_c(P)$, and the line of equilibrium A to B transitions, $T_{AB}(P)$ within the PLTS temperature scale. The fitted equations for $T_{c,PLTS}$ and $T_{AB,PLTS}$ are provided in Equations 7, 8. In addition, the values of $T_{\mathrm{AB,PLTS}}$ and $T_{\mathrm{c,PLTS}}$ are listed in the accompanying Tables 2, 3.

$$
\begin{aligned}
&T_{\mathrm{c,PLTS}} = \sum_{i=0}^{5} d_i P^i, \\
&\text{with} \\
&d_0 = 0.90972399274531, && d_1 = 0.14037182852625, \\
&d_2 = -0.0074017331747577, && d_3 = 2.8617547367067 \times 10^{-4}, \\
&d_4 = -6.5064429600510 \times 10^{-6}, && d_5 = 6.0754459040296 \times 10^{-8}.
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
&T_{\mathrm{AB,PLTS}} = \sum_{i=0}^{5} c_i P^i, \\
&\text{with} \\
&c_0 = -26.864685876026, && c_1 = 5.2647866128370, \\
&c_2 = -0.37617826876151, && c_3 = 0.013325635880953, \\
&c_4 = -2.3510107585468 \times 10^{-4}, && c_5 = 1.6519539175010 \times 10^{-6};
\end{aligned}
\tag{8}
$$

P (bar)	$T_{\mathrm{c,G}}$ (mK)	$T_{\mathrm{c,PLTS}}$ (mK)
34.338	2.491	2.443
34	2.486	2.439
33	2.474	2.427
32	2.463	2.416
31	2.451	2.404
30	2.438	2.392
29	2.425	2.380
28	2.411	2.366
27	2.395	2.351
26	2.378	2.334
25	2.360	2.316
24	2.339	2.296
23	2.317	2.275
22	2.293	2.251
21	2.267	2.226
20	2.239	2.199
19	2.209	2.170
18	2.177	2.139
17	2.143	2.106
16	2.106	2.071
15	2.067	2.033
14	2.026	1.992
13	1.981	1.949
12	1.934	1.903
11	1.883	1.854
10	1.828	1.800
9	1.769	1.743
8	1.705	1.680
7	1.636	1.613
6	1.560	1.539
5	1.478	1.458
4	1.388	1.370
3	1.290	1.272
2	1.181	1.164
1	1.061	1.043
0	0.929	0.908

Table 2 The superfluid transition temperatures as a function of pressure, using P, T_{c} from Reference [2], and Eq. (5) to generate $T_{\mathrm{c,PLTS}}$.

P (bar)	$T_{\mathrm{AB},G}$ (mK)	$T_{\mathrm{AB,PLTS}}$ (mK)
P_{AB}	1.932	1.901
34	1.941	1.910
33	1.969	1.937
32	1.998	1.965
31	2.027	1.994
30	2.056	2.021
29	2.083	2.048
28	2.111	2.074
27	2.137	2.100
26	2.164	2.127
25	2.191	2.153
24	2.217	2.178
23	2.242	2.202
22	2.262	2.221
21.22	2.273	2.232

Table 3 The equilibrium T_{AB} as a function of Pressure, using P, T_{AB} from Reference [2], and Eq. (5) to generate $T_{\mathrm{AB,PLTS}}$. P_{AB} specifies the pressure of the equilibrium A-B transition at melting pressure.

4 Continuation of the melting curve below T_N.

The lowest temperature on the PLTS scale is the solid ordering temperature or T_N, where there is a discontinuous drop in entropy [7]. The excitations and entropy in the liquid are limited, and the solid rapidly loses its entropy below T_N. Thus, the extent of the pressure variation is small. Nevertheless, with a reasonable design one can use a compact melting curve thermometer to measure the temperature down to $\approx 0.7\ T_N$.

Osheroff and Yu [3] measured the temperature dependence of the melting curve in this region. The publication quotes the value of T_A to be 2.752 and $T_N = 1.03$ mK. Below T_N, the reported pressure dependence is

$$\Delta P = 0.58T^8 - 1.2T^6 + 2.4T^4 - 0.002 \tag{9}$$

with T in mK and $\Delta P = P(T=0) - P(T)$ in mbar. [3]. In order to match the value of T_N, we scaled their temperatures by a factor of the two Neel temperatures $(0.902/1.03 = 0.875728)$ to match the PLTS value of T_N, 0.902 mK, and by setting the pressure of the solid ordering at the PLTS value of 34.3934 bar. Thus, we obtain

$$\Delta P_{PLTS} = 1.67677T^8 - 2.66051T^6 + 4.080694T^4 - 0.002 \tag{10}$$

Having obtained this scaled relation, we need to invert it so that an input of the pressure yields the temperature in mK. The expression that we find works well is

$$T = 0.0537(\Delta P)^{1.5} - 0.2773(\Delta P) + 0.793(\Delta P)^{0.5} + 0.184 \tag{11}$$

with T in mK and $\Delta P = P(T) - P_N$ in mbar.

5 Caveats due to Pressure Differences

As was stated in the introduction, there are significant discrepancies between the values of the pressure "fixed points" [6,2]. We summarize the reported values of these fixed points in Table 4. The result is that a measurement that referenced $P - P_A$ under the Greywall scale can be readily converted to the PLTS scale using Equations 5 or 6.

Since the two values for P_A in the Greywall and PLTS measurements are different, a calculation of the temperature from the Greywall scale (Equations 3, 4) from a pressure measured without reference to P_A will incur an additional error in conversion to T_{PLTS}. For example in Table 5, Column 1, we list the values of pressure corresponding to temperatures, T_G, in Column 2. That same value of pressure with the P_{PLTS} value for P_A substituted into Equations 1, 2) yields Column 4 (T_{PLTS}). Note that $P - P_A$ for the PLTS scale will be 2.7 mb smaller than that for the Greywall scale. Importantly, equation 6 assumes that the same value of $P - P_A$ is used to calculate T_G and T_{PLTS} (Column 2 converted to Column 4). If there were no access to the A transition and pressures were converted to temperatures directly (without reference to P_A) an additional error could accrue to the conversion (Table 5). If an older data set is being converted to the T_{PLTS}, and there was no access to P_A, one should assume a conversion error as shown in Table 5.

While taking new data, a researcher using the PLTS scale would calibrate their melting curve thermometer using a room temperature gauge. If the melting curve thermometer was mounted to a dilution refrigerator equipped with a nuclear demagnetization stage, the researcher would proceed to measure the values of P_{min}, P_{A}, $P_{\mathrm{A-B}}$, and P_{N}. Corrections due to the pressure head of ^{3}He in the fill line (typically of order 10-20 mbar) would lead to a correction being applied to the value of P_{A}. Values of $P - P_A$ obtained would then serve as inputs to calculate the corresponding temperatures with high accuracy. An apparatus without a nuclear demagnetization stage would have to rely on corrections to the pressure head referenced to the value of P_{min}, inherently fraught because the value read off is dependent on the purity of the ^{3}He sample, as well as a tendency to flatten the P, T dependence due to overfilling that leads to a complete conversion of the bulk sample (not contained in the sinter) into solid. A further complication is the small variation of the modulus of commonly used construction materials with temperature [8,9] resulting in errors in the inferred pressure. Therefore, P_{A} remains the best choice of reference pressure. A caveat needs to be expressed that researchers who might use the melting curve thermometer to cover the entire temperature range between 0.9 mK upto 100 mK should satisfy themselves that their melting curve thermometer's pressure calibration is consistent with the values of the pressure fixed points in the PLTS scale.

Fixed points	P_{PLTS} (bar)	T_{PLTS} (mK)	P_G (bar)	T_G (mK)
Minimum	29.3113	315.24	29.4061	280.33
A	34.3407	2.444	34.3380	2.491
A-B	34.3609	1.896	34.3580	1.932
Néel	34.3934	0.902	34.3905	0.931

Table 4 Values of fixed points along the Greywall and PLTS scales.

P (bar)	T_G (mK)	T_{PLTS} (mK)	T_{PLTS}[Eq.(6)] (mK)	ΔT (mK)	$\Delta T/T_{\mathrm{PLTS}}[P_G]$
34.0396	10	9.8137	9.8801	0.0664	0.67%
33.4607	25	24.6813	24.7541	0.0688	0.28%
32.6122	50	49.5916	49.6782	0.0866	0.17%
31.3057	100	99.3856	99.5097	0.1241	0.12%

Table 5 Column 1 shows the pressure associated with a particular temperature (Column 2) on the Greywall scale. The same pressure is used to calculate $P - P_{A,PLTS}$ and then using Eq. 2 we calculate Column 3. Conversion from T_G to T_{PLTS} using Eq. 6 yields Column 4. Columns 5 and 6 show the difference and percentage difference between the two methods of calculating T_{PLTS} from T_G.

6 Conclusions

We have presented a methodology that we believe should prove useful for implementing thermometry below 100 mK using the highly transferable ^{3}He melting curve thermometer. Providing a device can be reliably calibrated, and the pressure read off with mbar precision or better, thermometry in this important range can be reproducibly implemented.

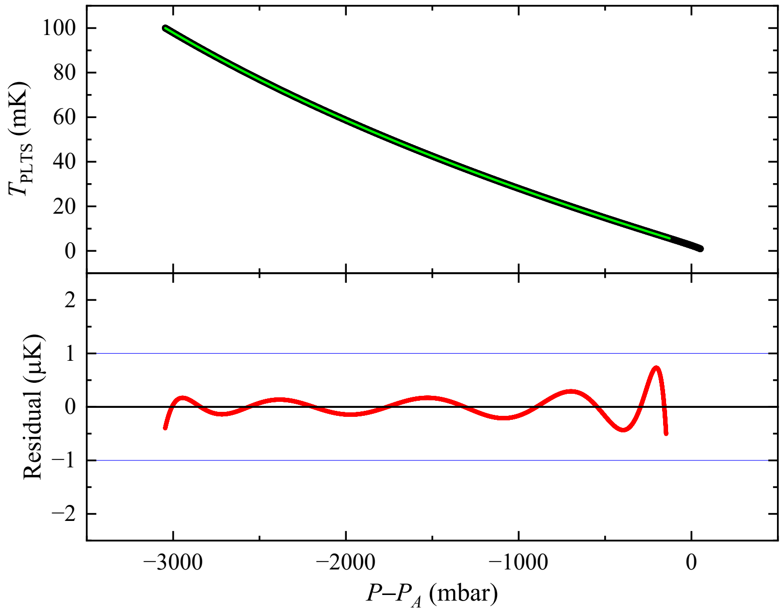

Fig. 1 (Color online) The PLTS temperature T^h_{PLTS} [1] and fit from Eq. (1) in the temperature range 5.6-100 mK against $P - P_A$ (mbar). The lower panel shows the residuals of the fit.

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8 Data Availability

All data is contained within this article and no repository is needed or available.

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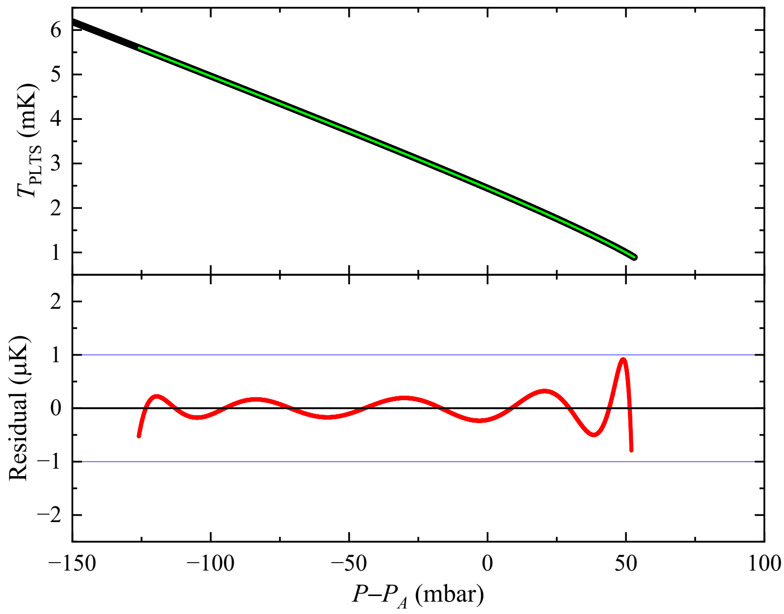

Fig. 2 (Color online) The PLTS temperature T^l_{PLTS} [1] and fit from Eq.2 in the temperature range 0.9-5.6 mK against $P - P_A$ (mbar). The lower panel shows the residuals of the fit.

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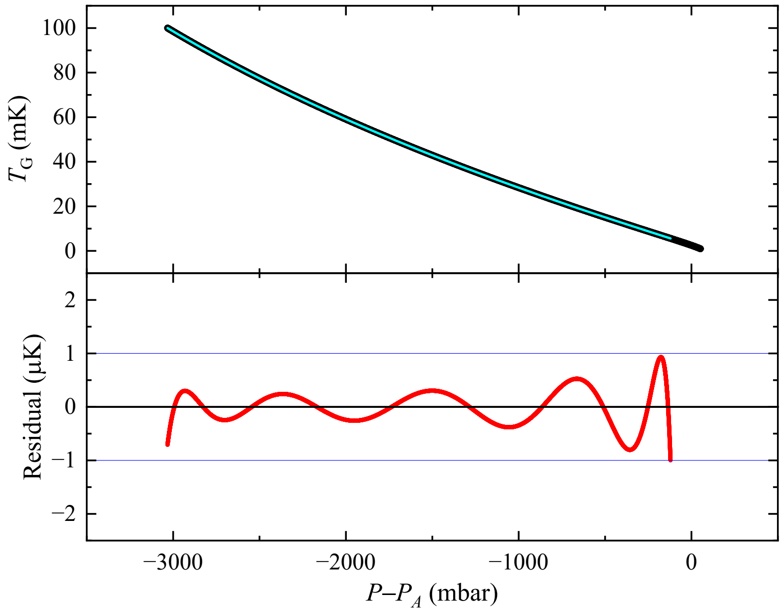

Fig. 3 (Color online) The Greywall temperature T_G^h [2] and fit from Eq. (1) in the temperature range 5.6-100 mK against $P - P_A$ (mbar). The lower panel shows the residuals of the fit.

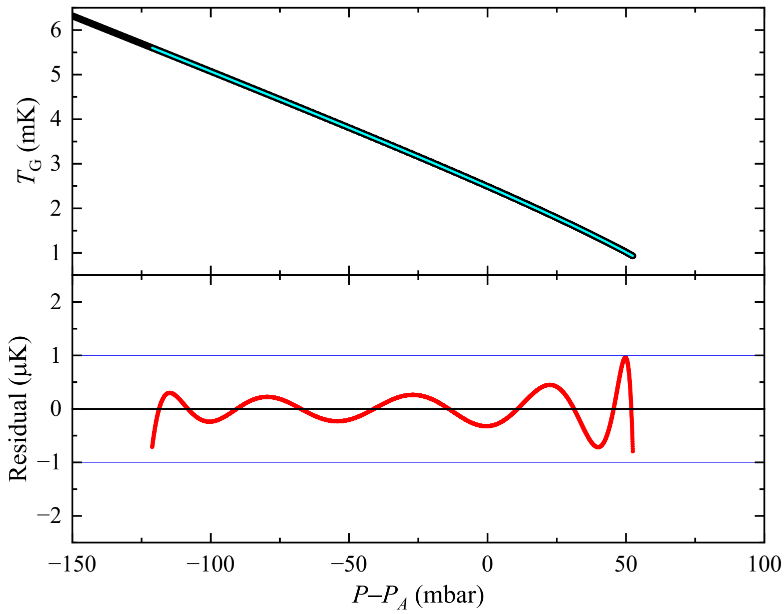

Fig. 4 (Color online) The Greywall temperature T_G^l [2] and fit from Eq. (1) in the temperature range 0.9-5.6 mK against $P - P_A$ (mbar). The lower panel shows the residuals of the fit.

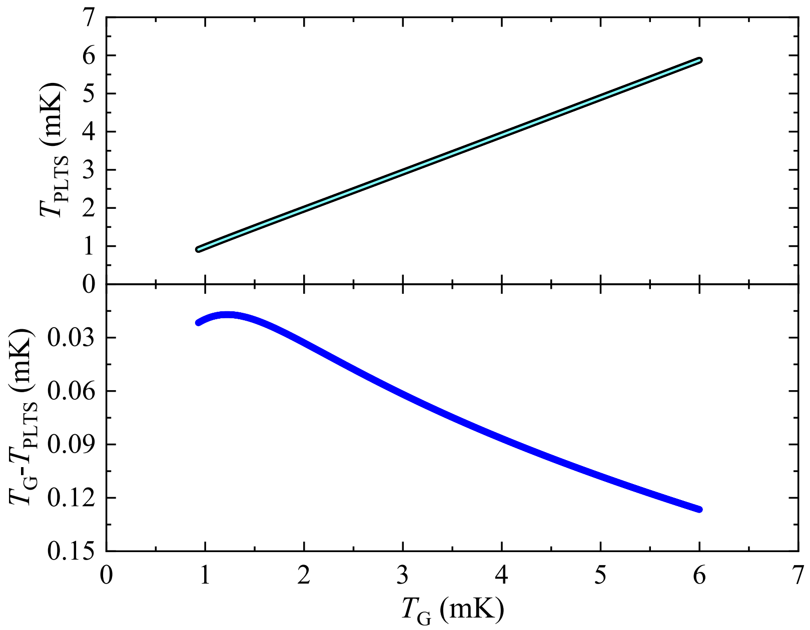

Fig. 5 (Color online) The top panel shows the near linear relationship between T_{PLTS}, and T_{G} below 6 mK. The results of the low temperature fitting function from Eq. (6) is shown as the solid green line. Each set of temperatures was related to one another through the assigned pressures $P - P_A$ in the publications [1,2]. The lower panel illustrates the differences between the PLTS2000 scale and the Greywall melting curve scales, plotted against the temperature (in mK) derived from the Greywall melting curve scale.

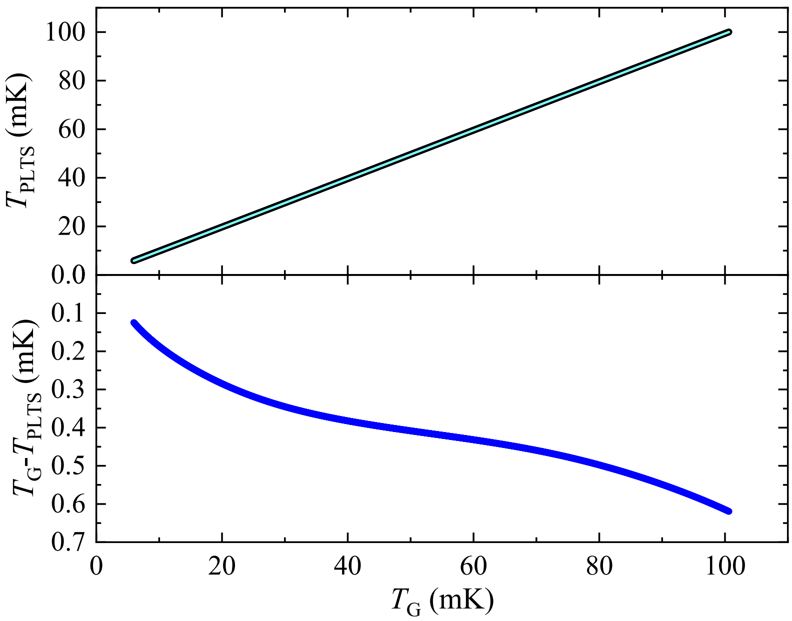

Fig. 6 (Color online) The top panel shows the near linear relationship between T_{PLTS}, and T_{G} between 6 and 100 mK. The results of the low temperature fitting function from Eq. (6) is shown as the solid green line. Each set of temperatures was related to one another through the assigned pressures $P - P_A$ in the publications [1, 2]. The lower panel illustrates the differences between the PLTS2000 scale and the Greywall melting curve scales for identical values of $P - P_A$, plotted against the temperature (in mK) derived from the Greywall melting curve scale.

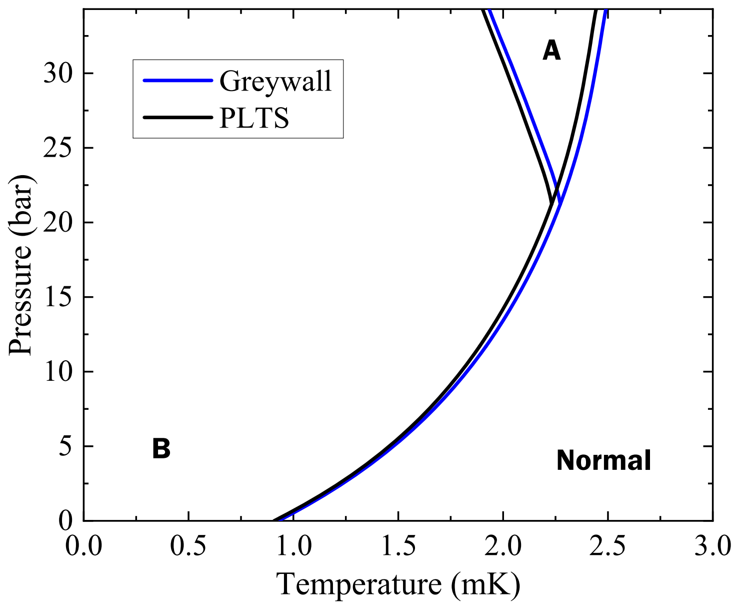

Fig. 7 (Color online) The phase diagram for superfluid ^{3}He plotted using the Greywall scale[2] and according to the PLTS scale after conversion using Eq.6

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