Distribution of AoI in EH-powered Multi-source Systems with Source-aware Packet Management

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***Abstract*—This paper considers a *multi-source* updating system in which a transmitter powered by energy harvesting (EH) sends status updates about multiple sources of information to a destination, where the freshness of status updates is measured in terms of *Age of Information (AoI)*. The harvested energy packets and the status updates of each source are assumed to arrive at the transmitter according to independent Poisson processes, and the service time of each status update is assumed to be exponentially distributed. Our focus is on understanding the *distributional properties* of AoI under a *source-aware preemptive in service queueing discipline* (which only allows preemption between the status updates generated by the same source to enhance fairness). In particular, we use the stochastic hybrid systems (SHS) framework to derive closed-form expressions of the moment generating function (MGF) and average of AoI. To the best of our knowledge, this paper is the first to characterize the AoI performance under a source-aware preemptive policy for the generic case where the transmitter has an arbitrary number of sources. The generality of our results is demonstrated by recovering several existing results for EH-powered single-source systems as special cases. Our results demonstrate that the proposed source-aware preemptive policy strikes a balance between minimizing the sum of average AoI values associated with different sources (average sum-AoI) and achieving fairness among the average AoI values of different sources.**

***Index Terms*—Age of information, energy harvesting, queueing systems, communication networks, stochastic hybrid systems.**

I. Introduction

The ongoing large-scale deployment of Internet of Things (IoT) will enable a class of real-time updating systems in which a *transmitter node* aims to deliver timely status updates about some physical process(es) of interest to a *destination node* [1]. To quantify the freshness of information at the destination, the authors of [2] introduced the concept of AoI and derived its average for single-source systems in which *a non-EH transmitter* (i.e., it is powered by a reliable energy source) has a single source of information. While AoI has been extensively analyzed in single-source systems, the analysis of AoI in multi-source systems is quite challenging, and hence the prior work in this direction is relatively sparse [3]–[7]. The average AoI was characterized under non-preemptive and preemptive in service/waiting queueing disciplines in [3]–[6], whereas the distribution of AoI was numerically characterized for various discrete time queues in [7]. Different from [2]–[7] that considered a non-EH transmitter, our focus in this paper is on the analytical characterization of the distributional properties of AoI in EH-powered multi-source systems.

The analyses of the above works were mainly based on identifying the properties of the AoI sample functions and applying geometric arguments, which often involve tedious calculations of joint moments. This has motivated the authors of [8] and [9] to build on the SHS framework of [10], and derive promising results allowing the use of the SHS approach for the queueing-theoretic analyses of AoI. The analysis of AoI using the SHS approach becomes much more challenging when we consider an EH-powered transmitter. This is because the process of decision-making (i.e., the decisions of discarding or serving new arriving status updates at the transmitter) is dependent on the joint evolution of the battery state and the system occupancy with respect to the status updates.

Studies on AoI in multi-source systems with a non-EH transmitter have shown the superiority of the source-agnostic preemptive queueing disciplines (which allow status updates of different sources to preempt each other in service/waiting) over the non-preemptive ones in terms of the achievable average sum-AoI. However, this comes at the expense of having unfair achievable average AoI values among different sources since the updates of sources with small update arrival rates are more likely to be preempted by the updates of sources with higher update arrival rates. To resolve this issue, the authors of [11] have recently characterized the average AoI for a two-source system with a non-EH transmitter under several source-aware preemptive strategies. Different from [11], this paper is the first to characterize the AoI performance under a source-aware preemptive in service queueing discipline for the generic case where the transmitter has an arbitrary number of sources.

For the case where the transmitter is powered by EH, there are a handful of prior works [12]–[16] analyzing AoI by applying geometric arguments [12], [13], and by using the SHS approach [14]–[16]. However, the analyses of [12]–[15] have been limited to the evaluation of the average AoI in single-source systems, and the analysis of [16] was focused on the characterization of the distributional properties of AoI in single-source systems. Different from these, this paper makes the first attempt at deriving the average/MGF of AoI in multi-source systems with an EH-powered transmitter.

Contributions. This paper presents a novel queueing-theoretic analysis to derive closed-form expressions for the MGF and average of AoI in EH-powered multi-source systems with source-aware packet management. In our analysis, we use the SHS framework where the system discrete state is modeled as a two-dimensional continuous-time Markov chain (CTMC) to track both the numbers of update and harvested energy packets in the system. We analytically demonstrate that as the aggregate generating rate of status updates from all the sources other than the source of interest approaches zero, the MGF/average AoI expression derived in this paper reduces to its counterpart in [16] for EH-powered single-source systems. Our numerical results demonstrate the necessity of incorporating the higher moments of AoI in the implementation/optimization of multi-source updating systems.

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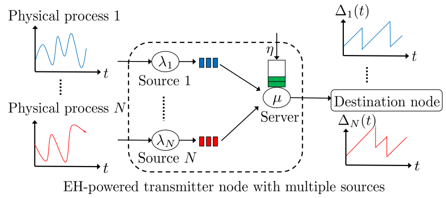

Fig. 1. An illustration of the system setup.

II. System Model

We consider a real-time updating system in which an EH-powered transmitter observes N physical processes, and sends its measurements to a destination in the form of status update packets. As shown in Fig. 1, the transmitter consists of N sources and a single server; each source generates status updates about one physical process, and the server delivers the status updates generated from all the sources to the destination. In particular, each status update packet generated by source i carries some information about the value of the i-th physical process and a time stamp indicating the time at which that process was sampled. This system setup can be mapped to many scenarios of practical interest, such as an IoT network in which an aggregator (represents the transmitter in our model) delivers measurements sensed/generated by the N IoT devices (represent the sources) in its vicinity to a destination.

Status update packets generated by the i-th source are assumed to follow a Poisson process with rate λ_i. Further, the energy packets are assumed to arrive at the transmitter according to a Poisson process with rate η, and are stored in a battery queue of length B packets at the server (for serving the update packets generated by the different sources). We consider that each energy packet contains the energy required for sending one status update from any of the sources [12]–[15], and hence the length of the energy battery queue reduces by one whenever a status update is successfully received at the destination. Given that the transmitter node has at least one energy packet in its battery queue, the time needed by its server to send a status update packet is assumed to be a rate μ exponential random variable [2]–[4]. Let $\rho = \frac{\lambda}{\mu}$ and $\beta = \frac{\eta}{\mu}$ respectively denote the server utilization and energy utilization factors, where $\lambda = \sum_{i=1}^{N} \lambda_i$. Further, we have $\rho_i = \frac{\lambda_i}{\mu}$, $\lambda_{-i} = \sum_{j=1,\, j\neq i}^{N} \lambda_j$, and $\rho_{-i} = \frac{\lambda_{-i}}{\mu}$.

We quantify the freshness of information about each physical process at the destination (as a consequence of receiving status update packets from the transmitter) using the concept of AoI. Formally, AoI is defined as follows [2].

Definition 1. *Let $t_{i,k}$ denote the arrival time instant of the k-th update of source i at the transmitter. Further, define $L_i(t)$ to be the index of the source i's latest update received at the destination by time t. Then, the AoI associated with the physical process observed by source i at the destination (referred henceforth as the AoI of source i) is defined as the following random process: $\Delta_i(t) = t - t_{i,L_i(t)}$.*

We analyze the AoI performance at the destination under the last-come-first-served with source-aware preemption in service (LCFS-SA) queueing discipline. Under this queueing discipline, a new arriving status update at the transmitter enters service upon its arrival if the server is idle, given that the battery contains at least one energy packet. Further, when the server is busy, a new arriving status update preempts the update in service only if the two updates are generated from the same source; otherwise, the new arriving status update is discarded.

With regards to the EH process, we consider that the transmitter can harvest energy only if its server is idle. This case corresponds to the scenario where the transmitter is equipped with a single radio frequency chain, and thus can either transmit a status update or harvest energy at a certain time instant. The case where the transmitter can harvest energy anytime (i.e., even when its server is busy) is left as a promising direction of future work.

III. SHS: A Very Brief Introduction

Our analysis is focused on deriving the distributional properties of AoI through the characterization of its MGF. To do so, we resort to the SHS framework of [10], which was first tailored for the analysis of AoI in [8] and [9]. In the following, we provide a very brief[1] introduction of the SHS framework, which will be useful in understanding our AoI MGF analysis in the next section. The SHS is represented by a hybrid state $(q(t), \mathbf{x}(t))$, where $q(t) \in \mathcal{Q} = \{1, \cdots, m\}$ is a finite-state CTMC modeling the system discrete state and $\mathbf{x}(t) = [x_0(t), \cdots, x_n(t)] \in \mathbb{R}^{1\times(n+1)}$ describes the evolution of the system continuous state over time. In the CTMC $q(t)$, a transition $l \in \mathcal{L}$ from state q_l to state q'_l occurs with a rate $\lambda^{(l)}\delta_{q_l,q(t)}$ and causes $\mathbf{x}$ to reset to $\mathbf{x}' = \mathbf{x}\mathbf{A}_l$, where $\mathbf{A}_l \in \mathbb{B}^{(n+1)\times(n+1)}$ is a binary reset map matrix and the Kronecker delta function $\delta_{q_l,q(t)}$ ensures that l occurs only when $q(t) = q_l$. Further, $\dot{\mathbf{x}}(t) \triangleq \frac{\partial \mathbf{x}(t)}{\partial t} = \mathbf{1}$ holds as long as the state $q(t)$ is unchanged, where $\mathbf{1}$ is the row vector $[1, \cdots, 1] \in \mathbb{R}^{1\times(n+1)}$. Denote by $\mathcal{L}'_q = \{l \in \mathcal{L} : q'_l = q\}$ and $\mathcal{L}_q = \{l \in \mathcal{L} : q_l = q\}$ the sets of incoming and outgoing transitions for state q. Further, let $\mathbf{v}_q(t) = [v_{q0}(t), \cdots, v_{qn}(t)] \in \mathbb{R}^{1\times(n+1)}$ denote the correlation vector between $q(t)$ and $x(t)$, and $\mathbf{v}^s_q(t) = [v^s_{q0}(t), \cdots, v^s_{qn}(t)] \in \mathbb{R}^{1\times(n+1)}$ denote the correlation vector between $q(t)$ and the exponential function $e^{s\mathbf{x}(t)}$, where $s \in \mathbb{R}$. Thus, we have

$$\mathbf{v}_q(t) = [v_{q0}(t), \cdots, v_{qn}(t)] = \mathbb{E}[\mathbf{x}(t)\delta_{q,q(t)}],\ \forall q \in \mathcal{Q}, \quad (1)$$

$$\mathbf{v}^s_q(t) = [v^s_{q0}(t), \cdots, v^s_{qn}(t)] = \mathbb{E}[e^{s\mathbf{x}(t)}\delta_{q,q(t)}],\ \forall q \in \mathcal{Q}. \quad (2)$$

Using the above notations, it has been shown in [9, Theorem 1] that under the ergodicity assumption of the CTMC $q(t)$, if we can find a non-negative limit $\bar{\mathbf{v}}_q = [\bar{v}_{q0}, \cdots, \bar{v}_{qn}]$, $\forall q \in \mathcal{Q}$, for the correlation vector $\mathbf{v}_q(t)$ satisfying

$$\bar{\mathbf{v}}_q \sum_{l\in\mathcal{L}_q} \lambda^{(l)} = \bar{\pi}_q \mathbf{1} + \sum_{l\in\mathcal{L}'_q} \lambda^{(l)} \bar{\mathbf{v}}_{q_l} \mathbf{A}_l,\ q \in \mathcal{Q}, \quad (3)$$

where $\bar{\pi} = [\bar{\pi}_0, \cdots, \bar{\pi}_m]$ is the unique state stationary vector satisfying

$$\bar{\pi}_q \sum_{l\in\mathcal{L}_q} \lambda^{(l)} = \sum_{l\in\mathcal{L}'_q} \lambda^{(l)} \bar{\pi}_{q_l},\ q \in \mathcal{Q},\ \sum_{q\in\mathcal{Q}} \bar{\pi}_q = 1, \quad (4)$$

then:

- The expectation of $x(t)$, $\mathbb{E}[x(t)]$, converges to the following stationary vector:

$$\mathbb{E}[x] = \sum_{q\in\mathcal{Q}} \bar{\mathbf{v}}_q. \quad (5)$$

[1]Interested readers are advised to refer to [8] and [9] for a detailed discussion about the use of the SHS approach in the analysis of AoI.

- There exists $s_0 > 0$ such that for all $s < s_0$, $\mathbf{v}_q^s(t)$ converges to $\bar{\mathbf{v}}_q^s$ that satisfies

$$\bar{\mathbf{v}}_q^s \sum_{l \in \mathcal{L}_q} \lambda^{(l)} = s\bar{\mathbf{v}}_q^s + \sum_{l \in \mathcal{L}'_q} \lambda^{(l)} [\bar{\mathbf{v}}_{q_l}^s \mathbf{A}_l + \bar{\pi}_{q_l} \mathbf{1} \hat{\mathbf{A}}_l], \; q \in \mathcal{Q}, \tag{6}$$

where $\hat{\mathbf{A}}_l \in \mathbb{B}^{(n+1)\times(n+1)}$ is a binary matrix whose elements are constructed as: $\hat{\mathbf{A}}_l(k,j) = 1$ if $k = j$ and the j-th column of $\mathbf{A}_l$ is a zero vector; otherwise, $\hat{\mathbf{A}}_l(k,j) = 0$. Further, the MGF of the state $\mathbf{x}(t)$, which can be obtained as $\mathbb{E}[e^{s\mathbf{x}(t)}]$, converges to the following stationary vector:

$$\mathbb{E}[e^{s\mathbf{x}}] = \sum_{q \in \mathcal{Q}} \bar{\mathbf{v}}_q^s. \tag{7}$$

From (5) and (7), when the first element of the continuous state $\mathbf{x}(t)$ represents the AoI at the destination node, the expectation and the MGF of AoI at the destination node respectively converge to:

$$\Delta_1 = \sum_{q \in \mathcal{Q}} \bar{v}_{q0}, \tag{8}$$

$$M(s) = \sum_{q \in \mathcal{Q}} \bar{v}_{q0}^s. \tag{9}$$

IV. Analysis of the MGF of AoI

It is clear from [9, Theorem 1] (stated in Section III) that in order to use (6) to derive the MGF of AoI, one needs to find a non-negative limit $\bar{\mathbf{v}}_q$ ($\forall q \in \mathcal{Q}$) satisfying (3). We have rigorously shown the existence of a non-negative $\bar{\mathbf{v}}_q$ satisfying (3) in the expanded journal version of this paper [17, Thoerem 3]. Note that the solution of the equations in (3) can be obtained along the same lines of the analysis presented in this paper for solving the equations in (6). Thus, for the sake of brevity, we next focus on evaluating $\bar{v}_{q0}^s, \forall q \in \mathcal{Q}$, satisfying (6), using which the MGF of AoI is obtained as in (9).

Without loss of generality, we consider that source 1 is the source of interest in the AoI analysis in the sequel. The AoI performance of the other sources can then be obtained using the same expressions derived for source 1, as will be evident shortly. While analyzing the AoI of source 1, the status update packets associated with the other sources are generated according to a Poisson process with rate $\lambda_{-1} = \sum_{j=2}^{N} \lambda_j$. Using the notations of the SHS approach (presented in Section III), the continuous process $\mathbf{x}(t)$ is given by $\mathbf{x}(t) = [x_0(t), x_1(t)]$, where $x_0(t)$ represents the value of the source 1's AoI at the destination at time instant t (i.e., $\Delta_1(t)$), and $x_1(t)$ indicates the value that the source 1's AoI at the destination will become if the existing update packet in the system completes its service at time instant t (i.e., the packet is delivered to the destination at t). Recall from Section III that as long as there is no change in $q(t)$, the elements of $\mathbf{x}(t)$ increase linearly with time.

The CTMC $q(t)$ is depicted in Fig. 2. Recall that according to the mechanism of the LCFS-SA queueing discipline, a new arriving status update preempts the update in service only if the two updates are generated from the same source. Thus, $q(t)$ needs to not only account for the number of status updates in the system but also track the index of the source which generated the current update in service. Because of that, we observe from Fig. 2 that for a state $q = (e_q, u_q)$, we have $u_q \in \{0, 1, \cdots, N\}$. In particular, $u_q = 0$ indicates that the system is empty and hence the server is idle, and $u_q = i$ indicates that there is an update in service and the index of its generating source is i. Further, due to the finite capacity of the battery queue at the server, we have $e_q \in \{0, 1, \cdots, B\}$. Table I presents the set of transitions $\mathcal{L}$ and their impact on the values of both $q(t)$ and $\mathbf{x}(t)$. We denote the set of states in the $i-$th row of the CTMC by r_i. From Fig. 2, we observe that the states in r_1 correspond to the time when the system is empty, and thus the subset of transitions represented by $l = (3N+1)\,k - 3N$ in Table I occurs when a new energy packet is harvested by the transmitter when its server is idle. The rest of transitions in Table I occur due to either the arrival of a new status update at the transmitter or the delivery of the status update in service to the destination. Since the first component of $\mathbf{x}(t)$ is only impacted by the delivery of a status update generated from source 1 (with rate λ_1) to the destination, we further observe that the first component of updated age vector $\mathbf{x}\mathbf{A}_l$ only changes to x_1 (age of the update in service) when a transition from the subset of transitions $l = (3N+1)\,k - N + 1$ (corresponding to the reception of an update of source 1 at the destination) occur. Finally, we note that the second component of $\mathbf{x}\mathbf{A}_l$ is always 0. This happens since this component either becomes irrelevant (when the system becomes empty or the new arriving update does not belong to source 1) or represents the age of an update of source 1 upon its arrival at the transmitter ($l = (3N+1)k - 3N + 1$ and $l = (3N+1)k - 2N + 1$).

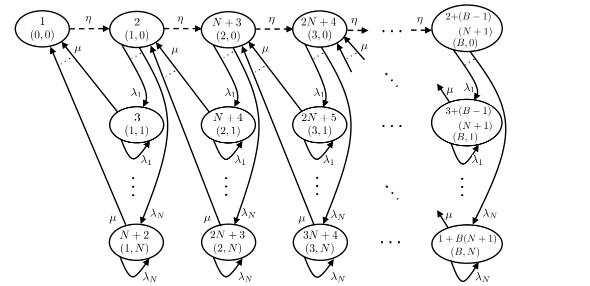

Fig. 2. The Markov chain modeling the discrete state in the LCFS-SA queueing discipline.

TABLE I
TRANSITIONS OF THE LCFS-SA QUEUEING DISCIPLINE IN FIG. 2
$(2 \leq i \leq N,\; 2 \leq k \leq B)$.

l	$q_l \to q'_l$	$\lambda^{(l)}$	$\mathbf{x}\mathbf{A}_l$	$\bar{\mathbf{v}}_{q_l}\mathbf{A}_l$	$\bar{\pi}_{q_l}\mathbf{1}\hat{\mathbf{A}}_l$
1	$1 \to 2$	η	$[x_0, 0]$	$[\bar{v}_{10}^s, 0]$	$[0, \bar{\pi}_1]$
2	$2 \to 3$	λ_1	$[x_0, 0]$	$[\bar{v}_{20}^s, 0]$	$[0, \bar{\pi}_2]$
$1+i$	$2 \to 2+i$	λ_i	$[x_0, 0]$	$[\bar{v}_{20}^s, 0]$	$[0, \bar{\pi}_2]$
$2+N$	$3 \to 3$	λ_1	$[x_0, 0]$	$[\bar{v}_{30}^s, 0]$	$[0, \bar{\pi}_3]$
$1+N+i$	$2+i \to 2+i$	λ_i	$[x_0, 0]$	$[\bar{v}_{2+i,0}^s, 0]$	$[0, \bar{\pi}_{2+i}]$
$2\,(N+1)$	$3 \to 1$	μ	$[x_1, 0]$	$[\bar{v}_{31}^s, 0]$	$[0, \bar{\pi}_3]$
$1+2N+i$	$2+i \to 1$	μ	$[x_0, 0]$	$[\bar{v}_{2+i,0}^s, 0]$	$[0, \bar{\pi}_{2+i}]$
$(3N+1)\,k-3N$	$2+(N+1)(k-2) \to 2+(N+1)(k-1)$	η	$[x_0, 0]$	$[\bar{v}_{q_l,0}^s, 0]$	$[0, \bar{\pi}_{q_l}]$
$(3N+1)\,k-3N+1$	$2+(N+1)(k-2) \to 3+(N+1)(k-2)$	λ_1	$[x_0, 0]$	$[\bar{v}_{q_l,0}^s, 0]$	$[0, \bar{\pi}_{q_l}]$
$(3N+1)\,k-3N+i$	$2+(N+1)(k-2) \to 2+i+(N+1)(k-2)$	λ_i	$[x_0, 0]$	$[\bar{v}_{q_l,0}^s, 0]$	$[0, \bar{\pi}_{q_l}]$
$(3N+1)\,k-2N+1$	$3+(N+1)(k-2) \to 3+(N+1)(k-2)$	λ_1	$[x_0, 0]$	$[\bar{v}_{q_l,0}^s, 0]$	$[0, \bar{\pi}_{q_l}]$
$(3N+1)\,k-2N+i$	$2+i+(N+1)(k-2) \to 2+i+(N+1)(k-2)$	λ_i	$[x_0, 0]$	$[\bar{v}_{q_l,0}^s, 0]$	$[0, \bar{\pi}_{q_l}]$
$(3N+1)k-N+1$	$3+(N+1)(k-1) \to 2+(N+1)(k-2)$	μ	$[x_1, 0]$	$[\bar{v}_{q_l,1}^s, 0]$	$[0, \bar{\pi}_{q_l}]$
$(3N+1)k-N+i$	$2+i+(N+1)(k-1) \to 2+(N+1)(k-2)$	μ	$[x_0, 0]$	$[\bar{v}_{q_l,0}^s, 0]$	$[0, \bar{\pi}_{q_l}]$

Now, in order to obtain $\bar{\mathbf{v}}_q^s$ satisfying (6), one needs to compute the state probabilities $\{\bar{\pi}_q\}$, and the vectors $\bar{\mathbf{v}}_{q_l}^s \mathbf{A}_l$ and $\bar{\pi}_{q_l} \mathbf{1} \hat{\mathbf{A}}_l$. The calculations of $\bar{\mathbf{v}}_{q_l}^s \mathbf{A}_l$ and $\bar{\pi}_{q_l} \mathbf{1} \hat{\mathbf{A}}_l$ are listed in Table I, and $\{\bar{\pi}_q\}$ are given by the following proposition.

Proposition 1. *The steady state probabilities $\{\bar{\pi}_q\}$ can be* *expressed as*

$$\bar{\pi}_{2+(k-1)(N+1)} = \left(\frac{\beta}{\rho}\right)^k \bar{\pi}_1, \quad (10)$$

$$\bar{\pi}_{2+i+(k-1)(N+1)} = \rho_i \left(\frac{\beta}{\rho}\right)^k \bar{\pi}_1, \quad (11)$$

where $1 \leq k \leq B$, $1 \leq i \leq N$, *and* $\bar{\pi}_1$ *is is given by*

$$\bar{\pi}_1 = \begin{cases} \dfrac{1}{1+B(1+\rho)}, & \text{if } \rho=\beta, \\ \dfrac{\rho^B(\beta-\rho)}{\rho^B(\beta-\rho)+\beta(1+\rho)(\beta^B-\rho^B)}, & \text{otherwise.} \end{cases} \quad (12)$$

Proof: The expressions in (10) and (11) follow directly from the solution of (4). ■

Having the steady state probabilities $\{\bar{\pi}_q\}$ in Proposition 1 and the set of transitions $\mathcal{L}$ in Table I, we are now ready to derive the MGF of AoI in the following theorem.

Theorem 1. *The MGF of AoI of source 1 under the LCFS-SA queueing discipline is given by*

$$\overset{\text{SA}}{M}_1(\bar{s}) = \frac{\rho_1\left[(1+\rho-\bar{s})\sum_{q\in \mathrm{r}_1\cup\, \mathrm{r}_2/\{1\}}\bar{\pi}_q + (1+\rho_1-\bar{s})\,\bar{v}^s_{10}\right]}{(1+\rho_1-\bar{s})\left[(1-\bar{s})(\rho-\bar{s})-\rho_{-1}\right]}, \quad (13)$$

where $\bar{s}=\frac{s}{\mu}$ *and* $\bar{v}^s_{10}$ *is given by*

$$\bar{v}^s_{10} = \frac{\mu\rho_1}{1+\rho_1-\bar{s}}\sum_{j=0}^{B-1}\frac{\bar{\pi}_{2+j(N+1)}+\bar{\pi}_{3+j(N+1)}}{\prod_{h=0}^{j}\bar{c}^s_{h-1}}\left(\frac{\mu\rho_{-1}}{1-\bar{s}}\right)^{j-1}, \quad (14)$$

where the set $\{\bar{c}^s_{-1},\bar{c}^s_0,\cdots,\bar{c}^s_{B-1}\}$ *is defined as*

$$\bar{c}^s_h = \begin{cases} \lambda - s, & h=B-1, \\ \eta+\lambda-s-\dfrac{\mu\eta\lambda_{-1}}{\bar{c}^s_{h+1}(\mu-s)}, & 0\leq h\leq B-2, \\ \dfrac{(\mu-s)(\eta-s)}{\mu\lambda_{-1}}-\dfrac{\eta}{\bar{c}^s_0}, & h=-1. \end{cases} \quad (15)$$

Proof: See Appendix. ■

Note that the MGF of AoI for source $i\in\{2,3,\cdots,N\}$ can be obtained directly using (13) by replacing λ_1 with λ_i (which results in replacing $\{\lambda_{-1},\rho_1,\rho_{-1}\}$ with $\{\lambda_{-i},\rho_i,\rho_{-i}\}$ as well). This argument applies to all the following results.

Corollary 1. *When* $\rho_{-1}=0$ *(i.e.,* $\rho_1=\rho$*),* $\overset{\text{SA}}{M}_1(\bar{s})$ *in (13) reduces to the following MGF of AoI derived in [16, Theorem 3] for EH-powered single-source systems under the LCFS with preemption in service (LCFS-PS) queueing discipline*

$$\overset{\text{PS}}{M}_1(\bar{s}) = \frac{\rho(1+\rho)\bar{\pi}_1\left[\bar{s}^2\theta-\bar{s}\theta(1+\rho+\beta)+\beta(1+\theta+\theta\rho)\right]}{(1-\bar{s})(\rho-\bar{s})(1+\rho-\bar{s})(\beta-\bar{s})}, \quad (16)$$

where θ *is given by*

$$\theta = \begin{cases} B, & \text{if } \rho=\beta, \\ \dfrac{\beta(\beta^B-\rho^B)}{\rho^B(\beta-\rho)}, & \text{otherwise.} \end{cases} \quad (17)$$

Proof: When $\rho_{-1}=0$, $\bar{v}^s_{10}$ reduces to $\dfrac{\rho_1(1+\rho_1)\bar{\pi}_2}{(1+\rho_1-\bar{s})(\beta-\bar{s})}$ since we have from (15) that: $\bar{c}^s_h=\eta+\lambda-s, 1\leq h\leq B-1$, and $\bar{c}^s_{-1}=\infty$. The final expression in (16) can be obtained by defining $\sum_{q\in\mathrm{r}_1/\{1\}}\bar{\pi}_q=\theta\bar{\pi}_1$ and substituting $\bar{\pi}_2$ from Proposition 1 as $\frac{\beta}{\rho}\bar{\pi}_1$. ■

Corollary 2. *The average AoI of source 1 under the LCFS-SA queueing discipline is given by:* $\overset{\text{SA}}{\Delta}_{1,1}=$

$$\frac{1+\rho}{\mu\rho_1(1+\rho_1)}+\frac{(1+\rho)\sum_{q\in\mathcal{Q}/\mathrm{r}_2}\bar{\pi}_q}{\mu(1+\rho_1)}+\frac{\sum_{q\in\mathcal{Q}/\mathrm{r}_1}\bar{\pi}_q}{\mu}+\bar{v}_{10}, \quad (18)$$

where $\bar{v}_{10}$ *is given by*

$$\bar{v}_{10} = \frac{\bar{\pi}_1}{\bar{c}_{-1}\mu\rho_{-1}}+\sum_{j=1}^{B}\frac{\bar{\pi}_{2+(j-1)(N+1)}(\mu\rho_{-1})^{j-1}}{\prod_{h=0}^{j}\bar{c}_{h-1}} + \sum_{j=0}^{B-1}\frac{\frac{\bar{\pi}_{3+j(N+1)}}{1+\rho_1}+\sum_{m=4+j(N+1)}^{1+(j+1)(N+1)}\bar{\pi}_m}{\prod_{h=0}^{j}\bar{c}_{h-1}}(\mu\rho_{-1})^{j-1}, \quad (19)$$

where the set $\{\bar{c}_{-1},\bar{c}_0,\cdots,\bar{c}_{B-1}\}$ *is defined as*

$$\bar{c}_h = \begin{cases} \lambda, & h=B-1, \\ \eta\left(1-\dfrac{\lambda_{-1}}{\bar{c}_{h+1}}\right)+\lambda, & 0\leq h\leq B-2, \\ \eta\left(\dfrac{1}{\lambda_{-1}}-\dfrac{1}{\bar{c}_0}\right), & h=-1. \end{cases} \quad (20)$$

Proof: The result can be obtained from either the first derivative of the MGF of AoI in Theorem 1 or the solution of the set equations in (3) as in (8). ■

Corollary 3. *For the single source case where* $\rho_{-1}=0$ *(i.e.,* $\rho=\rho_1$*),* $\overset{\text{SA}}{\Delta}_{1,1}$ *in (18) reduces to:* $\overset{\text{SA}}{\Delta}_{1,1}=$

$$\begin{cases} \dfrac{B\rho^3+(3B+1)\rho^2+(3B+4)\rho+B+2}{\mu\rho(1+\rho)(\rho B+B+1)}, & \text{if } \rho=\beta, \\ \dfrac{\beta^{B+2}(1+\rho)^3-\rho^{B+2}\left[(\beta^2+\beta)(\rho+2)+1+\rho\right]}{\mu(1+\rho)\left[\beta^{B+2}(\rho^2+\rho)-\rho^{B+2}(\beta^2+\beta)\right]}, \end{cases} \quad (21)$$

where the second case in (21) holds when $\rho\neq\beta$*. Note that the expression of* $\overset{\text{SA}}{\Delta}_{1,1}$ *in (21) is identical to the average AoI expression derived in [16, Corollary 3] for EH-powered single-source systems under the LCFS-PS queueing discipline.*

Proof: We note from (20) that when $\rho_{-1}=0$, we have $\bar{c}_h=\eta+\lambda, 0\leq h\leq B-2$, and $c_{-1}=\infty$. Thus, $\bar{v}_{10}$ in (19) reduces to: $\bar{v}_{10}=\dfrac{\bar{\pi}_1+\frac{\bar{\pi}_3}{1+\rho_1}+\sum_{m=4}^{N+2}\bar{\pi}_m}{\eta}$. The final expression in (21) can be obtained by substituting $\{\bar{\pi}_q\}$ from Proposition 1, followed by some algebraic simplifications. ■

Let $\overset{\text{D}}{\Delta}_{i,j}$ denote the j-th moment of source i's AoI under D, where D = PS and D = WP refer to the source-agnostic preemptive in service LCFS and the non-preemptive LCFS queueing disciplines (studied in [17]), respectively.

Corollary 4. *When* $\beta\to\infty$, $\overset{\text{SA}}{\Delta}_{1,1}$ *in (18) reduces to*

$$\lim_{\beta\to\infty}\overset{\text{SA}}{\Delta}_{1,1}=\frac{1+\rho}{\mu\rho_1}+\frac{\rho_{-1}}{\mu(1+\rho)(1+\rho_1)}, \quad (22)$$

which indicates that $\lim_{\beta\to\infty}\overset{\text{PS}}{\Delta}_{1,1}\leq\lim_{\beta\to\infty}\overset{\text{SA}}{\Delta}_{1,1}\leq\lim_{\beta\to\infty}\overset{\text{WP}}{\Delta}_{1,1}$.

Proof: The result follows from noting that: $\lim_{\beta\to\infty}\bar{v}_{10} = 0$ and $\frac{(1+\rho)\sum_{q\in\mathcal{Q}/\mathrm{r}_2}\bar{\pi}_q}{\mu(1+\rho_1)} + \frac{\sum_{q\in\mathcal{Q}/\mathrm{r}_1}\bar{\pi}_q}{\mu} = \frac{(1+\rho)^2+\rho_{-1}}{\mu(1+\rho)(1+\rho_1)}$. ■

Remark 1. *Note that from Corollary 2, [17, Theorem 1] and [17, Theorem 2], we have*

$$\overset{\mathrm{WP}}{\Delta}_{1,1} - \overset{\mathrm{SA}}{\Delta}_{1,1} = \frac{\rho_1(1+\rho)\sum_{k=1}^{B}(\frac{\beta}{\rho})^k}{\mu(1+\rho_1)\left[1+(1+\rho)\sum_{k=1}^{B}\left(\frac{\beta}{\rho}\right)^k\right]} + \frac{\bar{\pi}_1\rho_1^2}{1+\rho_1}\sum_{j=0}^{B-1}\frac{\left(\frac{\beta}{\rho}\right)^{j+1}(\mu\rho_{-1})^{j-1}}{\prod_{h=0}^{j}\bar{c}_{h-1}}, \quad (23)$$

$$\overset{\mathrm{SA}}{\Delta}_{1,1} - \overset{\mathrm{PS}}{\Delta}_{1,1} = \frac{\rho_{-1}\sum_{k=1}^{B}(\frac{\beta}{\rho})^k}{\mu(1+\rho_1)\left[1+(1+\rho)\sum_{k=1}^{B}\left(\frac{\beta}{\rho}\right)^k\right]} + \frac{\bar{\pi}_1\rho_1\rho_{-1}}{(1+\rho_1)(1+\rho)}\sum_{j=0}^{B-1}\frac{\left(\frac{\beta}{\rho}\right)^{j+1}(\mu\rho_{-1})^{j-1}}{\prod_{h=0}^{j}\bar{c}_{h-1}}. \quad (24)$$

Since the set $\{\bar{c}_{-1},\bar{c}_0,\cdots,\bar{c}_{B-1}\}$ contains positive real numbers, we observe from (23) and (24) that $\overset{\mathrm{PS}}{\Delta}_{1,1} \leq \overset{\mathrm{SA}}{\Delta}_{1,1} \leq \overset{\mathrm{WP}}{\Delta}_{1,1}$ for any choice of values of the system parameters.

Remark 2. *Note that one can deduce from Theorem 1, [17, Theorem 4] and [17, Theorem 5] that $\overset{\mathrm{PS}}{\Delta}_{1,2} \leq \overset{\mathrm{SA}}{\Delta}_{1,2} \leq \overset{\mathrm{WP}}{\Delta}_{1,2}$ for any choice of system parameter values. Further, when $\rho_{-1}=0$ (i.e., $N=1$), we have $\overset{\mathrm{WP}}{\Delta}_{1,2}-\overset{\mathrm{PS}}{\Delta}_{1,2} = \overset{\mathrm{WP}}{\Delta}_{1,2}-\overset{\mathrm{SA}}{\Delta}_{1,2}$.*

V. Numerical Results and Concluding Remarks

Impact of β and B on the achievable AoI performance. We study the impact of B on the achievable pairs of average AoI $(\Delta_{1,1},\Delta_{2,1})$ in Fig. 3d when $N=2$ and ρ is fixed. We observe from Figs. 3a and 3d that the AoI performance improves with increasing B or β until it converges to its counterpart with a non-EH transmitter (as stated in Corollary 4 for the first moment). This happens since increasing B or β decreases the likelihood that the battery queue is empty upon the arrival of a new status update at the transmitter when the server is idle, and hence increases the likelihood of delivering new arriving updates to the destination.

Minimum average sum-AoI vs. Fairness between the achievable average AoI values by different sources. In Figs. 3b, 3c, 3e and 3f, we compare the LCFS-SA queueing discipline with the LCFS-PS and LCFS-WP queueing disciplines in terms of: i) the average sum-AoI $\Delta_{1,1}+\Delta_{2,1}$, and ii) the Jain's fairness index, which is defined as $\mathrm{JFI} = \frac{\left(\sum_{i=1}^{N}\Delta_{i,1}\right)^2}{N\sum_{i=1}^{N}\Delta_{i,1}^2}$ [18]. Note that the $\mathrm{JFI}\in\left[N^{-1},1\right]$ is a measure of the fairness between the achievable average AoI values by different sources such that $\mathrm{JFI}=1$ when the average AoI values of different sources are equal (the best case scenario with respect to fairness). We observe that there is a fundamental trade-off between obtaining a minimum average sum-AoI and having fair achievable average AoI values among different sources. Further, the LCFS-SA queueing discipline achieves a balance between the two performance aspects compared to the two other disciplines. In particular, it achieves a close average sum-AoI to the achievable one by the LCFS-PS, and it is more effective (especially as N becomes large) than the LCFS-PS in terms of the achievable fairness performance.

Is it reasonable to solely rely on the average AoI in the implementation/optimization of multi-source real-time updating systems as has been mostly done in the existing literature? As was the case in [16] for EH-powered single-source systems, we observe from Figs. 3b and 3e that the standard deviation of AoI σ associated with each queueing discipline in multi-source systems is relatively large with respect to the average value. This indicates that the implementation of multi-source status update systems based on just the average value of AoI does not ensure reliability, and it is crucial to incorporate the higher moments of AoI in the design/optimization of such systems. This insight demonstrates the significance of the analytical distributional properties of AoI derived in this paper.

Appendix

Using Table I, the set of equations in (6) corresponding to $q\in\mathrm{r}_1$ can be expressed as

$$q_1: \quad (\eta-s)\,\bar{v}_{10}^s = \mu\bar{v}_{31}^s + \mu\sum_{j=4}^{N+2}\bar{v}_{j0}^s, \quad (25)$$

$$q_2: \quad (\eta+\lambda-s)\,\bar{v}_{20}^s = \eta\bar{v}_{10}^s + \mu\bar{v}_{N+4,1}^s + \mu\sum_{j=N+5}^{2N+3}\bar{v}_{j0}^s, \quad (26)$$

$$q_{2+k(N+1)}, 1\leq k\leq B-2: \quad (\eta+\lambda-s)\,\bar{v}_{2+k(N+1),0}^s = \eta\,\bar{v}_{2+(k-1)(N+1),0}^s + \mu\bar{v}_{3+(k+1)(N+1),1}^s + \mu\sum_{j=4+(k+1)(N+1)}^{N+2+(k+1)(N+1)}\bar{v}_{j0}^s, \quad (27)$$

$$q_{2+(B-1)(N+1)}: \bar{v}_{2+(B-1)(N+1),0}^s = \frac{\eta}{\lambda-s}\bar{v}_{2+(B-2)(N+1),0}^s. \quad (28)$$

Further, the set of equations in (6) corresponding to $q\in\mathrm{r}_{i+1}$, $1\leq i\leq N$, can be expressed as

$$q_{2+i+k(N+1)}, 0\leq k\leq B-1: \quad (\mu-s)\,\bar{v}_{2+i+k(N+1),0}^s = \lambda_i\bar{v}_{2+k(N+1),0}^s. \quad (29)$$

Summing the equations in (25)-(28) gives

$$(\lambda-s)\sum_{q\in\,\mathrm{r}_1}\bar{v}_{q0}^s = \mu\sum_{q\in\,\mathrm{r}_2}\bar{v}_{q1}^s + \mu\sum_{q\in\,\mathcal{Q}/(\mathrm{r}_1\cup\,\mathrm{r}_2)}\bar{v}_{q0}^s + \lambda\bar{v}_{10}^s, \quad (30)$$

where $\sum_{q\in\,\mathrm{r}_2}\bar{v}_{q1}^s = \frac{\lambda_1\sum_{q\in\,\mathrm{r}_1\cup\,\mathrm{r}_2/\{1\}}\bar{\pi}_q}{(\mu+\lambda_1-s)}$. In addition, by summing the equations in (29), we get

$$(\mu-s)\sum_{q\in\,\mathrm{r}_{i+1}}\bar{v}_{q0}^s = \lambda_i\sum_{q\in\,\mathrm{r}_1/\{1\}}\bar{v}_{q0}^s, \quad (31)$$

where $1\leq i\leq N$. From (30) and (31), $\sum_{q\in\,\mathrm{r}_1}\bar{v}_{q0}^s$ can be obtained as

$$\left[(1-\bar{s})(\rho-\bar{s})-\rho_{-1}\right]\sum_{q\in\,\mathrm{r}_1}\bar{v}_{q0}^s = (\rho_1-\rho\bar{s})\,\bar{v}_{10}^s$$

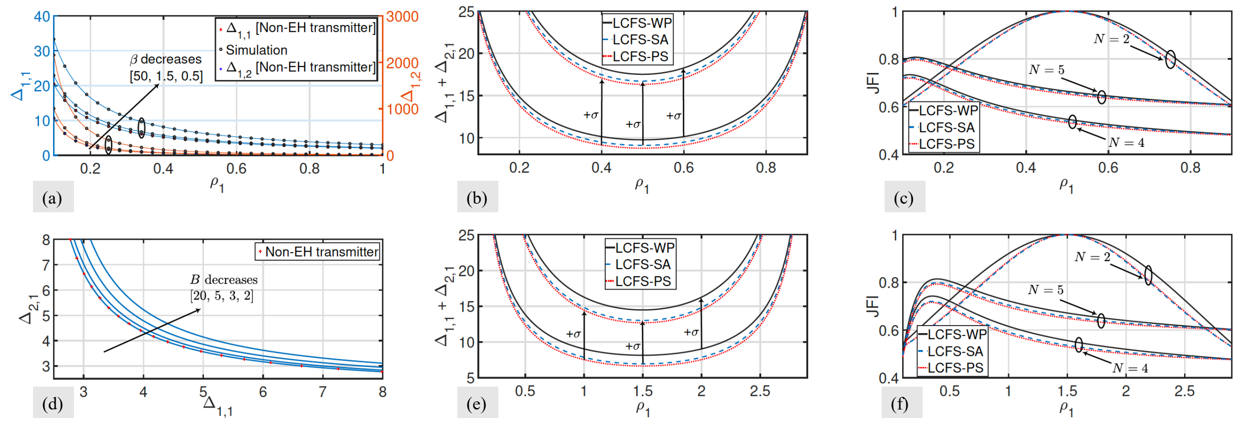

Fig. 3. Verification of the analytical results and impact of the system design parameters on the achievable AoI performance. Unless otherwise specified, we use the following values for different system parameters: $\mu=1$, $\beta=1.5$, $B=2$, $N=2$, and $\rho=1$ [$\rho=3$] in Figs. (a)-(d) [(e) and (f)]. Note that N can be chosen arbitrary in (a), and for $N\in\{4,5\}$ in (c) and (f), we set $\rho_2=0.1(\rho-\rho_1)$ and $\rho_i=\frac{0.9}{N-2}(\rho-\rho_1)$, $3\le i\le N$.

$$+\frac{\rho_1(1-\bar{s})\sum_{q\in\,\mathrm{r}_1\cup\,\mathrm{r}_2/\{1\}}\bar{\pi}_q}{1+\rho_1-\bar{s}}, \tag{32}$$

where $\bar{s}=\frac{s}{\mu}$. Hence, the MGF of AoI of source 1 can be evaluated as

$$\overset{\text{SA}}{M}_1(\bar{s})=\sum_{q\in\,\mathcal{Q}}\bar{v}^s_{q0}\overset{\text{(a)}}{=}\frac{(\lambda+\mu-s)\sum_{q\in\,\mathrm{r}_1}\bar{v}^s_{q0}-\lambda\bar{v}^s_{10}}{\mu-s}$$
$$\overset{\text{(b)}}{=}\frac{\rho_1\Big[(1+\rho-\bar{s})\sum_{q\in\mathrm{r}_1\cup\,\mathrm{r}_2/\{1\}}\bar{\pi}_q+(1+\rho_1-\bar{s})\,\bar{v}^s_{10}\Big]}{(1+\rho_1-\bar{s})\Big[(1-\bar{s})(\rho-\bar{s})-\rho_{-1}\Big]}, \tag{33}$$

where step (a) [step (b)] follows from substituting (31) [(32)] into (33). Now, what only remains is to show how $\bar{v}^s_{10}$ can be expressed as in (14). From (28), $\bar{v}^s_{2+(B-1)(N+1),0}$ is given by

$$\bar{v}^s_{2+(B-1)(N+1),0}=\frac{\eta}{\bar{c}^s_{B-1}}\bar{v}^s_{2+(B-2)(N+1),0}, \tag{34}$$

where $\bar{c}^s_{B-1}=\lambda-s$. By noting that $\bar{v}^s_{3+k(N+1),1}=\frac{\lambda_1\left(\bar{\pi}_{2+k(N+1)}+\bar{\pi}_{3+k(N+1)}\right)}{\mu+\lambda_1-s}$, $0\le k\le B-1$, (27) can be rewritten as

$$\bar{v}^s_{2+k(N+1),0}=\frac{\eta\bar{v}^s_{2+(k-1)(N+1),0}}{(\eta+\lambda-s)}+\frac{\mu\lambda_{-1}\bar{v}^s_{2+(k+1)(N+1),0}}{(\mu-s)(\eta+\lambda-s)}$$
$$+\frac{\mu\lambda_1\left(\bar{\pi}_{2+k(N+1)}+\bar{\pi}_{3+k(N+1)}\right)}{(\mu+\lambda_1-s)(\eta+\lambda-s)}, \tag{35}$$

where $1\le k\le B-2$, and $\sum_{j=4+(k+1)(N+1)}^{N+2+(k+1)(N+1)}\bar{v}^s_{j0}$ in (27) was substituted from (29). Repeated application of (35) gives

$$\bar{v}^s_{2+k(N+1),0}=\frac{\eta\bar{v}^s_{2+(k-1)(N+1),0}}{\bar{c}^s_k}+\frac{\mu\lambda_1}{\mu+\lambda_1-s}$$
$$\sum_{j=1}^{B-k-1}\frac{\bar{\pi}_{2+(k+j)(N+1)}+\bar{\pi}_{3+(k+j)(N+1)}}{\prod_{h=1}^{j}\bar{c}^s_{k+h-1}}\left(\frac{\mu\rho_{-1}}{1-\bar{s}}\right)^{j-1}, \tag{36}$$

$$\bar{v}^s_{2,0}=\frac{\mu\lambda_1}{\mu+\lambda_1-s}\sum_{j=1}^{B-1}\frac{\bar{\pi}_{2+j(N+1)}+\bar{\pi}_{3+j(N+1)}}{\prod_{h=1}^{j}\bar{c}^s_{h-1}}\left(\frac{\mu\rho_{-1}}{1-\bar{s}}\right)^{j-1}$$
$$+\frac{\eta\bar{v}^s_{1,0}}{\bar{c}^s_0}, \tag{37}$$

where $1\le k\le B-1$ and $\{\bar{c}^s_h\}$ is defined in (15). Finally, $\bar{v}^s_{10}$ in (14) can be obtained by solving (25) and (37) while noting that $\bar{v}_{31}=\frac{\lambda_1(\bar{\pi}_2+\bar{\pi}_3)}{\mu+\lambda_1-s}$ and $\sum_{j=4}^{N+2}\bar{v}^s_{j0}=\frac{\lambda_{-1}\bar{v}^s_{20}}{\mu-s}$. ∎

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