Proceedings of the ASME 2022
Pressure Vessels & Piping Conference
PVP2022
July 17-22, 2022, Las Vegas, NV, USA

PVP2022-84546

DANCING MANHOLE-COVER BASICS

Niels F. van de Meulenhof
Department of Mathematics and Computer Science
Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven
The Netherlands
Email: n.f.v.d.meulenhof@student.tue.nl

Arris S. Tijsseling
Department of Mathematics and Computer Science
Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven
The Netherlands
Email: a.s.tijsseling@tue.nl

Jose G. Vasconcelos
Department of Civil Engineering
Auburn University
Auburn, AL 36849
U.S.A.
E-mail: jgv@auburn.edu

ABSTRACT

A manhole is a shaft that functions as an access point to the underground infrastructure and is covered with a very heavy lid, sometimes weighing more than 100 kg. Occasionally a strange phenomenon occurs in which such a manhole cover is lifted above its opening and sort of dances on or above its supporting ring without any human intervention. This usually happens when it is stormy with heavy rainfall, but it is not tied to one specific location. Videos from all over the world can be found on the internet showing such 'dancing manhole covers'. Sometimes air seems to be the main driving force behind the behavior, sometimes water, and sometimes both. Although the videos are funny, the behavior can create a very dangerous situation for both traffic and pedestrians. In this report the cause of these 'dancing manhole covers' is studied.

The 'dancing' is simplified into two different problems: one with an overflow of air and one with an overflow of water. For both problems a simple model consisting of differential equations is proposed and the numerical results are studied. The problem with an overflow of air is driven by an influx of air into the manhole from below, resulting in an increase in pressure, which lifts up the cover, until air is allowed to escape, and the pressure decreases again. Two different approaches for the escaping discharge of air are tried. The overflow of water is driven by a constant pressure that is exerted on a water column inside the manhole. Furthermore, a solution to the dancing problem is proposed: attaching the manhole cover to the ground with a hinge. This solution is tested by using a similar model as the one used for the overflow of air.

Keywords: manhole cover, air flow, water flow, filling, leaking, vent, hinge, sewer

INTRODUCTION

Displaced manhole covers pose a danger to pedestrians, bicycles, cars, and other traffic. Yamamoto [1] presented one of the first studies on the subject. Wang and Vasconcelos [2] investigated the likelihood of manhole cover displacement caused by either the pressurized air in the vertical shaft headspace or water impact when a stormwater system is undergoing rapid filling. Walski et al. [3] and Chen et al. [4] included cover weight and displacement in their mathematical models. The problem may be so urgent that tilt and vibration sensors are attached to all manhole covers in a (smart) city to monitor their displacements [5,6] and to warn pedestrians via their smartphones [7].

In this study the so-called phenomenon of ‘dancing manhole covers’ is approached in two different ways. The first problem considers an overflow of air. What is a realistic way to model the influx of air and how does the air escape when the cover is lifted above the ground? The second problem considers an overflow of water. What is a realistic way to model the leakage of water when the cover is opened? How does the manhole cover behave for different driving pressures?

Both problems are described by models containing equations similar to ones used in [8] and [9]. The current paper is based on [10].

In a very exploratory fashion the numerical results of both models are studied, as well as the difference in behavior. The results are compared to the behavior of manhole covers as seen on videos.

A possible solution to lessen the impact of the ’dancing’ is to attach the manhole cover to the ground with a hinge. Hinged manhole covers are used already in some places, but are they indeed a safer alternative? For this solution a simple model is introduced and its dynamics is studied. The hinged cover model is compared with the other models.

AIR-DRIVEN MODEL

In the first proposed model the manhole cover with mass m_c, diameter D_c and depth d_c moves up and down as a result of a change in pressure P inside the manhole. The manhole itself is a vertical cylinder with length L and diameter D, as portrayed in Fig. 1. The system is driven by an influx of air into the manhole from below. The air pressure inside the manhole acts on the area of the cover $A_c = \pi D_c^2/4$. The cover will start to move up with vertical displacement x_c when $(P - P_{atm})A > m_c g$, where $A = \pi D^2/4$ is the cross-sectional area of the manhole. However, when the cover is lifted up slightly, the air pressure acts on the area of the cover A_c instead of A. This gives rise to the differential equation:

$$\frac{\mathrm{d}^2 x_c}{\mathrm{d}t^2} = \frac{\mathrm{d}v_c}{\mathrm{d}t} = -g + \frac{A_c}{m_c}(P - P_{atm}) \tag{1}$$

Herein v_c is the vertical velocity of the manhole cover. Note that there is a nonlinear constraint on the displacement x_c, that is

$$x_c \geq 0 \tag{2}$$

since the cover will always rest on its supporting ring when $x_c = 0$ m.

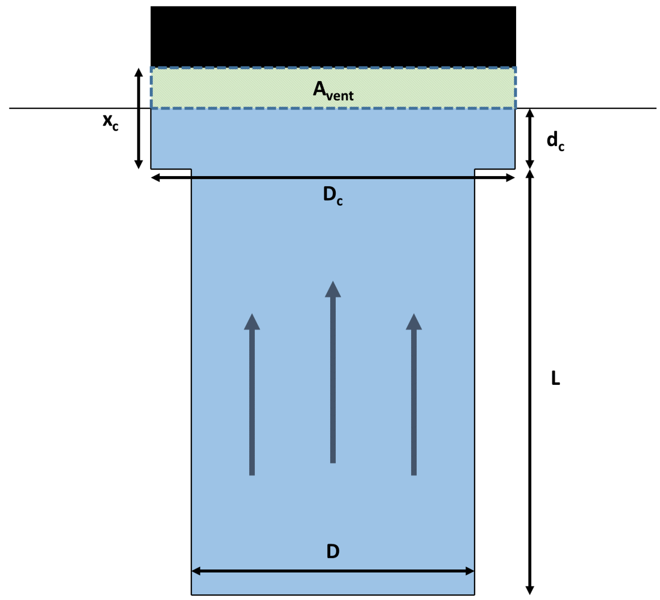

FIGURE 1. Manhole cover hovering above the ground

In the process the ideal gas law is assumed to hold. Furthermore the process is assumed to be isentropic, as the heat conduction through the walls and cover is assumed to be too slow with respect to the time it takes for the air to be compressed, move the cover, and expand due to the leakage. These assumptions imply that the system has to satisfy the differential equation:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{nP}{m}\frac{\mathrm{d}m}{\mathrm{d}t} \tag{3}$$

where m is the mass of the air inside the manhole and n is the polytropic index.

When the manhole cover is lifted above ground level entirely, i.e. $x_c > d_c$, air is allowed to escape and the pressure inside the manhole will decrease. Two different approaches are considered to describe the relation between air pressure P and velocity of leaking air U. The first one is a linear relation as given in [11]:

$$U = K_1(P - P_{atm}) \tag{4}$$

The second one is an equation based on the valve flow relation with discharge coefficient as given in [12]:

$$\rho U^2 = K_2(P - P_{atm}) \tag{5}$$

Herein K_1 and K_2 are constants and ρ is the density of the air. The latter is assumed to be uniform with respect to position within the manhole, so it can be written as

$$\rho = \frac{m}{\pi(D/2)^2 L + \pi(D_c/2)^2 x_c} \tag{6}$$

The velocities above are used in determining the mass flow rate of escaping air. At one end air flows with velocity v_{in} into the manhole through a cross-sectional area $A_{in} = \pi D^2/4$, compressing the air inside the manhole. At the other end air escapes, decompressing the air, through an area

$$A_{vent} = \begin{cases} 0 & \text{if } x_c \leq d_c \\ \pi D_c(x_c - d_c) & \text{if } x_c > d_c \end{cases} \tag{7}$$

For very large values of x_c this equation does not hold anymore and the maximum outflow area is simply A_c. This maximum has not been implemented herein, because the model is intended for small cover displacements only. These can be combined to get the differential equation for the rate of change of the air mass inside the manhole:

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \rho(Av_{in} - A_{vent}U) \tag{8}$$

where U is given by either Eq. (4) or (5), ρ by Eq. (6), and A_{vent} by Eq. (7). The behavior of the manhole cover is described by the four variables: air mass m, air pressure P, cover displacement x_c and cover velocity v_c. They have to satisfy a system of four differential equations, respectively Eq. (8), Eq. (3), Eq. (1) and

$$\frac{\mathrm{d}x_c}{\mathrm{d}t} = v_c \tag{9}$$

Friction is neglected within the manhole.

WATER-DRIVEN MODEL

The second model considers the same setup as shown in Fig. 1, only now with water inside the manhole instead of air. Furthermore, the areas A and A_c are taken equal for simplicity.

The system consists of two masses, the water column m and the manhole cover m_c, which are considered as two separate systems coupled via an intermediate pressure P_1 at the top of the manhole. A constant pressure P_{in} is exerted on the water column from below. Since the water inside the manhole is assumed to have constant density ρ, the water column will rise and lift the cover when $(P_{in} - P_{atm})A > (m + m_c)g$. The initial situation of the system is such that the manhole is filled entirely with water. As water enters the manhole from below, the water column moves up. Furthermore the assumption is made that air ventilation is ideal and at all times there is no air between water column and cover, so the two are always connected. The mass of the water column is $m = \rho A(L + x_c)$. For both water column and manhole cover a separate equation of motion is defined to describe the behavior in terms of their respective velocities v and v_c:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{P_{in} - P_1}{\rho(L + x_c)} - g \tag{10}$$

$$\frac{\mathrm{d}v_c}{\mathrm{d}t} = \frac{(P_1 - P_{atm})A}{m_c} - g \tag{11}$$

In the model two distinct situations can be distinguished based on the position of the manhole cover: one in which water cannot leak, i.e. $x_c \leq d_c$, and one in which water is able to leak, i.e. $x_c > d_c$. In the first case the problem is described by the system of equations consisting of Eq. (9) and the following:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{(P_{in} - P_{atm})A}{m_c + m} - g \tag{12}$$

$$v = v_c \tag{13}$$

By subtracting Eq. (10) from Eq. (11) and applying Eq. (13) the intermediate pressure P_1 in this case is found to behave like:

$$P_1 = \frac{m_c P_{in} + m P_{atm}}{m + m_c} \tag{14}$$

In the second case the cover is lifted above the ground and water is allowed to leak. The system is then decoupled in the sense that the water column and the manhole cover each have their own equation of motion, i.e. Eq. (10) and Eq. (11). The area through which the water can escape is treated like an orifice. It is described by the equation for the pressure difference on the cover, which is based on [9]:

$$P_1 - P_{atm} = \left(\frac{A}{C_d A_{vent}}\right)^2 \frac{1}{2}\rho(v - v_c)^2 \tag{15}$$

Note the relative velocity $v - v_c$. Taking v_c equal to zero would give a quasi-steady formula based solely on x_c (and excluding its time-derivative v_c) via A_{vent}.

HINGED COVER MODEL

In this section, a new model is proposed as a solution to the 'dancing' and hovering behavior of the manhole cover for air outflow. In this model the manhole cover is attached to the ground with a hinge, as shown in Fig. 2. This means that the cover can only rotate around the hinge, making the cover a sort of lever with the hinge as its pivot point. The behavior of the cover can be described by the angle ϕ between the manhole cover and the ground.

An influx of air at the bottom of the manhole increases the pressure P inside the manhole. The air pressure inside exerts a force on the manhole cover from below, resulting in a net torque τ if this force is larger than the gravity acting on the cover. The torque depends on the perpendicular component of the force $F_\perp$ acting on the cover like $\tau = rF_\perp$, where r is the distance to the pivot point. This distance is assumed to be $r = D_c/2$. Besides the gravitational force the only other force on the cover is assumed to be the force exerted by the air pressure F_{in}. Both are directed vertically, resulting in the equation for the net torque:

$$\tau = \frac{D_c}{2}(F_{in} - m_c g)\cos\phi. \qquad (16)$$

The net torque on the manhole cover directly relates to the derivative of the cover's angular momentum $I\omega$, where I denotes the moment of inertia and ω denotes the angular velocity. By the cylindrical shape of the cover and Steiner's parallel axis theorem the moment of inertia I for the cover around the hinge is found to be approximately:

$$I = \frac{5}{16}m_c D_c^2 + \frac{1}{12}m_c d_c^2 \qquad (17)$$

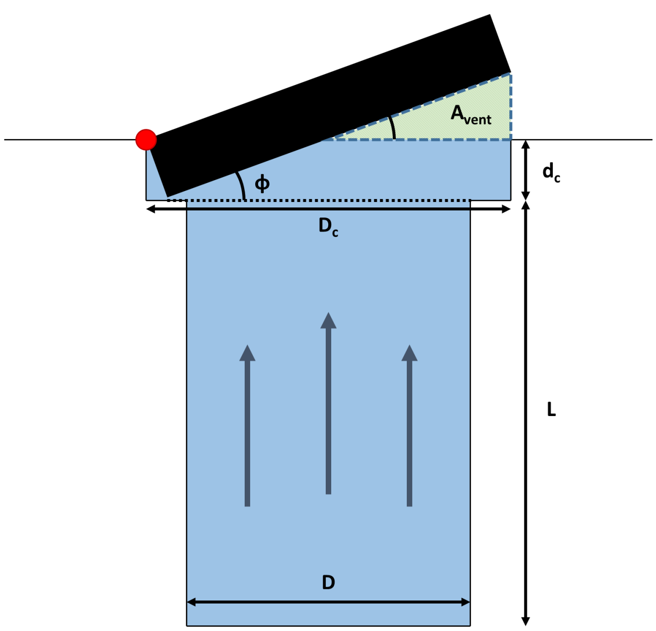

FIGURE 2. Manhole cover rotated around a hinge

Since the moment of inertia is constant, the time derivative of the angular momentum of the cover can be written such that:

$$I\frac{d\omega}{dt} = \tau \qquad (18)$$

Hence the system has to satisfy the following system of differential equations:

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{D_c}{2I}(A_c(P - P_{atm}) - m_c g)\cos\phi \qquad (19)$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \omega \qquad (20)$$

$$\frac{dm}{dt} = \rho\pi\frac{D^2}{4}v_{in} - A_{vent}\sqrt{\rho K_2(P - P_{atm})} \qquad (21)$$

$$\frac{dP}{dt} = \frac{nP}{m}\frac{dm}{dt} \qquad (22)$$

where the density of the air inside the manhole ρ is:

$$\rho = \frac{m}{\pi(D/2)^2 L} \qquad (23)$$

The variable A_{vent} is more complex in the hinged system, as the area through which air can escape is not simply an open cylinder. When the cover is pushed upwards infinitesimally little, the area A_{vent} increases with the area of an infinitesimally small disk, so essentially a circle. Doing this for an angle of $\phi = 2\pi$ would thus yield a torus, with major radius R equal to minor radius r. The surface area of a torus is $4\pi^2 Rr$, so in this system the escape area is approximated depending on the angle ϕ as such:

$$A_{vent} = \frac{\phi}{2\pi}\cdot 4\pi^2\left(\frac{Dc}{2}\right)^2 = \frac{\pi}{2}D_c^2\phi \qquad (24)$$

For high opening angles ϕ this approximation is not valid anymore and the maximum outflow area is simply A_c.

NUMERICAL RESULTS

The data for the manhole and its cover are based on [8] and [13]. The following parameters are used for all models: $D = 0.5$ m, $D_c = 0.55$ m, $d_c = 0.055$ m, $P_{atm} = 1$ bar, $m_c = 100$ kg, $L = 50$ m and $g = 9.81$ m/s^2.

Air-Driven Model

Equations (1), (3), (8) and (9) together form an autonomous nonlinear first-order ordinary differential system of equations.

Heun's method is used to solve the system of equations, with time-step $\Delta t = 10^{-4}$ seconds.

The parameters are the same as mentioned above, together with specifically for this model: the polytropic index $n = 1.4$ and unity constants $K_1 = 1$ m^{2}kg^{-1}s^{-1} and $K_2 = 1$.

The studied input parameter for this model is v_{in}. The initial values for cover displacement x_c and velocity v_c are $x_{c,0} = 0$ m and $v_{c,0} = 0$ m/s. The initial pressure inside the manhole is atmospheric, i.e. $P_0 = 1$ bar. The initial air mass is $m_0 = 11.78$ kg.

For a constant influx velocity $v_{in} = 1$ m/s, the numerical results are shown in Figs 3-6. The blue line represents the results of the first model (with K_1). The red line represents the numerical results of the second model (with K_2). The dotted blue line in Figure 5 represents ground level d_c.

The mass of the air inside the manhole m and the resulting pressure P are clearly coupled, as shown in Figs 3 and 4. This makes sense looking at Eq. (3). Both m and P start increasing immediately and constantly due to the constant influx of air. The manhole cover starts moving up when the pressure is large enough, i.e. $P > P_{atm} + gm_c/A_c \approx 1.041$ bar. The manhole cover rises in exactly the same fashion in both models, because the models only differ when air is allowed to leak.

After approximately 2 seconds the cover is lifted above the ground and air is allowed to escape. The mass decreases rapidly to its initial state and as a result the pressure drops to atmospheric level. For these values of K_1 and K_2, the air mass m, air pressure P and cover velocity v_c decrease slower in the second model, since the leaking law (Eq. (4) or (5)) determines the rate of escaping air. Therefore the cover is lifted 6.57 cm above the ground in the second model, whereas it is only lifted 1.74 cm in the first model, as shown in Fig 5. Both of these values are not unrealistic, however the difference between the two results is going out of phase. In general, the periods of the numerical solutions are approximately 1.91 seconds for model 1 and 2.05 seconds for model 2.

The behavior overall is very simple and stable, the same process repeats itself eternally. This is to be expected, as the model is fairly simple and a constant inflow of air is assumed.

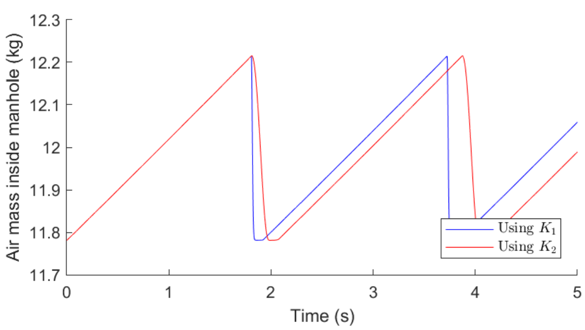

FIGURE 3. Air mass, $v_{in} = 1$ m/s

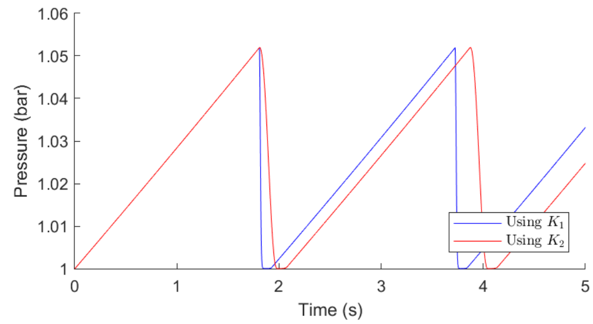

FIGURE 4. Air pressure, $v_{in} = 1$ m/s

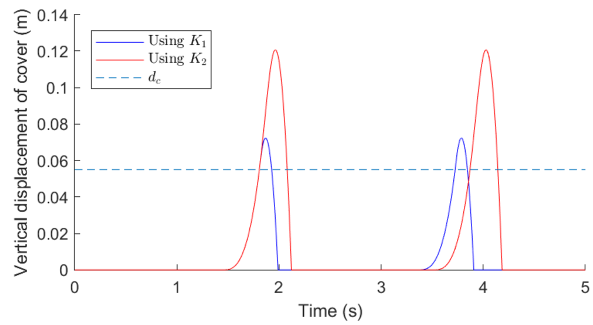

FIGURE 5. Cover displacement, $v_{in} = 1$ m/s

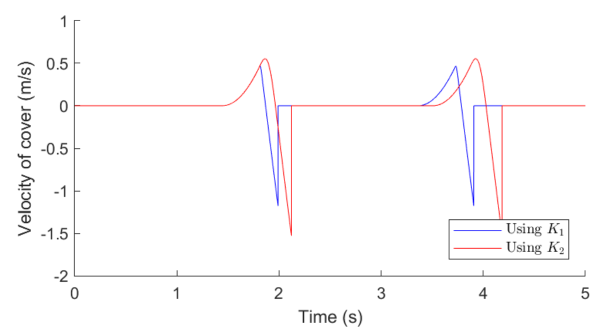

FIGURE 6. Cover velocity, $v_{in} = 1$ m/s

More interesting behavior is found when the driving velocity v_{in} is increased or taken as a function of time. Figure 7 shows the cover displacement for an unrealistically high driving velocity $v_{in} = 50$ m/s. Dancing behavior should definitely not be expected for such velocities, it is more like geysering or an explosion [14]. The manhole cover is blown off more than 2 meters high for the second model, so that Eq. (7) is not valid anymore. The behavior in the first model is a bit milder, as the cover only reaches 0.5 m. Interestingly, the numerical solution for both models is a sum of sinusoids, with two alternating amplitudes.

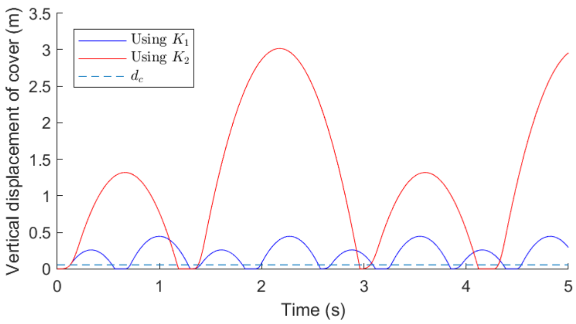

FIGURE 7. Cover displacement, $v_{in} = 50$ m/s

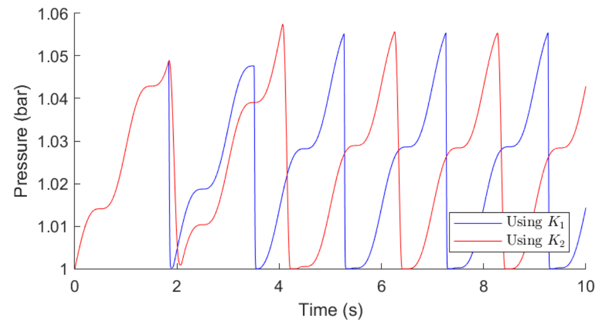

FIGURE 8. Air pressure, $v_b = 1$ m/s, $B = 1$ m/s, $T = 1$ s

The same models are also considered with an inflow velocity that oscillates over time. The following cosine function is used:

$$v_{in}(t) := v_b + B\cos\left(\frac{2\pi}{T}t\right) \tag{25}$$

where v_b is the baseline velocity in m/s, B is the amplitude in m/s and T the period in seconds. Because of the time-dependence of v_{in}, the ordinary differential equation that describes the system is not autonomous anymore. The results for $v_b = 1$ m/s, $B = 1$ m/s and $T = 1$ s can be found in Figs 8-10.

Clear oscillatory behavior can be seen over the main oscillation from earlier results, as shown in Figs 4-6. The pressure inside the manhole increases in oscillatory fashion, and when it is high enough, the manhole cover is lifted up. As soon as air is allowed to escape, the pressure drops like in previous results. The same holds for the displacement and the velocity. In Fig. 10 the dotted line represents the driving velocity v_{in}. In Fig. 9 the height to which the manhole cover is lifted differs per oscillation. However, especially in the other figures, it is immediately noticeable that after 3 oscillations, the system becomes stable and keeps following the same trajectory. It is interesting that the solution seems to converge to a fixed frequency pretty quickly. Even more interesting is the value of this frequency. The observed period for both of the models converges to exactly 2 seconds. This implies that the angular frequency for both models is π Hz and is independent of the type of model or value of the flow coefficients K_1 and K_2.

The frequency of the oscillation is completely determined by the driving frequency, yet the behavior of the system is not always convergent; in fact it is very sensitive to the parameters of the driving velocity, especially the period T. The cover displacement x_c for parameter values $v_b = 1$ m/s, $B = 1$ m/s and $T = 0.5$ s and $v_b = 1$ m/s, $B = 1$ m/s and $T = 1.5$ s is shown in Fig. 11 and 12 respectively. For the former the periods of the two solutions are approximately 1.93 and 2.04 seconds. These are the same periods as for the model with constant influx $v_{in} = 1$ m/s in Fig.

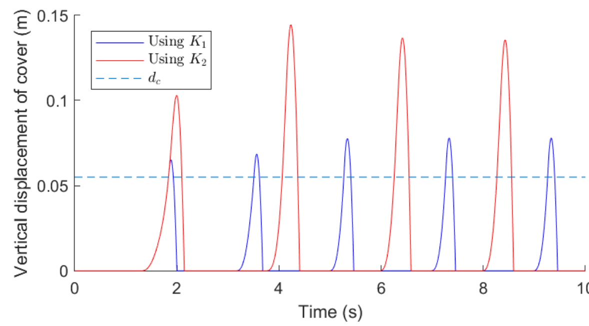

FIGURE 9. Cover displacement, $v_b = 1$ m/s, $B = 1$ m/s, $T = 1$ s

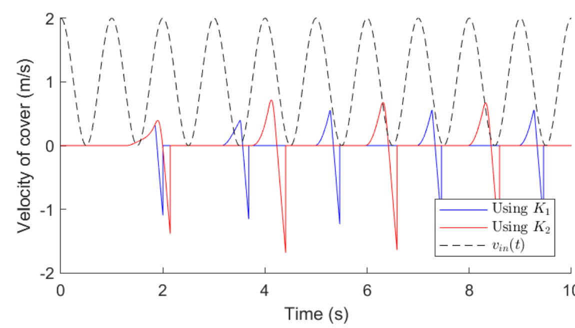

FIGURE 10. Cover velocity, $v_b = 1$ m/s, $B = 1$ m/s, $T = 1$ s

9. Now however the peaks themselves oscillate in height, like a beat, for both models. This takes approximately 17.5 seconds for the first model and 24.5 seconds for the second model. Figure 12 shows similar behavior, only even more chaotic. This highlights the sensitivity of the model and its oscillations.

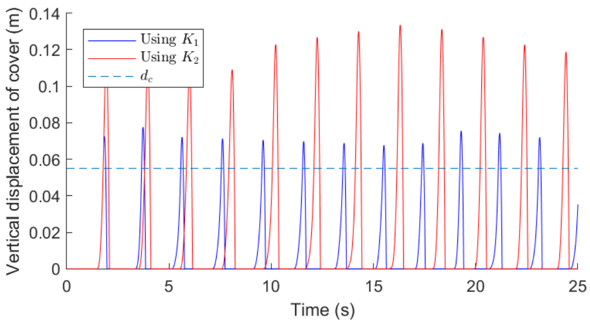

FIGURE 11. Cover displacement, $v_b = 1$ m/s, $B = 1$ m/s, $T = 0.5$ s

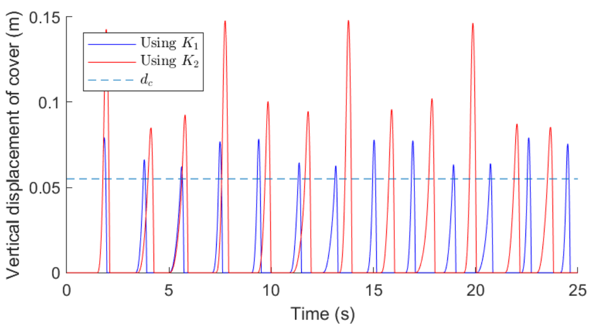

FIGURE 12. Cover displacement, $v_b = 1$ m/s, $B = 1$ m/s, $T = 1.5$ s

Water-Driven Model

Equations (9-13) together form a nonlinear first-order ordinary system of differential equations. It is solved by the forward Euler method with a numerical time-step of $\Delta t = 10^{-4}$ seconds. Which equations are used at a time-step depends on the cover displacement x_c.

The parameters are the same as mentioned above, only now with constant density $\rho = 998$ kg/m^3 and $C_d = 1$ in Eq. (15). The studied input parameter for this model is P_{in}. The driving pressure needs to be large enough to lift the water column and cover, i.e. $P_{in} > P_{atm} + g(m_c/A + \rho L) \approx 5.945$ bar.

The initial values of the variables are displacement $x_{c,0} = 0$ m, cover velocity $v_{c,0} = 0$ m/s and water velocity $v_0 = 0$ m/s. Because $A_{vent} = 0$ m^2 introduces a singularity, a numerical buffer of $\varepsilon = 10^{-3}$ m^2 is implemented on A_{vent}. This implies that the water starts leaking when $x_c \approx 0.05564$ m instead of $x_c = d_c = 0.055$ m. For $P_{in} = 5.96$ bar the results are shown in Figs 13 and 14.

The water column and cover start moving up, until the cover is lifted above the ground after 2 seconds. At this moment the intermediate pressure P_1 drops to 1 bar as the term $(v - v_c)$ in Eq. (15) is 0, before rising back quickly again. Because of this the velocity of the cover drops down and water begins to leak through the gap between ground and cover. The cover itself slows down in acceleration, but remains moving up very slightly. The velocity of the water column keeps rising as well, in contrast to the cover velocity, implying the water flow out of the system keeps rising as well.

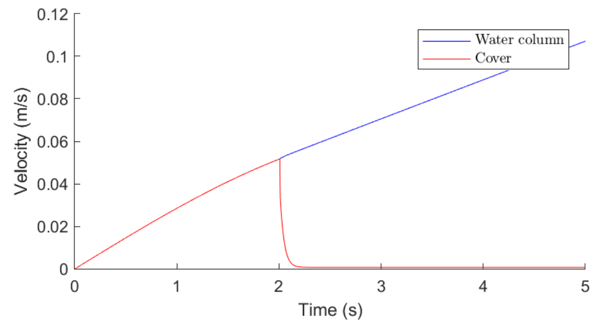

FIGURE 13. Water column and cover velocities, $P_{in} = 5.96$ bar

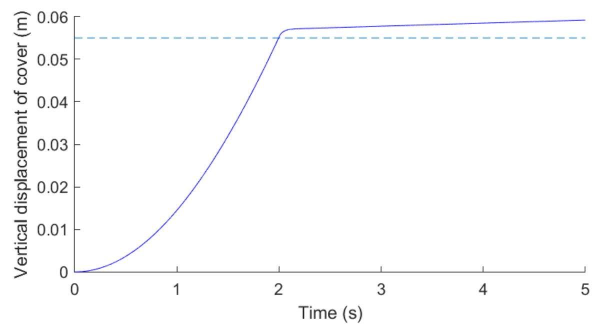

FIGURE 14. Cover displacement over 5 seconds, $P_{in} = 5.96$ bar

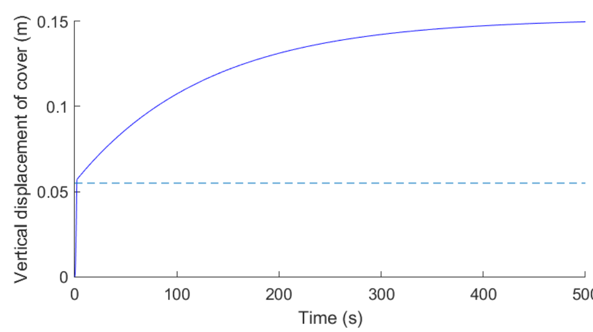

FIGURE 15. Cover displacement over 500 seconds, $P_{in} = 5.96$ bar

The cover displacement for a longer time period of 500 seconds is shown in Fig. 15. The cover slowly converges to its equilibrium point $x_c^* = 0.1517$ m. A full equilibrium analysis can be found in [10]. The cover is eventually floating 10 cm above the ground on top of the water column, which is not too unrealistic given the right conditions. However, no 'dancing' or even oscillatory behavior can be seen in these results.

For a slightly lower driving pressure, more interesting behavior happens. Figures 16 and 17 show the results for driving pressure $P_{in} = 5.9505$ bar. The cover starts increasing like in the results for $P_{in} = 5.96$ bar. However, at the moment the cover is lifted above the ground and the pressure drops, the velocity of the water column v starts decreasing instead of increasing. As a result the manhole cover drops very slightly in height (note that Fig. 17 is zoomed in). The equilibrium point is $x_c^* = 0.055$ m, but due to the numerical buffer this point is not reached exactly. Around $x_c \approx 0.556$ m, the numerical boundary as mentioned before, the system keeps oscillating between the two situations, implying the cover keeps closing and opening for the remainder of the time. The pressure drops every time the cover opens. This results in the cover velocity dropping below 0 m/s, and thus the cover itself drops as well. When the cover closes, the velocities of the water column and cover are equal by definition, so the velocity jumps back up and the cover gets lifted up again. It takes the cover 1.31 seconds to open and close again. The behavior resembles a kind of burping, albeit on a very small scale. It is an artifact of this most elementary model.

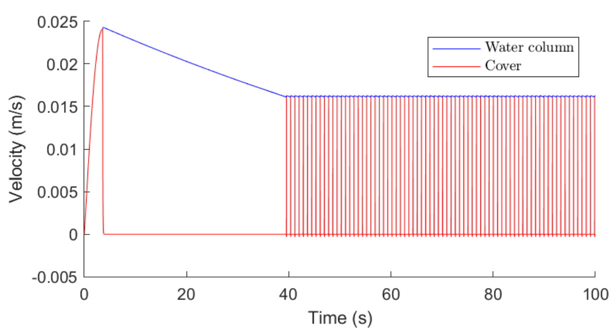

FIGURE 16. Water column and cover velocities. $P_{in} = 5.9505$ bar

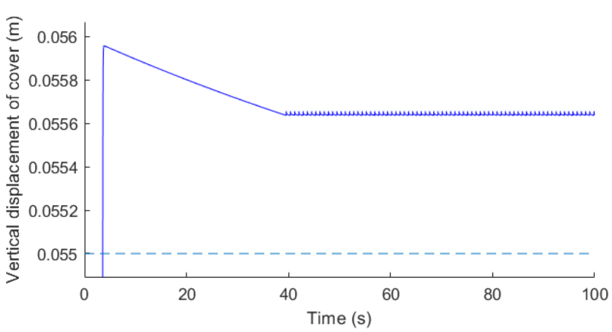

FIGURE 17. Cover displacement, $P_{in} = 5.9505$ bar

Hinged Cover Model

Equations (19-22) together form an autonomous nonlinear first-order ordinary differential system of equations. Heun's method is used to solve the system with time-step $\Delta t = 10^{-4}$ seconds. The parameters are the same as for the air model. The initial values are $m_0 = 11.78$ kg, $P_0 = 1$ bar, $\phi_0 = 0$ rad, $\omega_0 = 0$ rad/s. The angle ϕ is computed in radians, however for convenience the results are shown in degrees. For a driving velocity $v_{in} = 1$ m/s, the results are shown in Figs 18-20.

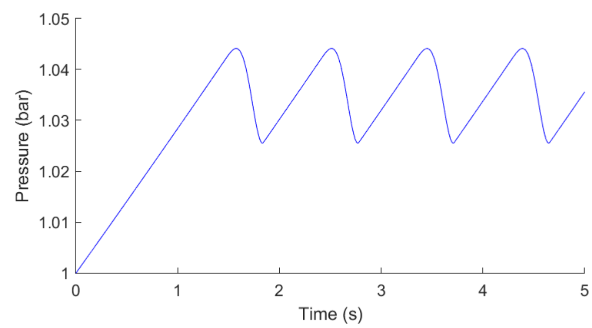

FIGURE 18. Air pressure, $v_{in} = 1$ m/s

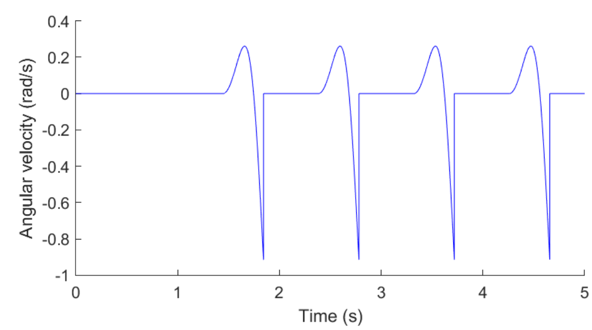

FIGURE 19. Angular velocity of the cover, $v_{in} = 1$ m/s

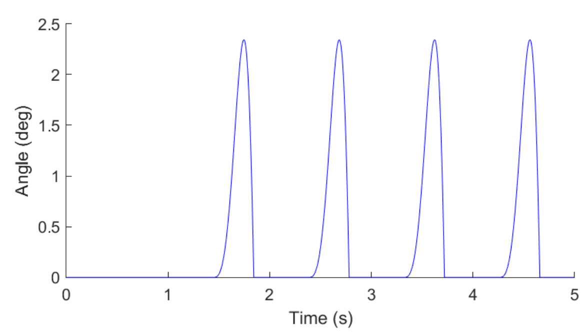

FIGURE 20. Cover angle in degrees, $v_{in} = 1$ m/s

Air flows into the manhole. The pressure P rises and when it is large enough, i.e. $P > P_{atm} + m_c g/A_c \approx 1.041$ bar, the cover is lifted up slightly. In contrast to the model with a loose cover, the air can escape immediately through the little gap between cover and ground. This results in the pressure dropping soon after the cover starts opening. The cover reaches an angle of $\phi_{max} \approx 2.5$ degrees, after which it falls back down quickly. At the far side the cover reaches a height of at most $D_c \sin\phi_{max} \approx 0.0222$ m above the ground. This process repeats itself. In the oscillation the pressure drops to at lowest 1.025 bar.

Two important differences are found with respect to the model in which the cover is not attached to the ground. For the same initial values and parameters the cover is lifted up 0.0657 m above the ground for the air model using K_2 (as seen in Fig. 5). Hence, the hinged cover is lifted up less, which might make it less dangerous to its surroundings. On the other hand, the hinged cover is able to oscillate more frequently. For the hinged cover the period is roughly 0.94 seconds, instead of the 2 seconds found for the air model using K_2. Thus it oscillates twice as fast.

For very high input velocities, there is a point at which the cover gains so much momentum that it will flip over and end on its top. This happens for driving velocity $v_{in} = 24.5$ m/s, which can be seen in Figs 21-23. As a lot of air starts to flow into the manhole from below, the pressure inside the manhole immediately rises to 1.1 bar. The cover gains angular velocity, and is lifted up. The air inside can now escape and the pressure drops quickly back to 1 bar. The velocity of the cover decreases, however the cover has angular momentum and is lifted up so far that the perpendicular component of the gravitational force is too small to bring the cover back down. The cover surpasses a 90° angle, and the cover falls back down on its top side, lying on the street with a dangerous hole next to it. After this the cover is at rest at the wrong position allowing a huge flow of air to leave the shaft.

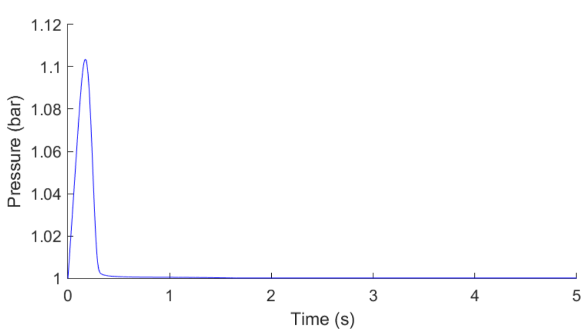

FIGURE 21. Air pressure, $v_{in} = 24.5$ m/s

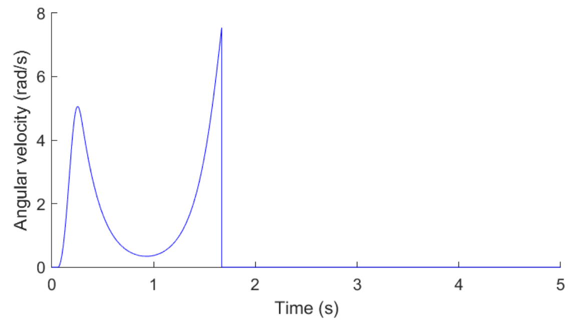

FIGURE 22. Angular velocity of the cover, $v_{in} = 24.5$ m/s

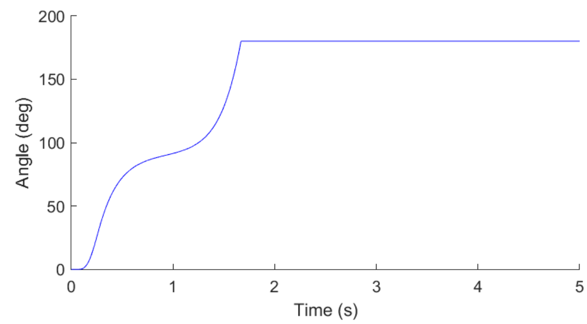

FIGURE 23. Cover angle in degrees, $v_{in} = 24.5$ m/s

DISCUSSION

The models proposed and discussed in the paper appear to be very sensitive. They contain nonlinearities and singularities. Despite this, the obtained results are stable in most cases, and sometimes asymptotically stable. For the numerical results either Heun's method or forward Euler is used. Although the latter is a very basic explicit method, it is used because it is convenient for modeling the system and its nonlinearities. The used time-step proved to be small enough.

Both the air-driven and water-driven model do not result in very chaotic behavior. This is due to the simplistic nature of the model. Another addition might be to include friction in the model like in [8], which is now neglected in the models in general. Nevertheless, both the air and water model show some similarities to real-life behavior of manhole covers.

In the air model two different relations for the flow rate of leaking air are tried. There is no clear result on which model performs better with respect to manhole cover behavior in real life. However, because K_2 is dimensionless, it is preferred, as it can be universally used. Most likely a choking flow model as in [8] would be a better suited option to model the outflow of air.

The water-driven model is very sensitive and has some other issues. Walski et al. [3], based on their unique experimental data, already warned for sensitivity and potential instability. Further research has to be done on an appropriate way to model the leaking outflow of water. It is a difficult aspect of the model, since the leak is initially perpendicular to the flow direction, forcing the water column to flow sideways. Jet-type behavior may be expected. Furthermore, the area through which the water can leak changes over time, and the model shows singular behavior around $A_{vent} = 0$ m^2. The numerical buffer used to work around this is also sensitive. Lastly, the constant pressure from below as driving force is not realistic.

The last proposed model, attaching the cover to the ground with a hinge, has its advantages and disadvantages. Hinged manhole covers are used in practice and have the advantage that the cover cannot separate from the manhole. However, they may clatter or even flip over. A simple model has shown that the latter can occur. The hinged cover model does give some promising results, perhaps in combination with other features, such as a permanent orifice to release pressure, or a chain or latch to limit displacement and rotation.

CONCLUSION

Dancing manhole covers are observed worldwide. Herein both dancing on air and dancing on water has been studied via basic mathematical models. Dancing on air seems possible, but dancing on water not. Floating on water is possible, but unstable. Hinged covers have been modeled for the first time and it is shown that they may flip over. A remedy to the latter is to limit rotation by means of a chain or a string. In practice, alternating air and water outflow is often observed, which asks for a combination of the proposed models. This study should be complemented in the future by experimental measurements and by additional modeling techniques. In particular, the FSI model coupling outflow and cover velocity needs attention.

ACKNOWLEDGMENT

The first and second author would like to thank Georgios Skantzaris for his support and insights.

This material is based upon work supported in part by the U.S.A. National Science Foundation under Grant No. 2048607.

REFERENCES

[1] Yamamoto, S., 2002. "Mechanism Behind Manhole Cover Ejection Phenomenon and Its Prevention Measures". In Ninth International Conference on Urban Drainage (9ICUD), American Society of Civil Engineers, pp. 1–16.

[2] Wang, J., and Vasconcelos, J. G., 2020. "Investigation of Manhole Cover Displacement during Rapid Filling of Stormwater Systems". *Journal of Hydraulic Engineering,* ***146***(4), art. no. 04020022, p. 04020022.

[3] Walski, T., Whitman, B., Kelly, B., Leaman, M., and Loughran, D., 2011. "Predicting head loss vs. flow relationship for overflowing manholes". In Urban Water Management: Challenges and Oppurtunities - 11th International Conference on Computing and Control for the Water Industry, CCWI 2011, Vol. 1.

[4] Chen, A. S., Leandro, J., and Djordjević, S., 2016. "Modelling sewer discharge via displacement of manhole covers during flood events using 1D/2D SIPSON/P-DWave dual drainage simulations". *Urban Water Journal,* ***13***(8), pp. 830–840.

[5] Jia, G., Han, G., Rao, H., and Shu, L., 2018. "Edge computing-based intelligent manhole cover management system for smart cities". *IEEE Internet of Things Journal,* ***5***(3), art. no. 8234581, pp. 1648–1656.

[6] Zhang, H. S., Li, L., and Liu, X., 2020. "Development and Test of Manhole Cover Monitoring Device Using LoRa and Accelerometer". *IEEE Transactions on Instrumentation and Measurement,* ***69***(5), art. no. 8963652, pp. 2570–2580.

[7] Zhou, Z., 2015. "HeadsUp: Keeping pedestrian phone addicts from dangers using mobile phone sensors". *International Journal of Distributed Sensor Networks,* ***11***(5), art. no. 279846.

[8] Tijsseling, A. S., Vasconcelos, J. G., Hou, Q., and Bozkus, Z., 2020. "Venting manhole cover: A nonlinear spring-mass system". In ASME 2020 Pressure Vessels and Piping Conference. Volume 4: Fluid-Structure Interaction, Vol. 4, pp. 1–7.

[9] Tijsseling, A. S., Hou, Q., Svingen, B., and Bergant, A., 2012. "Acoustic Resonance in a Reservoir - Double Pipe - Orifice System". In Proceedings of the ASME 2012 Pressure Vessels & Piping Conference. Volume 4: Fluid-Structure Interaction., pp. 183–191.

[10] van de Meulenhof, N. F., 2021. "Dancing Manhole Covers". B.Sc. thesis, Department of Mathematics and Computer Science, Eindhoven University of Technology, Eindhoven, The Netherlands.

[11] Mitsuyasu, H., 1966. "Shock Pressure of Breaking Wave". In Tenth International Conference on Coastal Engineering, American Society of Civil Engineers (ASCE), pp. 268–283.

[12] White, F., 2011. *Fluid Mechanics (seventh ed.).* The McGraw-Hill Companies, New York, NY, USA.

[13] Wang, J., and Vasconcelos, J. G., 2018. "Manhole cover displacement caused by the release of entrapped air pockets". *Journal of Water Management Modeling,* ***26***, art. no. C444, pp. 1–6.

[14] Li, P., Huang, P., Liu, Z., Du, B., and Li, M., 2019. "Experimental study on vented explosion overpressure of methane/air mixtures in manhole". *Journal of Hazardous Materials,* ***374***, pp. 349–355.