ANALYSIS OF TEACHER ACTIONS TO PROMOTE GENERALIZING

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This study analyzes the instruction of one teacher in two classroom episodes. We analyzed the teacher's instruction using a framework for whole-class discourse and a framework for identifying activity that supports generalizing. Across both episodes when priming and particularly generalizing-promoting activity increased, students' generalizing activity also increased. An increase in the number and quality of questions and student contributions occurred with more student generalizations. Similarly, the responsibility for questioning and thinking shifted from teacher to student as more students responded to requests for justification. Future research should identify productive small group practices to support generalization.

Keywords: Algebra and Algebraic Thinking, Preservice Teacher Education, Discourse

Algebraic thinking appears as early as Kindergarten in the Common Core State Standards (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010) and is often linked to the development of critical thinking and problem-solving skills (Schoenfeld, 1995). We focus on the algebraic practice of generalizing, the situated activity of "lifting" and communicating reasoning to a level where the focus is no longer on a particular instance, but rather on patterns and relationships of those particular instances (Kaput, 1999, p. 137). Algebraic thinking involves reasoning with generalizations--constructing generalizations and justifying and applying them (Blanton et al., 2011; Cooper & Warren, 2011; Kieran, 2007; Kieran et al., 2016; Mason, 1996). We consider how instruction can support students in engaging in the mental activity that leads to generalizing about functional relationships represented in a pattern task. Prior research has mapped out how students develop functional thinking (e.g., Stephens et al., 2017), but we do not have a clear understanding of general ways to support students in engaging in generalizing about functions. By examining the actions of one preservice teacher as she worked to support students' generalizations of a visual pattern, we have identified features of teacher actions and classroom activity that support students in generalizing.

Conceptual Framework

Central to fostering algebra learning is understanding how to support students in generalizing, thus we conducted our analysis using a framework (viz., Strachota, 2020) built on a teaching experiment (Ellis, 2011) that identified seven types of generalizing-promoting actions. The framework (Strachota, 2020) used here was designed to identify students' generalizing and the activity that supported them in developing those generalizations, which is described as *generalizing-promoting activity*, the moves and interactions that promote generalizing, and *priming activity*, the moves and interactions that typically prepare students to engage in a later generalizing-promoting activity (see Table 1 for a description of priming activities and Strachota (2020) for descriptions of generalizing promoting). Understanding the context of generalizing is needed to support students in developing and refining generalizations. Ellis (2011) argues that generalizing is tied to a specific socio-mathematical context through which people construct generalizations. Generalizing is demonstrated through an individual's activity and discourse (Ellis, 2011; Kaput, 1999), so we use the Math-Talk framework (Hufferd-

Ackles et al., 2004) to capture how students and teachers use discourse to support mathematics learning. We focused on questioning and mathematical explanations to capture the back and forth of teacher-student interactions to see how teachers' questions drew students' attention and explanations towards aspects of the mathematics that may have contributed to their generalizing.

Table 1: Priming Activities

Priming Activities	
Naming a phenomenon, clarifying critical terms, reviewing critical tools	Offering a common way to reference a phenomenon or emphasizing the meaning of a critical term or tool.
Constructing or encouraging constructing searchable and relatable situations	Creating or identifying situations or objects that can be used for searching or relating. Situations that can be used for searching or relating involve particular instances or objects that students can identify as similar in some way.
Constructing extendable situations	Identifying situations or objects that can be used for extending. Extending involves applying a phenomenon to a larger range of cases than that from which it originated.

Methods

Ms. Patton was enrolled in a one-year master's program for individuals with STEM degrees and was in the first semester of a year-long placement in an Algebra II classroom. She planned, taught, and reflected on her video data from two episodes of teaching a pattern task with grades 9-10 students as an assignment for her mathematics teaching methods course. Data included planning documents, video recordings, and reflections of Ms. Patton's teaching episodes of the pattern task. With her mentor and supervisor, Ms. Patton co-reflected on her teaching, and then retaught the same task. She then analyzed the video of her teaching using several frameworks, described in the data analysis section, to determine the effectiveness her promoting students' generalizations. In our analysis, we coded the video data in 15-second segments by noting: the types of interaction; who was directing the activity; questioning and mathematical explaining (Hallman-Thrasher, 2017; Huffered-Ackles et al., 2004); and instances of priming activity (PA), generalizing-promoting activity (GPA), or generalizing activity (GA). Each instance of generalizing was coded when it initially occurred only even though a single instance may have spanned multiple segments. Interrater reliability was established by all members of the research team reviewing and coding all video data and transcripts. Disagreements were discussed until consensus was reached (Syed & Nelson, 2015). The quantity, density, and distribution of codes across a lesson helped us determine the relationship between PA and GPA to GA, and connections among generalizing and math-talk.

Results

We compare two episodes of Ms. Patton teaching the same lesson. In the first generalizations were more sparse than in the second. We focus on what different activities occurred during these episodes and how those activities may have been linked to or contributed to students' generalizations. Hereafter, we refer to the generalization sparse class as Episode 1 and the generalization dense class as Episode 2. We highlight Episode 2 because Ms. Patton presents a positive example, especially for a novice, that is worthy of investigation. Overall, Ms. Patton used more instances of priming activity (PA) and generalizing-promoting activity (GPA) and, as noted, had more generalizing activity in Episode 2 (Table 2). While this finding validates prior work (e.g., Strachota, 2020), we also identified patterns in the ways the practices were used. Moreover, while the difference between six and ten generalizations may seem insignificant, we highlight that these kinds of sophisticated statements are fairly rare and result from intentional instruction and students engaging in highly complex cognitive mental activity. Across both episodes, we observed that when priming and particularly generalizing-promoting activity increased students' generalizing activity also increased. In Episode 2, there were repeated instances of GPA-PA, PA-GPA, or PA-GPA-GPA, before a generalization occurred. Further once student generalizations were made, they were often closely followed by additional generalizations. Repeated instances of GPA and PA were nearly always necessary to produce GA. For example, Ms. Patton asked students to share expressions to represent the fourth image in Figure 1. Ms. Patton elicited several examples of expressions and represented them on the board, which was one way she constructed relatable situations (PA). Once students provided an example she followed up with a GPA, such as encouraging justification or encouraging relating. In one instance when she encouraged relating and in turn justification, she asked the student to explain how their expression was represented in the fourth image. In response, one student explained "(you) have two 4 by 4 perfect squares." When Ms. Patton helped the student clarify the student concluded, "(they) have two 4 by 4 squares, plus 2." In response Ms. Patton encouraged reflection (GPA) by asking, "How would you represent that algebraically?" The student who described seeing two 4 by 4 squares said, "It would be $2 \times 4 \times 6 + 2$."

Table 2: Summary of Priming, Generalizing-Promoting, and Generalizing Activities

	Teaching Episode 1*			**Teaching Episode 2****		
	Instances	% of Episode	% of Discussion	Instances	% of Episode	% of Discussion
PA	9	5%	9%	11	7%	9%
GPA	17	10%	17%	27	18%	21%
GA	6 St, 2 Tr	5%	8%	10 St, 2 Tr	8%	9%

*Note episode 1 was 43 minutes, 26 of which were discussion. **Episode 2 was 38 minutes, 32 of which were discussion.

We share this excerpt to show how Ms. Patton set students up for situations that could be built on in a way that supported generalizing. It succinctly illustrates the generative nature of priming, generalizing-promoting, and generalizing activity.

Ms. P: What about another one? **(PA)**
Jen: 2s and 4s.
Ms. P: Where's that at? **(GPA)**
Jen: Huh? I mean split in to 32 with 2s and 4s.
Ms. P: How would you do that? So you're talking about like this 32 [points to 8 by 4 rectangle]? **(GPA)**
Jetta: Yes.
Ms. P: What do you mean by 2s and 4s? Like how would you split the picture? **(GPA)**
Jen: Columns of twos.
Addie: $2 \times 4^2 + 2$

Ms. P: How would you show that in the picture? **(GPA)**
Addie: Just have two 4 by 4 perfect squares.
Ms. P: So you have two 4 by 4s, plus 2. Lets go back to what you were just saying Jen.
Jen: Yeah. Would it be rows of two and columns of four?
Ms. P: So how would you represent that algebraically? **(GPA)**
Addie: It would be $2 \times 4 \times 6 + 2$.

The quality of students' contributions, including when students explain their ideas to each other, and the quality of teachers' questions seemed to play a critical role. We noted an increase in instances and quality of teacher questioning and eliciting students' ideas (Table 3) that we identified using the modified Math Talk rubric (Author, 2017). Within the rubric, lower levels are associated with teacher-generated and answered questions (levels 0 and 0.5), whereas the higher levels are associated with richer justifications that are prompted by the teacher (levels 1 and 1.5). As the levels increase, the responsibility of generating questions shifts towards the students, in turn student-to-student dialogue increases (level 2 and above).

In Episode 2, when students' generalizing increased, there was a shift of responsibility for questioning and thinking from teacher to student. The number of questions and student contributions of thinking increased as did the quality of those interactions (Table 3). The number of questions Ms. Patton posed doubled in the second episode and Ms. Patton used more follow-up questions that pressed students to justify their ideas (level 1.5). Similarly, there were more than double the instances of eliciting student contributions in the second episode and those instances involved a greater number of students sharing their thinking (level 1) and responding to follow-up requests for justification (level 1.5 and 2). The increase in frequency and quality of questions and explanations provided opportunities for generalizing in the second episode. The increase in the quality of student contributions and teacher moves to facilitate those contributions made student thinking and ideas more visible to the class so that ideas with the potential to be generalized were public to the work of the class.

Table 3: Instances of Questioning and Explaining for each Level of the Math Talk Rubric

	Teaching Episode 1		**Teaching Episode 2**	
	Questioning	Explaining	Questioning	Explaining
Level 0	6	8	9	11
Level 0.5	6	0	9	6
Level 1	4	1	7	30
Level 1.5	18	6	43	15
Level 2	0	4	4	13
Level 2.5	1	0	0	0

Conclusion

Within the complex task of teaching, this study provides more evidence that intentionality leads to better outcomes for students, specifically in supporting students in developing mathematical generalizations. We argue that teachers should be purposeful in the questions they ask and the structure of interactions. Our study highlights that small pedagogical moves can have a big impact, and we illustrate some of those moves in practice. Future research should aim to better understand supporting teachers in implementing those practices.

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