

Revising Assessments to Address UDL & Standards

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Think back to a recent mathematics lesson. Now, reflect on the ways you instructed or assessed students during that lesson. Consider this question: “How did I effectively assess each and every learner in ways that allowed them to demonstrate what they know?” As teachers ourselves, we felt confident that our instruction typically addresses students’ diverse learning needs through a Universal Design for Learning (UDL) lens. We regularly spend time thinking about ways to give students multiple opportunities to engage with content. However, we felt less confident that our assessment practices were diverse in nature. That lack of confidence led to this article. Our goal in this article is to describe a process to take textbook word problems, or math problems that we already use, and modify them to be more accessible to diverse groups of learners through UDL and in turn, gather assessment data about students’ problem solving.

Historically, NCTM (2000, 2014; 2020) has promoted problem solving through instructional tasks across K-12. Students should explore mathematical and statistical structures and their uses while solving problems drawn from real life (Authors, 2016a, 2016b; CCSSI, 2010). They are expected to “make sense of quantities and their relationships in problem situations” ([SMP2] CCSSI, 2010, p. 6) and “step back for an overview and shift perspective” ([SMP7] CCSSI, 2010, p.8). Assessment provides feedback about how students learn, and in today’s educational environment, we are urged to, “maximize time spent on assessing student understanding and developing the key mathematical ideas that are embedded within the essential learning” (NCTM, 2020, p. 12). An important mathematical practice is problem solving (e.g., NCTM, 2000; CCSSI, 2010). The first Standard for Mathematical Practice (SMP1) states that

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3 students are expected to “make sense of problems and persevere in solving them” (CCSSI, 2010,
4 p. 6). Many word problems we found in our textbooks did not effectively promote sense making
5 or perseverance while problem solving. By revising these textbook word problems that were
6 expected to be use as formative assessments, we believed that we could gather rich data about
7 students’ mathematical problem solving and knowledge development. In turn, these revisions to
8 textbook word problems also allowed us to better promote UDL through our assessment
9 practices.

Universal Design for Learning

21 A UDL framework can support integration between assessment and instruction (Author,
22 2021b). It was adopted in 2015 as an evidence-based, valid framework for use with all learners
23 (Every Student Succeeds Acts, 2015). Each and every learner – those with and without
24 disabilities – has capacity to do rich mathematics, and UDL provides a strength-based approach
25 to meet the needs of a diverse classroom (Kobett & Karp, 2020). UDL fosters purposeful and
26 motivated learners who are resourceful and knowledgeable while functioning in a strategic and
27 goal-directed manner (CAST, 2018). A central tenet of UDL is that instruction, including
28 assessment, should happen in ways that address the variability in a classroom, and does not
29 expect students to adapt to a one-size-fits-all learning experience (CAST, 2018). Figure 1
30 displays the three principles and three overarching guidelines.

31 A goal within the UDL framework is to use the three principles, and support students as
32 they move through more demanding guidelines (i.e., move left to right in Figure 1). Ultimately,
33 each and every child can benefit from UDL instruction and assessment (Mislevy et al., 2013).
34 Hence, we use UDL as the foundation for describing a means to revise assessments so that they
35 better address problem solving and the SMPs. The three UDL principles are: (a) multiple means
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of engagement, (b) multiple means of representation, and (c) multiple means of action & expression. Within each UDL Principle, there are levels for fostering expertise, which can be viewed from left to right across the three UDL Guidelines. These Guidelines consist of (i) accessing tasks, (ii) building skills, and (iii) internalizing expertise. The horizontal domains suggest the types of learning happening (How, Why, What) while the vertical aspects represent the actions that students take to do so (Access, Build, Internalize). Readers interested in learning more about UDL are strongly encouraged to read Authors (2021a) and check out the supplementary materials such as videos from teachers.



Note: Figure content adapted from CAST (2018), the UDL Guidelines Graphic Organizer 2.0 available at udlguidelines.cast.org

Figure 1. UDL framework adapted from CAST (2018).

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5 We leveraged the UDL framework to create realistic word problems for our assessments using
6 our textbook word problems. These revised tasks better aligned with our problem-solving
7 instruction, gave us more useful data about what our students knew and were able to do, and
8 provided greater access for all students. Although different Guidelines or Principles might be
9 chosen based on what the teacher or student needs, we focus in this article on three UDL
10 Guideline. We highlight one Principle for each, discuss what we did, and why we made those
11 choices. Those Guidelines are shown below:
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- 21 ● Multiple Means of Engagement: Recruiting interest
- 22 ● Multiple Means of Representation: Perception
- 23 ● Multiple Means of Action & Expression: Physical Action

24 25 26 27 28 29 **The Setup**

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31 Our instruction included connections to content and practice standards; however, the
32 word problems found in the textbook assessments were not as robust. We had trouble locating
33 assessment tasks like those we used during instruction, so we designed word problems for our
34 assessments that had instructional relevance and drew upon our students' lived experiences.
35 Revisions aimed to address the UDL Principle of Engagement: Interest. We wanted an
36 assessment task to look similar to those from our instruction. To clarify, our goal was not to
37 present tasks on assessments that were identical to those used during instruction. We believed the
38 tasks should be connected to similar content and practices from a lesson or series of them. We
39 wanted students to feel confident they learned relevant content and practices that they might be
40 able to demonstrate on assessments.
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We started thinking about Engagement, then Representation, and finally Action & Expression topics from UDL. Some questions around each of those topics are shown in Figure 2.

<u>UDL Guideline</u>	<u>How to address</u>	<u>Questions to Ask</u>	<u>Modification</u>
Engagement	Provide a context that is relevant/authentic for your students.	Who are my students? What communities do my students come from? What is relevant to my students?	The cat task from the text is a bit unrealistic, so for this modification we chose to move to a bake sale context where students may have experience with one, or have at least seen one.
Representation	Provide audio clips of the problem being read and create ample space for student work. Change the problem to promote representations in a solution.	What are at least two ways that the problem is communicated to students (reading words or listening)? How might students share their thinking in ways that demonstrate understanding? Within online environments, what training with online tools might students need before they can effectively use them? What technologies are students able to access?	Changing or decomposing numbers in a way that makes a solution less immediate allows for opportunity for students to make sense of the context, and process different solutions. Provide plenty of space for students to perform different strategies such as drawing, mathematics work, words, etc.
Action & Expression	Allow for multiple solution strategies that help students justify their work. Create an "open-endedness" that allows for those solution strategies.	Does the problem allow students to solve it in a variety of developmentally appropriate ways? How many solutions does the problem have? To what degree is the problem written so that students want to engage with it?	Put together the relevant student context and the new or decomposed numbers in a way that makes sense, and is realistic, to students reading or listening to the problem. Asking students to justify their answer or solution creates an open-ended style that promotes students' display of understanding of the standard or concept.

Figure 2. A table to organize the revision process

For Engagement, we took time to get to know our students, chatting with them often about their interests, starting at the beginning of the year. These informal chats helped us consider viable contexts for word problems that drew in community-based and realistic elements. For Representation, our goal was low-floor/high-ceiling problems designed with UDL in mind (see Authors, 2021a for more information). We considered how many districts changed from in-person to hybrid to online instruction because of the COVID-19 pandemic. Technology and

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3 access to it informed the means by which students might demonstrate their knowledge and skills.
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5 Finally, as part of the Action and Expression Principle, we reflected on viable mathematical
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7 strategies that students learned during lessons learned prior to an assessment task. This reflection
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9 on strategies led to ways our assessment might promote multiple representations and
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11 decontextualization/contextualization. Our goal was to create assessments that might allow us to
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13 see a variety of student solution pathways, and learn more about how students think and reason
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15 mathematically. Each of us revised a textbook task rather than starting anew. All of us agreed
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17 that starting with word problems was important because we saw a strong connection between
18
19 reading and mathematical problem solving (Authors, 2021b). Outlined in detail here are
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21 problems for grades five and seven. Readers interested in early childhood mathematics teaching
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23 might consider the tasks in Figure 3 and reflections from the Kindergarten teacher, [Shay](#), who
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25 revised one task to better suit her students. [Megan](#), a high school Algebra teacher shares her
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27 thoughts about revising word problems in a supplementary video, which may help educators
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29 thinking about secondary students.
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36 Original task: "Jody used yellow counters to show the number of dolls she has. Kerry used red
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38 counters to show her dolls. Which set has a number of dolls less than the other set?" Students are
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40 expected to answer this question while looking at two five frames with yellow and red counters.
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42 There are four red counters and two yellow counters." (Houghton Mifflin-Harcourt, 2015)

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42 Revised task: "Jody had a collection of yellow counters to show the number of books she has.
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44 Kerry used red counters to show her books. Kerry had fewer books than Jody. What is one way
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46 to show the number of books each child has?"

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46 Figure 3. Original and revised mathematical tasks for Kindergarten students

47 48 **Just One Problem: Fast Felines**

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50 Recent fifth-grade instruction focused on "5.NBT.5: fluently multiply multi-digit whole
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52 numbers using a standard algorithm" (CCSSI, 2010, p. 35). The task shown in Figure 4 asked
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54 students to write the answer below it.
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Domestic cats can run up to 44 feet per second on land. At this rate, how many feet could a cat run in 12 seconds?

Figure 4. Multiplication problem from McGraw Hill My Math (2014)

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One strategy that might come to mind for students is to multiply 44 by 12, which leads to the total number of feet that the cat runs in 12 seconds. Another strategy is repeated addition. This task could potentially foster multiple representations; however, it seems to promote operating symbolically with the two numbers in the task (i.e., multiplication). Finally, this task does not necessarily give students multiple options to represent their strategy and answer besides writing it. Students in our classrooms had previously experienced instruction on this standard so we gave it to a few students to see how they might solve it. A sample solution strategy from Luke, is shown below in Figure 5.

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Domestic cats can run up to 44 feet per second on land. At this rate, how many feet could a cat run in 12 seconds?

Figure 5. Luke's work on the Fast Felines task.

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3 Luke used repeated addition until he reached the fifth addend. Luke said “I can break 12 into
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5 $5+5+2$. So, I’ll add $220+220+88$. This expectation of seeking structure (SMP #7, CCSSI, 2010)
6
7 is commendable. Surprisingly, Author #2 heard Luke ignore the task situation during this online
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9 instruction and only use the numbers. Later, when Luke was asked about that he said “I didn’t
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11 even think about the situation but wait! That means Harley [his cat] can run across an entire
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13 football field in 12 seconds? No way! This [situation] doesn’t make any sense.” Reflecting on
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15 UDL and this task, it does not necessarily recruit students interest with relevant or authentic
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17 information. We felt that we were not getting all of the information about our students’ learning
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19 out of this task that we could. In its current state, it is unlikely that we might see varying student
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21 solutions, and this task does not effectively promote mathematical reasoning about the problem
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23 situation and mathematical quantities. We struggled to find ways in which this task might
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25 encourage our students to engage in the SMPs. In an attempt to know more about what and how
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27 students were thinking, we set out to revise the task to better align with the UDL Guidelines and
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29 attend to content-focused problem solving.
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35 The questions and strategies outlined in Figure 2 guided our thinking during task
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37 revisions to the word problem. We started with Engagement opportunities, then considered
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39 Representation aspects (i.e., how materials are presented to students), and followed up with
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41 Action & Expression elements. Using the questions and the mentioned modifications, a new
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43 assessment item and its directions stated:
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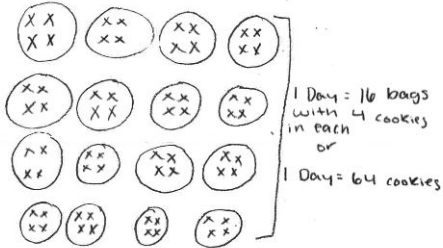
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47 Directions: Justify your answer with words or multiple appropriate strategies leading to
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49 the same result. You may write your work, audio-record it, submit a screencast, and/or
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51 draw your thinking.

52 Carter bakes cookies for a fundraiser at Falcon Elementary School. He sells bags of
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54 cookies that have 4 cookies in each bag. Carter will sell at least 16 bags each day. The
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56 fundraiser lasts for 12 days. How many cookies should Carter bake?
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3 In this revised problem, students considered the number of cookies needed each day prior to
4 determining the total needed for the 12-day bake sale. We had previously talked with our
5 students to optimize relevance, value, and authenticity. They routinely mentioned our school's
6 community-based activities. Numerous students expressed having participated in a bake sale or
7 similar fundraiser within the school, places of worship, or as part of after-school activities such
8 as sports teams or organizations. Connecting with a relevant context better addressed the
9 Engagement principle. In this revised task, students must make sense of the number of cookies
10 needed for the bake sale in one day before finding the total number of cookies over the course of
11 twelve days. Task revisions led to promoting the Engagement guideline, as well as SMP#1
12 "Make sense of problems and persevere in solving them" and SMP2 "Reason abstractly and
13 quantitatively" (CCSSI, 2010, p. 6). Reading can be a challenge for some students. They
14 expressed that having the opportunity to listen to the words read-aloud helped them know how to
15 pronounce them. We created an audio recording reading the revised task, which supported the
16 representation guideline. We expanded the workspace from one line to half of the page. This
17 provided adequate space for students to draw pictures and work symbolically. We expected
18 students to use a standard multiplication algorithm, an area model, or partial products (see Figure
19 6 for student work from students that met face-to-face; see Figure 7 for student work from Luke,
20 a student enrolled in online instruction) but we also anticipated that they might want to draw a
21 picture of the situation or check their work with a different strategy. For the Action & Expression
22 guideline, the original cat task did not necessarily imply that students should use multiple
23 strategies when multiplying multi-digit numbers, whereas the new task makes it an expectation.
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Directions: Justify your answer with words or multiple appropriate strategies leading to the same result. You may write your work, audio-record it, submit a screencast, and/or draw your thinking.

Carter bakes cookies for a fundraiser at Falcon Elementary School. He sells bags of cookies that have 4 cookies in each bag. Carter will sell at least 16 bags each day. The fundraiser lasts for 12 days. How many cookies should Carter bake? Justify your answer with words or multiple appropriate strategies leading to the same result.



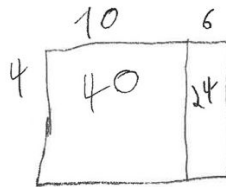
12 Days = ? cookies

$$\begin{array}{r} 64 \\ \times 12 \\ \hline 128 \\ + 640 \\ \hline 768 \end{array}$$

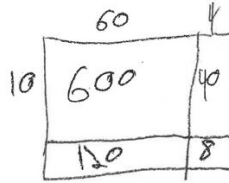
Carter needs 768 cookies.

Directions: Justify your answer with words or multiple appropriate strategies leading to the same result. You may write your work, audio-record it, submit a screencast, and/or draw your thinking.

Carter bakes cookies for a fundraiser at Falcon Elementary School. He sells bags of cookies that have 4 cookies in each bag. Carter will sell at least 16 bags each day. The fundraiser lasts for 12 days. How many cookies should Carter bake? Justify your answer with words or multiple appropriate strategies leading to the same result.



$$\begin{array}{r} 40 \\ + 24 \\ \hline 64 \text{ cookies} \end{array}$$



$$\begin{array}{r} 600 \\ 120 \\ 40 \\ 8 \\ \hline 768 \text{ cookies total} \end{array}$$

Directions: Justify your answer with words or multiple appropriate strategies leading to the same result. You may write your work, audio-record it, submit a screencast, and/or draw your thinking.

Carter bakes cookies for a fundraiser at Falcon Elementary School. He sells bags of cookies that have 4 cookies in each bag. Carter will sell at least 16 bags each day. The fundraiser lasts for 12 days. How many cookies should Carter bake? Justify your answer with words or multiple appropriate strategies leading to the same result.

4 cookies in each bag
Sell 16 bags each day so in one day
 $\frac{16}{4} \rightarrow (10 \times 4) + (6 \times 4) = 40 + 24 = 64$ cookies sold in one day.

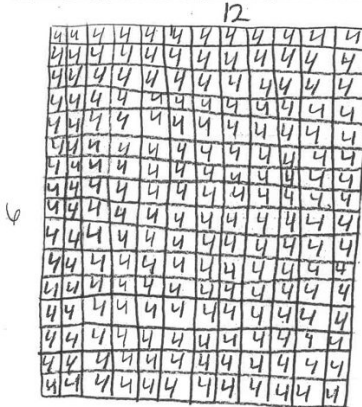
64 cookies sold each day
Fundraiser lasts 12 days.

$$\begin{array}{r} 64 \\ \times 12 \\ \hline (60 \times 12) + (60 \times 2) + (10 \times 4) + (2 \times 4) = \\ (600) + (120) + (40) + (8) = \\ 720 + 48 = 768 \end{array}$$

Carter should bake at least 768 cookies. 768 cookies would be exactly enough but if a cookie fell on the floor then Carter might need another one.

Directions: Justify your answer with words or multiple appropriate strategies leading to the same result. You may write your work, audio-record it, submit a screencast, and/or draw your thinking.

Carter bakes cookies for a fundraiser at Falcon Elementary School. He sells bags of cookies that have 4 cookies in each bag. Carter will sell at least 16 bags each day. The fundraiser lasts for 12 days. How many cookies should Carter bake? Justify your answer with words or multiple appropriate strategies leading to the same result.



= 768

Figure 6. Students written work on the revised task.

Justify your work with words or multiple appropriate strategies leading to the same result. You may write your work, audio-record it, submit a screencast, and/or draw your thinking.

Carter bakes cookies for a fundraiser at Falcon Elementary School. He sells bags of cookies that have 4 cookies in each bag. Carter will sell at least 16 bags each day. The fundraiser lasts for 12 days. How many cookies should Carter bake?



Figure 7. Luke's partial-product strategy for the revised task.

The first student (Figure 6) was able to draw what they were thinking, showing 16 “bags” or groups of 4, which led to the idea of 64 cookies per day. Then, they used this intermediate result to multiply it by the total number of days to find a total of 768 cookies using the standard algorithm. Although they knew how to implement the standard algorithm, it was clear that the drawing helped this student make sense of exactly what two numbers to multiply to reach a solution. The second student solution shows an area model where they decomposed the 16 “bags” or groups into 10 and 6, again allowing them to find the total for one day. They then did the same with 64, 60 and 4, and 12, 10 and 2, to find the total of 768, which told us that this student recognized numbers as composed of 10s and 1s, and it was easier for them to manipulate when they broke those numbers apart. The third student was able to write exactly what they were thinking and then use partial products, similar to the second student. Lastly, the fourth student was a little more deconstructed in their approach, showing a column of 16 fours, and then 12 rows to represent each day. Although they visualized the entire scenario, it is still unclear on their drawing whether they simply multiplied the 12 by 16 as they saw the grid, or if they added together each of the fours. Luke's work from online instruction confirms some of the same ideas we saw from face-to-face instruction as he used partial products like fifth-grade students from a

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3 different district. Taken collectively, we created a means to meet the needs of each and every
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5 student and leverage their mathematical strengths.
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8 Prior to being given the revised cookie task during online instruction, Luke said,
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10 “Normally I stick to one strategy,” which we saw in his work for the cat task. After given the
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12 revised problem, the student attested to it being more difficult due to needing to find two
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14 different strategies, but also expressed that it was more interesting because it involved more than
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16 one step to find an answer. Luke was given both problems through an online interface where he
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18 was able to draw, write, and type with his computer as he was thinking about each problem. As
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20 we were moving between face-to-face and online instruction, students had the option to take
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22 pictures of their work, use a writing tool online, or submit a screencast explaining their solutions.
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24 Giving students options for submission better aligned with many UDL guidelines.
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27 28 **The Marshmallow Task** 29

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31 Extending these questions and strategies from the fifth-grade revisions, we worked
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33 together to revise an item for the seventh-grade class focusing on Common Core State Standards
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35 "7.NS.A.2: Apply and extend previous understandings of multiplication and division and of
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37 fractions to multiply and divide rational numbers", and "7.NS.A.3: Solve real-world and
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39 mathematical problems involving the four operations with rational numbers" (CCSSI, 2010) to
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41 show consistency and effectiveness across multiple grade levels. The original task (Figure 8) is a
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43 multi-step task involving operations with mixed numbers. It provides appropriate distractors in
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45 the form of different trip options provided by the company and uses a real-world situational
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47 context. However, this context was not meaningful to our students. While some students had
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49 knowledge of kayaks and canoes, very few students expressed having personal experience with
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51 these activities. Additionally, the situational context is somewhat superficial as students simply
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need to identify the correct fraction from the table provided. It is possible that students can answer the item correctly while largely ignoring the situational context of the item, which did not promote making sense of quantities (SMP #2, CCSSI, 2010).

15. Great Adventures Canoe and Kayak company offers 3 different options as shown in the table. This afternoon a group of 6 friends is paddling the Adventure Mid Trip in individual kayaks. How many total yards will the friends paddle combined? (Hint: 1 mile = 1,760 yards)

Trip	Length (mi)
Beginner Short Trip	$2\frac{2}{5}$
Adventure Mid Trip	$4\frac{7}{8}$
Extreme Long Trip	$7\frac{2}{3}$

Figure 8. Multiplication problem from McGraw-Hill Reveal Math (2019)

We referred to the questions and strategies outlined in Figure 2 for our revisions. We started with Engagement opportunities, then considered Representation aspects, and followed up with Action & Expression elements. We used the same directions as in the fifth-grade task: “Directions: Justify your answer with words or multiple appropriate strategies leading to the same result. You may write your work, audio-record it, submit a screencast, and/or draw your thinking.” We named the revised problem the Marshmallow Task, which states:

Teresa wants to make marshmallow treats to give her neighbors. The recipe requires $3\frac{1}{8}$ cups of marshmallows, $4\frac{1}{4}$ cups of rice cereal, and $\frac{1}{8}$ cup of butter. The recipe serves nine people. How much of each ingredient will she need if she must make treats for 27 people?

Two teachers presented the Marshmallow Task to their seventh-grade students. In one scenario, students were asked to complete the item without the use of a calculator, and to provide evidence of how they solved the item. Students were provided a handout of the Marshmallow Task, and the item was read aloud as a class. The item was also available to students through Google

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3 Classroom. The task draws on the UDL guideline of Representation by presenting the task in a
4 variety of ways. Thus, students have choice in the way they engage with the task.
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8 Shepard (2000) emphasizes that student learning, grounded in a social-constructivist
9 approach, should be authentic and real-world. The situational context of the Marshmallow Task
10 was easily accessible to each and every student. Students were familiar with marshmallow treats,
11 although they may be accustomed to alternative names when referencing this common sweet
12 treat. The Marshmallow Task draws on the UDL guideline of Engagement by providing a
13 context that is relevant and meaningful to students. It promotes SMP1 “Make sense of problems
14 and persevere in solving them” and SMP2 “Reason abstractly and quantitatively” (CCSSI, 2010,
15 p. 6). We perceive two elements of the Marshmallow Task that qualify the item as a problem.
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17 The first element is that students must persevere through relative degrees of productive struggle.
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19 The situational context of the Marshmallow Task is cognitively demanding. Students must
20 navigate the situational context, and transfer knowledge of proportional relationships, to identify
21 that Teresa will need 3 times the original recipe to make enough treats for 27 people. Thus,
22 students are expected to engage with the item’s context. During administration of the item, a
23 typically high-performing student, Kelly, requested help in making sense of the problem. Kelly
24 was struggling to transfer her knowledge of proportional relationships to a new situational
25 context based on a recipe. Author #4 posed the question to Kelly, "If one batch of the recipe
26 serves 9 people, how many batches are needed to serve 27 people?" Kelly, like her peers, were
27 used to ignoring the situational context and combining numbers. She needed to step back and re-
28 examine the problem’s situation (decontextualization) to recognize the proportional relationship
29 and further reason about the quantities. Decontextualization is a process of peeling away a
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3 problem's situational context in order to focus on developing a mathematical representation, and
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5 this is a critical component of SMP2 "Reason abstractly and quantitatively" (CCSSI, 2010, p.6).
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8 Developing a mathematical representation to the Marshmallow Task is a second element
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10 that makes this task a problem, and also fosters meeting the Action & Expression aspect of UDL.
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12 Mathematical representations, or the ways in which students solve problems, is more closely
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14 related to the Action & Expression guideline (Authors, 2021a). The Representation guideline
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16 from UDL is about how *teachers* present information to students. The Marshmallow Task
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18 requires students to perform operations with mixed numbers. Students are introduced to
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20 multiplication of fractions and mixed numbers in fifth grade, but fraction operations are skills
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22 that many students struggle with. Misconceptions about multiplication with mixed numbers are
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24 common with seventh-grade students. These misconceptions are sometimes the result of an over-
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26 reliance on shortcuts and tricks to perform rote procedures. One student, Maggie, correctly
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28 converted the mixed numbers to improper fractions, but then decided to "flip" the fraction (see
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30 Figure 9). It is likely that Maggie's solution strategy was influenced by procedures she learned
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32 regarding fraction division. That is, Maggie could have been remembering elements of the
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34 saying "keep, change, flip," which provides students a shortcut to fraction division, but does not
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36 deepen students' understanding of fraction operations.
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Teresa wants to make marshmallow treats to give her neighbors. The recipe requires $3\frac{1}{8}$ cups of marshmallows, $4\frac{1}{4}$ cups of rice cereal, and $\frac{1}{8}$ cup of butter. The recipe serves nine people. How much of each ingredient will she need if she must make treats for 27 people?

① $3\frac{1}{8} \rightarrow \frac{25}{8} \times \frac{3}{1} = \frac{75}{8}$ or $9\frac{3}{8}$ she will need
 ② $4\frac{1}{4} \rightarrow \frac{17}{4} \times \frac{3}{1} = \frac{51}{4}$ or $12\frac{3}{4}$ 9 $\frac{3}{8}$ cups of marshmallows, 12 $\frac{3}{4}$ cups of rice cereal, and
 ③ $\frac{1}{8} \rightarrow \frac{1}{8} \times \frac{3}{1} = \frac{3}{8}$ $\frac{3}{8}$ cup of butter.

① work $\frac{25}{8} \times \frac{3}{1} = \frac{75}{8}$
 $\frac{25}{8} \times \frac{3}{1}$
 $\frac{75}{8}$
 ② work $\frac{17}{4} \times \frac{3}{1} = \frac{51}{4}$
 $\frac{17}{4} \times \frac{3}{1}$
 $\frac{51}{4}$
 ③ work $\frac{1}{8} \times \frac{3}{1} = \frac{3}{8}$
 $\frac{1}{8} \times \frac{3}{1}$
 $\frac{3}{8}$

Teresa wants to make marshmallow treats to give her neighbors. The recipe requires $3\frac{1}{8}$ cups of marshmallows, $4\frac{1}{4}$ cups of rice cereal, and $\frac{1}{8}$ cup of butter. The recipe serves nine people. How much of each ingredient will she need if she must make treats for 27 people?

Teresa wants to make marshmallow treats to give her neighbors. The recipe requires $3\frac{1}{8}$ cups of marshmallows, $4\frac{1}{4}$ cups of rice cereal, and $\frac{1}{8}$ cup of butter. The recipe serves nine people. How much of each ingredient will she need if she must make treats for 27 people?

$9 \times 3 = 27$
 Marshmallows $3\frac{1}{8} \times 3 = \frac{25}{8} \times \frac{3}{1} = \frac{75}{8} = 9\frac{3}{8}$ $\frac{24}{25}$ cups of marshmallows
 Cereal $4\frac{1}{4} \times 3 = \frac{17}{4} \times \frac{3}{1} = \frac{51}{4} = 12\frac{3}{4}$ $\frac{12}{17}$ cups of cereal
 butter $\frac{1}{8} \times 3 = \frac{3}{8}$ $\frac{24}{1}$ cups of butter

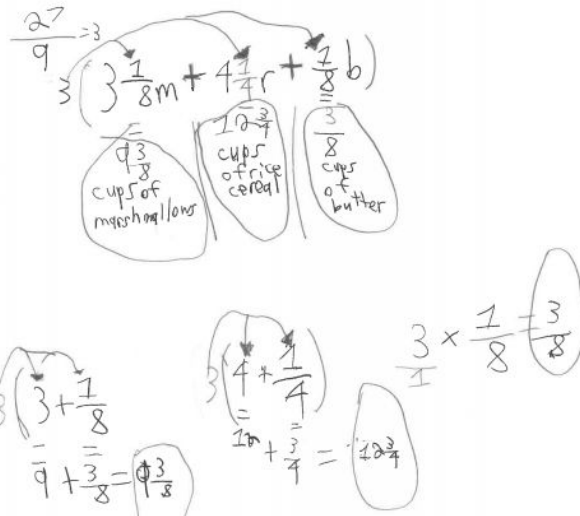


Figure 9. Students' written work on the revised marshmallow task.

In Author #5's class, students were given the Marshmallow Task in context of a whole-class think-aloud (see Authors, 2021b). Students in this classroom displayed many facets of mathematical reasoning as well as voicing the ways in which they were making sense of the problem, as outlined by SMP 1. One student, Isaiah, stated, "The recipe says originally 9 people,

1
2
3 and now she wants to serve 27 people. So, I was thinking 27 divided by 9, and then that's 3."

4
5 With more time to think, Isaiah later realized, "When she is doing the recipe, she would have to
6
7 do $3\frac{1}{8}$ cups of marshmallows, three times. 3 times $3\frac{1}{8}$ cups of marshmallows and then so on."

8
9
10 When given the opportunity to share her thoughts out loud, this problem encouraged Isaiah to
11
12 reason about the recipe, and make sense of how she could use the rate that she found, 3, to create
13
14 the new recipe that would serve 27 people. Another student, Maria, begins the problem by
15
16 stating, "So 9 times 3 makes 27. So we just multiply everything by 3, which made an improper
17
18 fraction." In the rest of this student's thought process, we heard Maria do the calculations for
19
20 increasing the recipe, and then think about how to convert those products that gave her improper
21
22 fractions into mixed numbers. Author #5 implemented the Marshmallow task as a think-aloud
23
24 assessment for a class that consists of mostly bilingual students. Some of them shared previously
25
26 that Spanish is their first and native language and are less comfortable using English. By
27
28 implementing the task as a think aloud, Author #5 not only was able to hear the same things that
29
30 we saw in the written examples given above, but also these students were given a better space to
31
32 think about the task and demonstrated perseverance while problem solving.
33
34
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37

38 Students are likely to engage in various solutions strategies when multiplying with mixed
39
40 numbers. The solution strategy that seventh grade students are likely to pursue, as shown in
41
42 Figure 9, is to convert all the mixed numbers to improper fractions and then multiply by 3. Other
43
44 possible solution strategies would be to apply the distribute property to multiply the mixed
45
46 numbers by a whole number and/or build visual fraction models to determine the product.
47
48 Supporting a variety of solution strategies and solution representations draws on the UDL
49
50 guideline of Action and Expression. The Marshmallow Task served two purposes. It was an
51
52 opportunity to assess what students knew and were able to demonstrate, but it was also a genuine
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1
2
3 learning opportunity for students. Students were able to experience the integration of two
4
5 different grade level domains. Authors #4 and #5 used instructional scaffolding to help students
6
7 transfer relevant knowledge from fifth-grade learning to solve a challenging word problem.
8
9

10 **Summary**

11
12 Our goal was to modify textbook word problems in ways that were better for our students
13
14 and better align our instructional and assessment practices. The UDL framework helped us –
15
16 teachers across multiple districts - create more relatable word problems and afforded more
17
18 opportunities for students to demonstrate what they know using multiple strategies, and gave
19
20 students more ways to access the task. By revising tasks successfully, we were able to see more
21
22 strategies than what was seen in the original task, (i.e., standard algorithm). We were able to use
23
24 this information shared by our students to help effectively plan our instruction, make
25
26 instructional decisions, and better match instruction and assessment. Additionally, revisions led
27
28 to assessment tasks that drew upon the UDL Guidelines and Principles. Each of our students
29
30 thinks differently, so it is important that we used assessment items that met their needs through
31
32 UDL, allows students freedom to be creative mathematical thinkers, and access to assessments
33
34 that encourage them to demonstrate their capabilities as mathematical problem solvers.
35
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41 **Acknowledgement**

42 <Information added after blind review has been completed.>
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Note: Figure content adapted from CAST (2018), the UDL Guidelines Graphic Organizer 2.0 available at udlguidelines.cast.org

Figure 1. This is an adapted UDL framework from CAST (2018).

UDL Guideline	How to Address	Questions to Ask
Engagement	<ul style="list-style-type: none"> • Provide a context that is relevant/authentic for your students. 	<ul style="list-style-type: none"> • Who are my students? • What communities do my students come from? • What is relevant to my students?
Representation	<ul style="list-style-type: none"> • Provide audio clips of the problem being read. • Offer ample space for students to show their work. • Change the problem to promote representations in a solution. 	<ul style="list-style-type: none"> • What are at least two ways that the problem is communicated to students (i.e., reading words or listening)? • How might students share their thinking in ways that demonstrate understanding? • Within online environments, what online training tools might students need before they can effectively use them? • What technologies are students able to access?
Action & Expression	<ul style="list-style-type: none"> • Allow for multiple solution strategies that help students justify their work. • Create an ‘open-endedness’ that allows for those solution strategies. 	<ul style="list-style-type: none"> • Does the problem allow students to solve it in a variety of developmentally appropriate ways? • How many solutions does the problem have? • To what degree is the problem written so that students want to engage with it?

Figure 2. A table to organize the revision process for assessments.

Original task: "Jody used yellow counters to show the number of dolls she has. Kerry used red counters to show her dolls. Which set has a number of dolls less than the other set?" Students are expected to answer this question while looking at two five frames with yellow and red counters. There are four red counters and two yellow counters." (Houghton Mifflin-Harcourt, 2015)

Revised task: "Jody had a collection of yellow counters to show the number of books she has. Kerry used red counters to show her books. Kerry had fewer books than Jody. What is one way to show the number of books each child has?"

Figure 3. The original and revised mathematical tasks that Shay used for Kindergarten students

Domestic cats can run up to 44 feet per second on land. At this rate, how many feet could a cat run in 12 seconds?

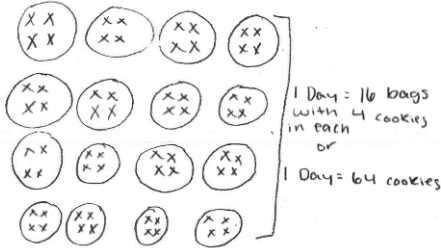
Figure 4. A multiplication problem from McGraw Hill *My Math* (2014)

Domestic cats can run up to 44 feet per second on land. At this rate, how many feet could a cat run in 12 seconds?

Figure 4. A multiplication problem from McGraw Hill *My Math* (2014)

Directions: Justify your answer with words or multiple appropriate strategies leading to the same result. You may write your work, audio-record it, submit a screencast, and/or draw your thinking.

Carter bakes cookies for a fundraiser at Falcon Elementary School. He sells bags of cookies that have 4 cookies in each bag. Carter will sell at least 16 bags each day. The fundraiser lasts for 12 days. How many cookies should Carter bake? Justify your answer with words or multiple appropriate strategies leading to the same result.



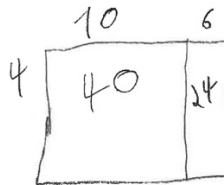
12 Days = ? cookies

$$\begin{array}{r} 64 \\ \times 12 \\ \hline 128 \\ + 640 \\ \hline 768 \end{array}$$

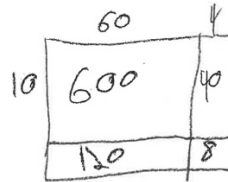
Carter needs 768 cookies.

Directions: Justify your answer with words or multiple appropriate strategies leading to the same result. You may write your work, audio-record it, submit a screencast, and/or draw your thinking.

Carter bakes cookies for a fundraiser at Falcon Elementary School. He sells bags of cookies that have 4 cookies in each bag. Carter will sell at least 16 bags each day. The fundraiser lasts for 12 days. How many cookies should Carter bake? Justify your answer with words or multiple appropriate strategies leading to the same result.



$$\begin{array}{r} 40 \\ + 24 \\ \hline 64 \text{ cookies} \end{array}$$



$$\begin{array}{r} 600 \\ 120 \\ 40 \\ 8 \\ \hline 768 \text{ cookies total} \end{array}$$

Directions: Justify your answer with words or multiple appropriate strategies leading to the same result. You may write your work, audio-record it, submit a screencast, and/or draw your thinking.

Carter bakes cookies for a fundraiser at Falcon Elementary School. He sells bags of cookies that have 4 cookies in each bag. Carter will sell at least 16 bags each day. The fundraiser lasts for 12 days. How many cookies should Carter bake? Justify your answer with words or multiple appropriate strategies leading to the same result.

4 cookies in each bag
sell 16 bags each day so in one day
 $\frac{16}{\times 4} \rightarrow (10 \times 4) + (6 \times 4) = 40 + 24 = 64 \text{ cookies sold in one day}$

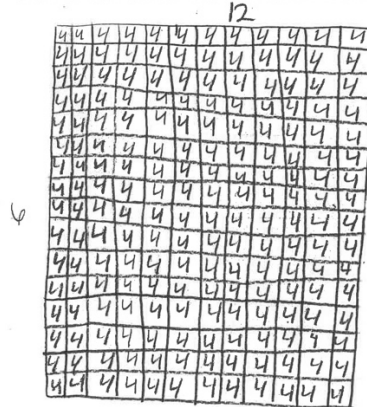
64 cookies sold each day
fundraiser lasts 12 days.

$$\begin{array}{r} 64 \\ \times 12 \\ \hline (60 \times 12) + (60 \times 2) + (10 \times 4) + (2 \times 4) = \\ (600) + (120) + (40) + (8) = \\ 720 + 48 = 768 \end{array}$$

Carter should bake at least 768 cookies. 768 cookies would be exactly enough but if a cookie fell on the floor then Carter might need another one.

Directions: Justify your answer with words or multiple appropriate strategies leading to the same result. You may write your work, audio-record it, submit a screencast, and/or draw your thinking.

Carter bakes cookies for a fundraiser at Falcon Elementary School. He sells bags of cookies that have 4 cookies in each bag. Carter will sell at least 16 bags each day. The fundraiser lasts for 12 days. How many cookies should Carter bake? Justify your answer with words or multiple appropriate strategies leading to the same result.



= 768

Figure 6. Students' written work on the revised task.

Justify your work with words or multiple appropriate strategies leading to the same result.
You may write your work, audio-record it, submit a screencast, and/or draw your thinking.

Carter bakes cookies for a fundraiser at Falcon Elementary School. He sells bags of cookies that have 4 cookies in each bag. Carter will sell at least 16 bags each day. The fundraiser lasts for 12 days. How many cookies should Carter bake?

The image shows two handwritten mathematical strategies for calculating 12×604 .

The first strategy is a grid method. It shows a grid with two rows and two columns. The top row is labeled '10' and the bottom row is labeled '2'. The left column is labeled '600' and the right column is labeled '40'. The grid contains the following numbers:

10	600	40
2	120	8

The second strategy is a vertical addition. It shows the numbers 200, 120, and 40 stacked vertically, with a plus sign to the left. A horizontal line is drawn below the numbers, and the sum 760 is written below the line.

$$\begin{array}{r} 200 \\ 120 \\ 40 \\ \hline 760 \end{array}$$

Figure 7. Luke's partial-product strategy for the revised Fundraiser task.

<u>UDL Guideline</u>	<u>How to address</u>	<u>Questions to Ask</u>	<u>Modification</u>
Engagement	Provide a context that is relevant/authentic for your students.	Who are my students? What communities do my students come from? What is relevant to my students?	The cat task from the text is a bit unrealistic, so for this modification we chose to move to a bake sale context where students may have experience with one, or have at least seen one.
Representation	Provide audio clips of the problem being read and create ample space for student work. Change the problem to promote representations in a solution.	What are at least two ways that the problem is communicated to students (reading words or listening)? How might students share their thinking in ways that demonstrate understanding? Within online environments, what training with online tools might students need before they can effectively use them? What technologies are students able to access?	Changing or decomposing numbers in a way that makes a solution less immediate allows for opportunity for students to make sense of the context, and process different solutions. Provide plenty of space for students to perform different strategies such as drawing, mathematics work, words, etc.
Action & Expression	Allow for multiple solution strategies that help students justify their work. Create an “open-endedness” that allows for those solution strategies.	Does the problem allow students to solve it in a variety of developmentally appropriate ways? How many solutions does the problem have? To what degree is the problem written so that students want to engage with it?	Put together the relevant student context and the new or decomposed numbers in a way that makes sense, and is realistic, to students reading or listening to the problem. Asking students to justify their answer or solution creates an open-ended style that promotes students’ display of understanding of the standard or concept.

Figure 8. Connecting modifications from the Fast Felines Task to UDL guidelines.

15. Great Adventures Canoe and Kayak company offers 3 different options as shown in the table. This afternoon a group of 6 friends is paddling the Adventure Mid Trip in individual kayaks. How many total yards will the friends paddle combined? (Hint: 1 mile = 1,760 yards)

Trip	Length (mi)
Beginner Short Trip	$2\frac{2}{5}$
Adventure Mid Trip	$4\frac{7}{8}$
Extreme Long Trip	$7\frac{2}{3}$

Figure 9. A multiplication problem from *Reveal Math* (McGraw-Hill, 2019)

Teresa wants to make marshmallow treats to give her neighbors. The recipe requires $3\frac{1}{8}$ cups of marshmallows, $4\frac{1}{4}$ cups of rice cereal, and $\frac{1}{8}$ cup of butter. The recipe serves nine people. How much of each ingredient will she need if she must make treats for 27 people?

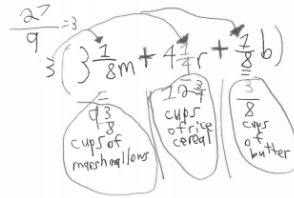
Teresa wants to make marshmallow treats to give her neighbors. The recipe requires $3\frac{1}{8}$ cups of marshmallows, $4\frac{1}{4}$ cups of rice cereal, and $\frac{1}{8}$ cup of butter. The recipe serves nine people. How much of each ingredient will she need if she must make treats for 27 people?

① $3\frac{1}{8} \rightarrow \frac{25}{8} \times \frac{3}{1} = \frac{75}{8}$ or $9\frac{3}{8}$

② $4\frac{1}{4} \rightarrow \frac{17}{4} \times \frac{3}{1} = \frac{51}{4}$ or $12\frac{3}{4}$

③ $\frac{1}{8} \rightarrow \frac{1}{8} \times \frac{3}{1} = \frac{3}{8}$

she will need
 $9\frac{3}{8}$ cups of marshmallows,
 $12\frac{3}{4}$ cups of rice cereal, and
 $\frac{3}{8}$ cup of butter.



① work

$$\frac{25}{8} \times \frac{3}{1} = \frac{75}{8}$$

$$\frac{25}{8} \times \frac{3}{1} = \frac{75}{8}$$

② work

$$\frac{17}{4} \times \frac{3}{1} = \frac{51}{4}$$

$$\frac{17}{4} \times \frac{3}{1} = \frac{51}{4}$$

③ work

$$\frac{1}{8} \times \frac{3}{1} = \frac{3}{8}$$

$$3 + \frac{1}{8} = \frac{24}{8} + \frac{1}{8} = \frac{25}{8}$$

$$4 + \frac{1}{4} = \frac{16}{4} + \frac{1}{4} = \frac{17}{4}$$

$$3 \times \frac{1}{8} = \frac{3}{8}$$

Teresa wants to make marshmallow treats to give her neighbors. The recipe requires $3\frac{1}{8}$ cups of marshmallows, $4\frac{1}{4}$ cups of rice cereal, and $\frac{1}{8}$ cup of butter. The recipe serves nine people. How much of each ingredient will she need if she must make treats for 27 people?

$27 \div 9 = 3$

Marshmallows $3\frac{1}{8} \times 3 = \frac{25}{8} \cdot \frac{3}{1} = \frac{75}{8} = 9\frac{3}{8}$ $\frac{24}{25}$ cups of marshmallows

Cereal $4\frac{1}{4} \times 3 = \frac{17}{4} \cdot \frac{3}{1} = \frac{51}{4} = 12\frac{3}{4}$ $\frac{12}{17}$ cups of cereal

butter $\frac{1}{8} \times 3 = \frac{3}{8}$ $\frac{1}{8} \cdot \frac{3}{1} = \frac{3}{8}$ 24 cups of butter

Figure 10. Students' written work on the revised marshmallow task.

Slide 7 / 16

Draw and label a picture of the classroom.

Example 2: Mrs. Santiago was asked to find the area of her rectangular classroom. The length is $(5a^2 - 3a + 2)$ feet and the width is $-6a^2$ feet. Write and simplify a polynomial that represents the area of Mrs. Santiago's classroom.

Not sure how to start?
Try drawing a picture of the

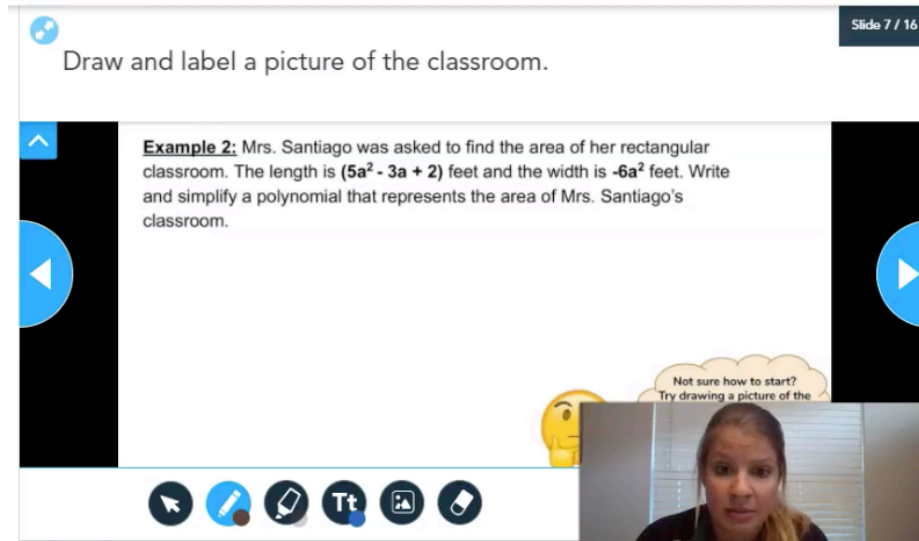


Figure 11. Megan uses Nearpod with some of her Algebra word problems.

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Thank you for your submission

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