Practice-based research on the teaching of mathematics: progress and imperatives for the future

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Professional fields face persistent challenges in connecting practice and theory. In particular, tensions exist as to how theory and knowledge are developed, as well as what constitutes authority for practice. Together the articles in this issue explore three elements of the turn toward "practice-based" research and professional education in mathematics education: designing teaching and learning in and for practice, learning mathematics teaching as a practice, and collaborating across professional roles and identities. In this commentary, we interrogate meanings of practice-based research on teaching and discuss themes across this collection of articles. We then argue for three imperatives for future efforts: (i) working on shared understandings of what the term "practice-based" might mean; (ii) developing more nuanced conceptualizations of "teaching"; and (iii) attending explicitly to justice in practice.

This thematic issue offers a helpful sense of the scope of "practice-based" work being done in Scandinavia. We are delighted to learn about progress being made to orient scholarship and professional work around notions of practice and are humbled to comment on it. We begin by reflecting on how the phrase "practice-based" is used in this collection and in the field more generally.

Calls for practice-based approaches are rooted in a laudable commitment to be useful for the practice of mathematics teaching and learning. However, the exact meaning of "practice-based research on teaching" remains underspecified. What distinguishes it from research that is not based in practice? Do the authors in this special issue share a common understanding of what they mean by the term? The description given in the original call for this thematic issue suggests that "practice-based"

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implies a particular orientation toward teaching as well as toward research on teaching:

> Instead of observing what teachers do, practice-based approaches tend to investigate the work that is to be done and the problems that are entailed in the teaching of mathematics. One important way to improve the impact of educational research on practice is that research pay closer attention to instructional problems teachers want to solve.

It also suggests that research problems need to be problems of practice. Looking further at the call, we note it uses practice-based to describe "research," "approaches," and "kinds of study." This differs from the prevailing use of the term in mathematics education, where it most often seeks to characterise a form of professional education. Examining what is meant by "practice-based research on teaching" seems therefore a good place to begin.

The term "practice-based" is not confined to education. It appears in other professional fields. In medicine, practice-based research often refers to research conducted by physicians in the context of their "practice." Similarly, dentistry in the United States and elsewhere has organised a network to support practitioners' practice-based research (Gilbert et al., 2008). In China, management education and research have turned to practice-based theory in response to critiques of being irrelevant (Zhang et al., 2018). In contrast, in the creative arts, practice-based research emphasizes understanding the nature of practice and how to improve it, while the creative arts emphasize the creative process and the works generated (Candy & Edmonds, 2018). In this range of work, scholars use "practice-based" as a descriptor of theory, evidence, approach, perspective, professional education, learning, design, and more. In education, it is most often used in reference to theory (e.g. Thompson et al., 2019), teacher education (e.g. Kavanagh et al., 2020), professional development (e.g. Osborne et al., 2019), and educational improvement (e.g. Peurach et al., 2019).

Given its popularity, and to support our thinking about contributions in this issue, we ask: How is the term "practice-based" being used? What does it mean? What do we want it to mean? What might care for the "integrity" of research mean in this context? And given how pervasively practice reflects and perpetuates systemic injustices, how might practice-based research confront patterns of harm in practice?

We turn to consider problems, methods, and claims (three key elements of research) and possible implications of the term "practice-based." One possibility is that practice-based research on teaching could be taken

to mean that the *problem* or focus of study is rooted in practice. Implicit here are questions about criteria for deciding what to study and who makes such decisions, as well as the extent to which critical lenses are deployed. Alternatively, it could mean that practice plays a central role in *methods*, as the source of empirical evidence and grounding for interpretation and analysis. Questions about practice-based methods might explore innovative approaches to studying practice, whether they are legitimate, appropriately critical, and who decides. A third possibility is that "practice-based" could refer to the nature of *claims*. The phrase could imply that claims need to be about practice, or useful to practice. Implicit here are questions about the basis for judging claims as worthwhile, whether they challenge taken-for-granted assumptions, and who decides. With these issues in mind, we summarise the articles, examining their research problems, methods, and findings, and then we offer perspectives on the development of practice-based research on teaching, with more explicit attention to our own perspective, including imperatives to conceptualize teaching and attend to justice.

Summaries and themes

Two articles in this collection identify and focus on issues of collaboration in practice-based research. Säfström and colleagues argue that teacher-researcher collaboration in design research is important for bridging the theory-practice divide. They distinguish symmetry (equal attention to the needs and conditions of teachers and researchers) and complementarity (recognizing the unique expertise of each group). They use these to make sense of and help navigate dynamics of power in collaborative work. Palmér and van Bommel use Kilpatrick's (1993) research-quality criteria (validity, predictability, rigour and precision, reproducibility, objectivity, originality, and relatedness) to examine quality differences over phases of a design-research project in which teachers and researchers had different roles. They identify tensions between collaboration and research quality, and tradeoffs, in particular between internal and external validity.

Several foundational questions arise from these studies. In both, teachers are collaborators, which the authors imply is a defining feature of practice-based research, but they are collaborators in design-research projects, not in the studies themselves. Teachers are not collaborators in the study of collaboration and the study of research quality, respectively. Although it is not necessary that teachers be collaborators in the actual research, this raises questions about what is meant by practice-based research. Perhaps these are meta-studies of practice-based research, but not themselves practice-based research. What here is practice-based and

why? Is it the professional development, the design research on it, or the study of the effort overall? Efforts to bridge the gap between theory and practice can lead to blurring the lines between theory and practice. This blurring may be desirable but also risks undermining the integrity of both. In addition, the issues of power and positionality raised in relation to teachers and researchers suggest additional questions about the voices of students and communities and deeper questions about the nature of potential harm being addressed and the theory of action in play.

Three other articles focus on the design of professional development. Björklund and Ekdahl argue that the design of professional development needs to draw on and target change in teachers' experiences of teaching. They situate teachers in an ecology of learning about variation theory as the teachers seek to understand and improve student learning. They argue that understanding teacher development in this way and using it to inform how they engage with teachers can lead to teachers' learning of theory and its use in teaching, and consequently to improving their practice. Fauskanger and Bjuland analyse participants' discourse moves during co-planning sessions. They find that expressing shared ideas, providing arguments, and raising challenges during co-planning develops teachers' skill in predicting student responses, recording students' ideas publicly for discussion, and aiming towards the lesson goal. Skott, Falkenberg and Honoré designed an induction programme to address problems new teachers experience and investigate what and how two teachers learn. They report that one teacher learned little, while the other teacher exceeded expectations. The authors argue that these differences in learning are shaped by the teachers' views of their own schooling, their training, and the schools where they teach.

All three studies investigate practice-based professional education, designed around cycles of planning, enacting, documenting, and reflecting. They make different assumptions and focus on different concerns. Fauskanger and Bjuland view teaching as professional work and teacher learning as skill development resulting from reasoned dialogue. Skott, Falkenberg and Honoré view teaching as a social practice and teacher learning as constituted by patterns of participation in school, local, and broader contexts. Björklund and Ekdahl combine elements of these. They foreground both a teacher's experience (one that "thrives in the constant encounter with others' both empirical and theoretical experiences") and the increased discernment of distinctions arising in those encounters and altering experience.

Fauskanger and Bjuland's analysis of opportunities to learn aligns with the original call for papers, where practice-based approaches investigate the work to be done in addressing instructional problems teachers need

to solve (though critical concern for deciding on instructional problems is not addressed). In contrast, Skott, Falkenberg and Honoré provide a helpful reminder that what teachers bring, how they take up professional development, and how they engage with others in social, institutional life, all influence their learning and their teaching, that teacher learning is not simply a matter of knowledge and skill development. This contrast in what is meant by teaching and teacher learning is visible in existing literature. For example, Grossman and colleagues' (2009) work on decomposition and recomposition in practice-based teacher education focuses on crucial analytic and dispositional tensions (such as which practices matter and skill versus will) but does not take up the issues of identity that often shape opportunities to learn, as considered by Battey and Franke (2008). This contrast is an important lesson for our field. Attending to multiple perspectives and inherent tensions is imperative. Practice-based research may help scholars notice and combine these foci.

Another foundational concern for practice-based research on teaching is which theory is best, or which types of theories. If the impetus for practice-based research is to prioritise its usefulness to practice, then the competing demands, dynamics, and realities of practice must be taken into account. Practice-based research should have as its goal to yield insights that inform the work of teaching. It should seek to help teachers examine their sense of themselves as actors in communities where they work, constructively and critically. In addition, such research must take into account what is to be learned (mathematics) and the goals and dynamics of the education enterprise in communities and society. Practice-based research on teaching must keep its eye on all of this.

The remaining three articles focus more squarely on teacher and student *learning* – in the context of professional education, but with greater attention to dynamics of learning teaching than in the studies above. Mårtensson and Ekdahl illustrate how integrating theory and practice can deepen pre-service teachers' knowledge about practice. In the context of a learning-study, they engage pre-service teachers in using variation theory both as a mathematics-task design tool as well as a lens for reflecting on use of the tasks. They then identify five types of tasks generated by their pre-service teachers and discuss how the pre-service teachers used what they were learning about variation theory as they prepared tasks and reflected on teaching those tasks. Tyskerud uses a commognitive lens to analyse changes in teaching as teachers participated in multiple cycles of lesson study. She observes that teachers develop skill in designing and enacting ritual and exploratory routines, but she echoes Nachlieli and Tabach (2019), cautioning that ritual routines (associated with traditional teaching) play an important, but inadequately understood

role. Eriksson, Fred, Nordin, Nyman and Wettergren discuss how students' tool-mediated collective reflections establish collective mathematical work in the classroom and what teachers need to do to support this. They describe two grade 7 lessons. The lessons combine problem situations that motivate student thinking by incorporating designed contradictions with instructional representations that support public deliberation. Together these features support collective reflection, where seeing their own and others' explanations in the light of public exchange leads students to awareness of their own thinking and consequent learning.

Although these studies differ in approach, they surface another important foundational concern for practice-based research on teaching, that the point is to inform practice, specifically teaching practice. Mårtensson and Ekdahl consider what teachers need to attend to and do with tasks to support student learning. They also note a limitation of their study in only examining practice in relation to instructional tasks, with little consideration of how this fits into practice as a whole. In analysing teaching routines, Tyskerud found that task design and asking questions support exploratory routines key to student-centred teaching. Eriksson and colleagues identify three didactical tools for supporting students' learning: attending to and using contradictions; seriousness in staging playfulness; and creating common workspace for explicit talk and ongoing documentation of work. Each of these studies draws implications for teaching, yet these are byproducts of their theoretical lenses and approaches to professional education. Even though they address related slices of the work of teaching (all are concerned with task design, eliciting thinking, and public recording), it is unclear how they "fit" with practice, how they might be effectively taken up, indeed how they "fit" with one another. In practice-based research, how can the integrity of teaching as a practice be honored, with its own logic and realities? Similar questions arise regarding research integrity: How can research claims be sensitive to the full set of realities at play in teaching, while maintaining a respectful sensitivity to teaching as a complex and contested practice that shapes and is shaped by the socio-political and historical environments in which it plays out?

As we see in this collection of articles, one challenge for practice-based research is the multiple layers and many competing concerns at play. Practice itself is complex, requiring attention to different objects from different perspectives. Practice-based research, too, is layered and needs to coordinate analysis across these differences, all the while maintaining primary allegiance to practice, including the experiences and perspectives of learners and teachers, the demands of their work, and the environments in which they are situated. In addressing these challenges with nuance and care, attending to the integrity of the research is not simple.

In discussing the integrity of practice-based research in the field of design, Biggs and Büchler (2007) describe a struggle for legitimacy and debate about whether practice-based research differs from academic research in the disciplines and should be held to a different standard. They conclude it is undesirable and unnecessary to create a special status and that, in addition to attending to problems, methods, and claims, the quality of research, practice-based included, depends on the strength of the chain of reasoning, judged in the context of problems and claims. To pursue practice-based research then, we need to attend to the nature of problems, methods, and claims and the quality of the chain of reasoning that links them. Quality is a complex notion, including transparency of the connections drawn and the types of evidence used and explained.

We created *table 1* as a tool to offer a snapshot of the problems, claims, and linkages for each of the eight articles and to consider challenges as they take shape in these studies. Starting with the first column, a research study must frame a problem, and justify not only the problem but also its significance. The researcher must be convinced, and convince others, that the problem, from a perspective of practice, is real, makes sense, and is worth studying and that the study holds promise for dealing with the problem in practice. Tensions can arise between relevance on one hand and study-ability on the other, but this challenge is one researchers must manage. For the first column, we found it helpful to reflect on three issues: the degree to which each frames a clear research problem, its merit when viewed from a practice perspective, and whether the approach for addressing the problem is consistent with the intent.

The second column of *table 1* provides an estimate of how well the theoretical framing aligns with what might be considered practice-based research. Some might argue that practices and their connections, not individuals or discourses, should comprise the theoretical building blocks for studying and understanding the human interactions of teaching and learning in schools. Gherardi (2019), an organizational theorist, characterises practice-based approaches as any that take a practice point of view, with the study of practice central. She writes:

> Why assume practices as the units of analysis of organizing? The simplest answer is that practices are loci – spatial and temporal – in which working, organizing, innovating, and reproducing occur. (p. 2)

Gherardi's conceptualization is certainly not the only way one might conceive of "practice" in practice-based, but it draws attention to whether conceptual and theoretical tools brought to bear in a study are suited for practice-based research. Her list of "working, organizing, innovating, and reproducing" conveys the significant scope that practice entails, where

Table 1. *Research components of each of the articles*

	Research Problem	Theoretical Framing	Research Questions	Research Design	Data Collection and Analysis	Claims
Initiating teacher-researcher collaboration Säfström, Palmberg, Granberg, Sidenvall and Lithner	Need good examples of teacher-researcher collaboration in design research	Symmetry and complementarity in all three parts of cycles of exploring a problem, designing a solution, and evaluating outcomes	How did symmetry and complementarity interplay and develop within the core processes during the first year of a TRC focusing on constructive design?	Design research, as an intervention in teaching, with analysis of symmetry and complementarity in project meetings, communication, and documentation.	8 researchers and 51 teachers from 7 primary and secondary schools in 3 cities. Analysis of symmetry and complementarity in approach and outcome for identified activities of collegial teams.	Identification of particular challenges and ways to address challenges. Conclusion that honouring symmetry and complementarity make sustained collaboration possible.
Teachers' participation in practice-based research Palmér and van Bommel	Need to understand how to include teachers in practice-based research without sacrificing quality	Design research, with cycles of hypothesis, testing, and refining solutions to problems of instruction requiring detailed understanding of context. Kilpatrick's criteria for research quality.	How do different kinds of collaboration between researchers and teachers coact with the quality of practice-based research on mathematics teaching and learning?	Design research with three phases in which collaboration varied, with analysis of research quality in each phase.	35 teachers; 145 students; 8 PD sessions. Student work, teacher interviews, student interviews. Subjective interpretation of quality based on Kilpatrick's criteria.	Teachers' increased roles led to increased internal reliability and predictability, along with decreased external validity, rigour, precisions, and reproducibility; Objectivity became more complex, originality was pre-determined, and overall relatedness holds.
Learning to teach mathematics in preschool Björklund and Ekdahl	Need to know how to support preschool teachers in developing a professional knowledge base useable in practice	Variation theory, in which (children's or teachers') learning is seen as a change in ways of experiencing a phenomenon resulting from more and new aspects being discerned.	How does one teacher's way of experiencing teaching numbers to preschool children change when participating in a practice-based professional development project?	Design research with analysis of changes in a teacher's teaching over time as she is engaged in a theory-driven PD intervention.	Selected teaching episodes and interviews from both commonly and individually planned number activities. Analysis of changes in principles for teaching evident in the teacher's teaching and interviews over time in PD.	Teachers learn about variation theory as they test it in practice, both as a design principle and as an interpretive lens, and these experiences reshape their teaching.
Opportunities to learn ambitious mathematics teaching Fauskanger and Bjuland	Need to develop better ways of supporting teacher learning of ambitious teaching practices.	Sociocultural learning and learning cycles.	1. Which ambitious mathematics teaching practices do teachers have opportunities to learn in reasoned dialogue in co-planning? 2. How do specific utterances of teachers' reasoned co-planning dialogues provide them with opportunities to learn ambitious mathematics teaching practices?	Analysis of interactions in a learning cycle PD intervention.	51 episodes from 2 learning cycles for each of 2 groups of 7 teachers. Analysis identifies thematic reasoned dialogue episodes and ambitious teaching practices discussed in these. These episodes were then analysed in relation to 5 dialogue moves identified in the literature.	Co-planning affords opportunities to learn to predict student responses, represent responses, and aim towards the lesson goal. Learning opportunities are enhanced in conversation that involves expressing shared ideas, providing arguments, and raising challenges (less so in moves of clarification or opinion and making supportive contributions).

	Research Problem	Theoretical Framing	Research Questions	Research Design	Data Collection and Analysis	Claims
New mathematics teachers' learning when participating in induction Skott, Falkenberg and Honoré	New teachers need better induction support and current understanding of how to support is lacking.	Situated perspective based on social practice and figured worlds.	What and how do new lower secondary mathematics teachers learn by participating in an ME-IP in terms of shifts among their practices and figured worlds?	PD intervention with pre- and post-analysis of practices and figured worlds.	2 teachers with 8 observed lessons and 8 interviews. Phased PD programme: identifying a shared problem, mentor support, cycles of lesson study. Analysis of practices and figured worlds.	While one teacher exceeds expectations for learning, the other learns very little. The two teachers' learning is reflexively related to school-local and broader contexts, which helps to explain the differences in their learning.
Development of mathematics teaching: a commognitive analysis of routines in the classroom Tyskerud	Need to improve teacher learning of student-centred teaching.	Commognitive theory.	No explicit research questions readily identifiable.	Longitudinal study of teacher learning across lesson study cycles.	4 teachers, 3 cycles over 1.5 yrs., 6 hrs. of video of mtgs. Analysis of whether object of study, process, and outcome indicate ritual or exploratory routines of interaction.	Changes in routines occur between lesson study cycles and teachers learn to switch routines, but without a clear sense of linear development or impact on student learning.
Variation theory and teaching experiences as tools Mårtensson and Ekdahl	Need illustrations of how the integration of theory and practice can deepen pre-service teachers' knowledge about practice	Learning study as cycles of lesson planning, analysis, and revision framed with a theory of learning. Variation theory as coming to see a thing in a new way by experiencing critical aspects not previously discerned.	In what different ways do the PS-Ts redesign mathematical tasks when using variation theory as a tool for reflecting on their planned and taught lessons?	Intervention in pre-service teacher education with categorization of the mathematics tasks produced and interpretation of how intervention led to these.	40 pre-service teachers in a common course; their final reports. Identify comments about changes to tasks, categorise them, and interpreted them.	Theory not just for developing practice or filling the "gap", but theory guiding practice and practice guiding theory – the theoretical tool was used for planning lessons and analysing student learning outcomes and practice seemed to change the PS-Ts' intentions about what the students must know, understand, and be able to do.
Tasks, tools, and mediated actions – promoting collective theoretical work Eriksson, Fred, Nordin, Nyman and Wettergren	Need to identify communicative actions for teachers to support students' agency in collective problem-solving for a specific learning outcome.	Learning activity with students' tool- mediated collective reflections central.	1: What indicates that collective theoretical work is established in discussions of algebraic expressions? 2: What in the content, design, staging, and tools of a task creates opportunities for students to engage in collective theoretical work on algebraic expressions?	Instructional intervention using a task based on learning-activity principles.	Part of a lesson. Construction of a narrative of students' learning actions and teacher's supporting actions, then analysis of who is doing what, with what tools, to what ends?	Tasks must capture core theoretical principles but retain problem identification for students to do. Teachers need three didactical tools: attend to and use contradictions; seriousness in staging playful format; and create common work space for explicit talk and ongoing documentation of work.

"reproducing" can invite the critical lens so necessary for recognizing injustice and "innovating" can open a door to new, more just ways of engaging in teaching and learning in society. Examining the theoretical framings for these articles, we find it interesting to consider potential matches and mismatches with the study of practice, as well as specific ways authors have made use of theories to study practice.

Similar practice-based considerations can be used to probe the other columns in the table. Are research questions significant from a practice perspective? What about claims? Again, we are not defining what ought to comprise a practice perspective. We are also not arguing that a practice perspective should serve as a criterion for the value or validity of the papers. Rather, we suggest such questions can help clarify what is meant by practice-based research on teaching. For instance, Björklund and Ekdahl investigate ways theory might support teachers in developing a useable knowledge base. They find teachers can learn about theory as they test it in practice and that doing so can change what they see and do when teaching, at least in the one case they examine. This is not a problem a group of teachers would likely pose, nor are the claims likely to be seen as directly informative for teaching. Alternatively, what makes this study practice-based might be the fact that it asks fundamental questions about what teaching is (as a practice) and how it might change.

In addition, it is important to examine the chain of reasoning for each row of *table 1*. If the aspirations of practice-based research are to be realised, studies must build transparent chains of reasoning from problems to claims. Of course, the chain of reasoning in a research article is neither simple nor straightforward, yet its construction is fundamental to a paper's quality. Further, it is likely that practice-based research faces additional complexity in its arguments, with nested objects of study and added concerns of useability in practice, practical effectiveness, and critical consideration, such as the positionality of the authors – that is, how their identities and experiences ground and shape the orienting perspectives, assumptions, evidence, and arguments of the research. It behooves practice-based researchers to justify their chain of reasoning and make visible how their positionalities shape their interpretations and conclusions.

Foundations for Practice-Based Research

Up to this point, we have summarised the articles and raised questions for collective consideration. We turn next to offer comments that draw on our own sensibilities and priorities, though still in a spirit of contributing to the common agenda of clarifying both the impetus for and meaning of practice-based research on teaching.

Although we are sympathetic with the aims of being relevant, useable, and effective, we encourage vigilance with these notions. Who determines relevance, or usefulness? And what might be limits to ideas about effectiveness? Focusing on research in light of a practice perspective need not preclude researchers' attention to these, but we urge care with their meanings and possible limitations. From our perspective, the fundamental orientation of "practice-based" research is the aim to treat teaching as a practice. As Gherardi argues, practice, as an object of study, includes the richness of situated human interaction while also providing enough focus to make sense of and inform action. Practice is flexible as a unit of analysis. It can illuminate a focused piece of the work or the comprehensive, contextualised whole. It supports zooming in on a specific aspect while maintaining sensibility for a broader perspective, and the conditions and contexts that contribute to and are affected by it. To repeat Gherardi's words, "practices are loci – spatial and temporal – in which working, organizing, innovating, and reproducing occur." In other words, practices are the means of work and the sites of production and reproduction. Teaching, teacher education, and research are human activities with inherent responsibilities. For us, education, at its core, is about building a better world. A focus on practice helps us attend to both the micro and the macro, consider how each is manifest in the other, be specific in ways that help people learn to teach, and keep in mind larger dynamics, values, and aims.

We see these issues take shape in this thematic issue. For instance, Skott, Falkenberg and Honoré remind us of the larger landscape, where the figured worlds of one's own schooling, teacher training, and a school's culture contribute to the practice teachers enact. Fauskanger and Bjuland identify specific practices and explore ways to support teachers in learning these practices. Tyskerud considers relationships between focused practices (discursive-routine activities) and broader characteristics of the practice (ritual teacher-centred work and exploratory student-centred work). Each foregrounds an aspect of teaching. Taken together, they begin to provide a picture of teaching.

For us, a practice-based approach to the study of teaching requires tools to attend flexibly and comprehensively to practice, even as it focuses on some aspect. We conceptualise the work of teaching as constituted over time through the relational interactions among learners and teachers, around some specific "stuff," and situated in broader sociopolitical and historical environments. These environments shape and are shaped by the individuals, in and through the larger patterns in which they are steeped, and the constraints and opportunities they experience in context (Ball, 2018; Chazan et al., 2016; Herbst & Chazan, 2017). Studies in this

thematic issue contribute to aspects of this conception. A central challenge is attending to a full picture of teaching both within single studies and across them. We do not mean to suggest that every study needs to be comprehensive, but that any practice-based study should seek to treat practice with integrity, to find ways of holding in mind the overall character of teaching as a practice – even as it focuses attention on some aspect. This is no small task. Having a functional, critical, comprehensive picture of teaching and being able to maintain regard for other aspects of teaching while focusing in on a particular aspect is essential to teaching. It is also essential for practice-based research on teaching, both in the approach used and in the sensibility with which it is conducted.

Moreover, connecting and relating across studies of teaching is crucial. This requires frames for making sense of studies. It asks of scholars that they consider and articulate how their work speaks to other practice-based research on teaching. The academy rewards individualism. It celebrates showing how one's work is distinguished from, refutes, or critiques other studies. In situating their work in relation to others' work, scholars typically name the theoretical issues to which they aim to contribute or those they seek to challenge, but less often point to the practice-centred problems with which their work connects. We wonder if practice-based research, with its primary interest in informing practice and secondary interest in advancing theory, might benefit from greater investment in working across theoretical camps, with a focus on relating and combining theoretically disparate work to develop more coherent implications for practice, ones that might build over time. We do not have well-formed solutions to offer but ask whether community effort along these lines might improve the collective work of the field, and indeed, contribute to a different view of "field" and of "community," and thus, of the construction of collective knowledge.

As initial support for such efforts, we offer three specific considerations to help build useful scholarship: (i) working on shared understandings of what the term "practice-based" might mean; (ii) developing more nuanced conceptualizations of "teaching"; and (iii) attending explicitly to justice in practice. The first follows from the discussions above. Greater clarity about what it is about the research that makes it based in practice would strengthen the notion of "practice-based" research on teaching. This phrase will not be useful if it becomes a ubiquitous descriptor that lacks shared meaning. In particular, using the term "practice-based" to characterize research, theory, professional education, teacher education, and learning will become meaningless unless scholars are more specific about what is meant. Are there advantages or disadvantages of basing practice-based intervention research on practice-based professional

education? Should practice-based learning serve as the basis for such professional education? Is practice-based research called thus because it uses practice-based theories, because it studies practice-based professional education, or for some other reasons? It is important that it be clear what is practice-based, what makes it practice-based, and why being practice-based is important. With this in place, scholars might then articulate the relationships among practice-based research, theory, professional education, and learning and might be in a better position to justify the research conducted.

Table 2 represents our attempt to characterise the meaning of practice-based research conveyed in each of the articles, often implicitly. We apologise for mischaracterizations and ask that readers and authors take these as our invitation to generate their own. We hope the table helps scholars consider what are essential characteristics of practice-based research. We note that some articles did not use the language of practice-based and for most articles we inferred meaning. As seen in the first column, several implied that their study was practice-based because the study was situated in practice-based professional education. One characterised practice-based research as integrating basic and applied research. Two studies implied that the research was practice-based because the study investigated teaching.

In addition to considering the meaning of practice-based, we argue that practice-based research on teaching needs to clarify the conceptualization of *teaching* being used. In most articles, the meaning is implicit, yet potential meanings differ in important ways. Research might provide normative views of how teaching should be. It might characterise good or effective teaching and advance a specific approach. It might be descriptive, portraying and analysing what teachers are doing in classrooms. In this case, teaching may be conceptualised as what teachers do, independent of notions of quality. Or research might be based in logical analysis of what the work entails and might use those analyses to identify key practices and practical issues central to learning or doing teaching. Some might view teaching only in relation to subject-matter learning, while others might see moral education, civic preparation, or attention to societal justice as central to teaching. Many studies focus only on what happens between teachers and students in classrooms, whereas some take a broader view of the work of teaching to include planning, communicating with families, or advocacy. A clearer sense of what is meant by teaching in diverse studies would make it easier to appraise studies and their claims, see relationships among them, and synthesise their implications for practice.

Table 2. *Explicit and implicit meanings of practice-based research, teaching, and social justice in each of the articles*

	Meaning of practice-based research	Meaning of teaching	Meaning of social justice
Initiating teacher-researcher collaboration Säfström et al.	Research situated in PD that engaged in cycles of exploring a problem of practice, designing and implementing solutions, and evaluating outcomes.	No explicit characterization. Normative approach focused on students' construction of solutions, where teaching involves diagnosis and feedback. Focused on student learning of content (without attention to social environment).	Power dynamics in professional education are addressed. Justice is not addressed in the research conducted or school teaching and learning.
Teachers' participation in practice-based research Palmér and van Bommel	Integration of basic and applied research, as in design research with cycles of instructional planning, enacting, and reflecting.	No explicit characterization. Constituted by cycles of preparing, implementing, and reflecting on instruction.	Not explicitly addressed in the research conducted, or in professional education or school teaching and learning.
Learning to teach mathematics in preschool Björklund and Ekdahl	Research situated in PD that intervened in practice, iteratively, with practitioners in cycles of planning, enacting, and reflecting, and attentive to theory and utility.	Constituted by dynamic interaction between learners and teacher, with an orientation towards a learning object (without attention to social environment).	Not addressed in research, professional education, or school teaching and learning.
Opportunities to learn ambitious mathematics teaching Fauskanger and Bjuland	Research investigating work of teachers and teacher educators in cycles of planning, enacting, and reflecting.	No explicit characterization. Draws from core practice literature and ambitious teaching literature, where teaching is a cycle of planning, enacting, and reflecting and is decomposable into practices. Focused on student learning of content (without attention to social environment).	Not addressed in research, professional education, or school teaching and learning.
New mathematics teachers' learning when participating in induction Skott et al.	Research situated in PD designed around a shared problem and engaged in lesson-study cycles of creating a lesson plan, using it, gathering data, and reflecting.	A social practice	Power dynamics in researcher-teacher relations are addressed. Justice is not addressed in professional development or school teaching and learning
Development of mathematics teaching: a commognitive analysis of routines in the classroom Tyskerud	Research investigating work of teachers and teacher educators in cycles of planning, enacting, and reflecting.	Defined as experienced interlocution in discursive-routine activity.	Not addressed in research, professional education, or school teaching and learning.
Variation theory and teaching experiences as tools Mårtensson and Ekdahl	Collaborative and iterative teacher-driven inquiry that seeks both to develop practice and generate knowledge about teaching and learning	No explicit characterization. What teachers do in schools, which is experienced by teachers and needs to be informed by theory to be more effective.	Not addressed in research, professional education, or school teaching and learning.
Tasks, tools, and mediated actions – promoting collective theoretical work Eriksson et al.	No mention of practice based. Implicitly, any intervention study or empirical interpretation.	No explicit characterization. Minimalist view of teaching as support for task implementation.	Not addressed in research, professional education, or school teaching and learning.

Central to a definition of teaching is a logic about how to support learning. It also has implications for what constitutes a curriculum of professional education. Tyskerud draws on Nachlieli and Tabach (2019), who draw on Sfard (2007, 2008), to define teaching as experienced interlocution in discursive-routine activity. The notion of discursive-routine activity provides a helpful tool for making sense of social interaction in teaching and learning, but as Nachlieli and Tabach point out, both ritual and exploratory routines are essential. What, then, is a teacher to do and a teacher educator to teach? Understanding that whatever the teacher does will constitute a discursive-routine activity is of little use in deciding what to do. The definition seems to beg the basic question of what professionally responsible interlocution is. What might be the references for determining this? Impact on learners, and if so, of what sorts? Relation to broader societal contexts? This brings us back to the question of what would improve teaching, and therein, what is conceived as teaching. In her analysis, Tyskerud finds that designing tasks and asking questions are important teaching practices – because they are key tools in managing discursive-routine activity. These are not new practices to identify, but beginning to understand how they can shape discursive-routine activity might contribute to other scholars' understanding of these practices. Our point is that in offering a definition, Tyskerud gives us the opportunity to consider it, ask whether we agree, propose alternatives, and advance our collective thinking and the field.

Many of the studies in this thematic issue might be better characterised as research on teacher education, rather than research on teaching. A key building block, though, is a conception of teaching, and given the meager theoretical work on conceptualizing teaching, it is not surprising that these studies attempt to grow their understanding of teaching as they go. In doing so, though, the imperative to define what is meant by teaching remains.

To give a fuller sense of potential distinctions among conceptions of teaching and how they might interact with features of practice-based research on teaching, consider *table 3*. It offers three questions to help elaborate how the distinctions might interact with how research problems are identified.

- Are problems and solutions likely to be seen as relevant and taken up?
- Are there risks to the integrity of research?
- Are solutions likely to inform practice?

Table 3. *Example studies for three ways of determining problems of study and three potential conceptions of teaching*

Basis for Determining Problems for Study	Meaning of Teaching		
	What Teachers Do Description of what practitioners do, perhaps as best practice but as currently done.	Work to be Done Work logically entailed in teaching content to learners as an educational endeavor in a society.	What Teachers Ought to Do Prescriptions for what should be done from a normative perspective or theoretical foundation.
Practicing teachers determine problems	A study of how teachers establish learning environments focused on mathematical reasoning	A study of what is required to develop a classroom environment that foregrounds the development of positive mathematical identities for children who are members of historically marginalized groups	A study of what works to establish a productive classroom learning environment
Concern for practice is used to determine problems	A study of what teachers do to develop a learning environment	A study that seeks to identify the key normative elements that require explicit disruption to develop a classroom environment that foregrounds the development of positive mathematical identities for children who are members of historically marginalized groups	A study that seeks to specify what teachers should do at the beginning of the year to create a positive classroom learning environment
Concern for theory is used to determine problems	A study focused on how language is used and shapes the mathematical discourse norms	A study that seeks to conceptualize what comprises positive mathematical identities in context	A study that examines the socio-mathematical norms and racial narratives that undergird different classroom learning environments

The questions help us consider how to fill in the cells of the table. For instance, looking at the row for *practicing teachers determine problems*, we see that the work is likely to be more relevant than in the bottom row, when *concern for theory is used to determine problems*, although the framing may lack criticality. Looking left to right across the row, with different conceptions of teaching, relevance is likely to decrease. Practicing teachers may not always be in the best position to frame problems related to the demands of approaches to teaching that are not common in current practice – for example, the entailments of teaching mathematics framed in ways very different from Western conceptions of the subject.

Considering the third question, about which configurations are likely to lead to improvement, it is worth noting that the source and mechanisms of improvement are different in each cell. In the first column, improvement is limited by the best of current practice, whereas in the third column it is limited by getting the right approach from the outset. Our own preference is for the middle cell, where teaching is viewed as being logically entailed and problems are determined in line with practice perspectives.

Finally, we argue that practice-based research on teaching must attend explicitly to issues of justice. Too often left invisible or ignored, pervasive injustice persists, without direct confrontation in research or in teaching and learning. As U.S. citizens, perhaps our attention to injustice is heightened, but white, patriarchal, economic dominion and violence have deep roots in Europe. Wealth and power in Scandinavia have their own dark histories and circumstances. In addition, for us, and we hope for mathematics educators broadly, personal and community commitments to just society are foundational to our professional work. We are educators because we value its possibilities, without being romantic about or blind to its potential for numbing, controlling, and oppressing. We do not mean this as ideological rhetoric, but as a matter-of-fact statement of our convictions and ongoing learning about the subtle dynamics of power and privilege.

Issues of power arise in the articles of Säfström et al. and Palmér and van Bommel. These authors attend to the tensions that arise in what they see as twin imperatives: addressing the power dynamics inherent in doing research while maintaining research integrity. We must not lose sight of the fact that research is a human endeavour situated in societies steeped in oppression and violence. The everyday choices we make and habits of action that we view as "normal" either perpetuate patterns of harm and oppression of particular groups of people or can disrupt these patterns (Ball, 2021; Gholson, 2021). Research affords opportunities and responsibilities to attend to the larger societal impacts of the work we do in contributing to or seeking to dismantle normalized patterns of practice. Säfström et al. and Palmér and van Bommel remind us of these responsibilities and the difficult terrain they represent. They engage us in figuring out challenges needing our attention and potential tools for navigating these challenges.

The dynamics of power and privilege and their concomitant responsibilities are not limited to conducting research. They are foundational to teaching and teacher education. Teachers have a distinctive role in society. They, too, have competing obligations. They are responsible for teaching subject matter, for teaching children, for serving society – and

they are responsible to their conscience. Classrooms are structured by "normal" practices rooted in power and control, and these patterns of domination contribute to reproducing societal structures, including oppression and violence. Classrooms are also our greatest aspiration for breaking these cycles. Any meaningful conception of teaching must account for the opportunities that reside in teaching, opportunities too often left implicit or invisible. Research on teaching, especially practice-based research on teaching, must contend explicitly with the fact that issues of social justice are central to both teaching and research on teaching. Justice and the disruption of injustice are not optional or special-interest issues, no less so than are attending to the subject matter or to student learning. It is not enough to build theories of teaching (which is fundamentally a social and institutional activity) solely on cognitive dissonance or social apprenticing. Theories of teaching and practice-based research on teaching must additionally address the political imperatives of social and ethical responsibility.

As evident in the third column of table 2, attention to issues of justice is largely absent in much of the research being reported. How larger institutional, historical, and societal patterns systematically shape the environments of practice is not considered. Neither are how classroom interactions are shaped in multiple ways by larger systems of oppression that find their way inside the daily work of teachers and learners. We encourage practice-based approaches to more deliberately and consistently interweave attention to these patterns, and to see the role that research can play in exposing and challenging them.

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