STRUGGLING WITH PRODUCTIVE STRUGGLE: IMPLICATIONS FOR STUDENTS WITH DIVERSE COGNITIVE RESOURCES

Angela R. Crawford
Boise State University
angelacrawford1@boisestate.edu

The perspectives in mathematics education and special education are in tension when it comes to productive struggle. This study describes how struggle surfaced for the students and teacher/researcher in teaching experiments using learning trajectories with three students with diverse cognitive profiles. The students' activity helps to illustrate the relationships between struggle and mathematics learning. I share how students' struggle led to my own challenge in navigating tensions between mathematics education and special education. I consider how my focus on productive struggle without attending to cognitive difference reflected ableist thinking. Finally, I suggest implications of these observations for reframing productive struggle.

Keywords: Students with Disabilities; Instructional Activities and Practices; Learning Trajectories and Progressions

The perspectives of mathematics education, with a commitment to student thinking, and special education, with a commitment to explicit teaching, are often in tension—no less so when it comes to the idea of productive struggle. However, there is agreement that combinations of instructional approaches are beneficial for students (Alfieri et al., 2010; NMAP, 2008; Woodward, 2004). A number of researchers are investigating the mathematical thinking of students with learning disabilities as they engage with constructivist-based tasks (e.g., Hunt & Tzur, 2017; Xin & Tzur, 2016), but questions remain as to how best combine approaches. I conducted teaching experiments (Confrey & Lachance, 2000; Steffe et al., 2000) using a learning trajectory (LT) approach (Sarama & Clements, 2009) with three students with learning challenges, herein referred to as cognitive differences. I explored how primarily constructivist tasks and productive struggle might be supplemented with explicit support to generate learning. I drew on Hiebert and Grouws' (2007) definition of productive struggle as expending effort to make sense of mathematics.

Method

Participants

A purposeful sample of three elementary-aged girls with different learning strengths and challenges participated in this research. Table 1 provides information about these students.

Table 1: Participant information

Name	Age	Parent's description	Neuropsychological evaluation
Miranda	10	Insightful, creative. Likes to plan time to include rewards and breaks. Miranda says she wants to be interested or intrigued	Strengths: fluid reasoning, auditory memory. Difficulties: attention, inhibiting behavior; compromises accuracy for speed.
Eva	9	Great sense-of-humor, honest, loves pets, very active. Has trouble with	Strength: auditory processing. Difficulties: anxiety, speech-sound

		self-control, appropriate behavior. Very anxious about math.	disorder, mixed expressive-receptive language disorder, ADHD.
Macey	11	Compassionate, fun. Does well with hands-on and informal situations. Learns slowly and gradually, does not have a-ha moments.	Strengths: fluid and visual-spatial reasoning. Difficulties: executive functions, abstract reasoning, expressive language.

As the teacher/researcher, I am a participant in this research. I am a white, cisgender woman with over 16 years working in general education and intervention. My disciplinary commitments tend toward the mathematics education perspective, but having taught many students who struggle, I know things are not simple in practice. Therefore, in this research, I committed to supporting student progress rather than rigid use of a single instructional approach.

Teaching Experiments

The teaching experiments (Confrey & Lachance, 2000; Steffe et al., 2000) involved 45-minute individualized sessions, once or twice weekly, over 2-6 months during the COVID-19 pandemic. The conjecture guiding the experiments (Confrey & Lachance, 2000) was instruction based on a LT can center on constructivist-based tasks and support efficient progress when supplemented appropriately with reflection prompts and explicit guidance. Data sources include video and transcripts, artifacts, and planning and reflection protocols. Rigor was ensured through regular consultation with critical colleagues for their interpretations.

Retrospective Analysis

I used a three-level analysis (Simon, 2019) beginning with coding each data source for activity and strategies as indicators of student thinking. The next level of analysis involved identifying patterns and change over time, and the final level involved using the previous analyses to make inferences informing the guiding conjecture. I ensured rigor and trustworthiness through regular discussion with critical colleagues and sharing interpretations with parents as a form of member checking.

Findings

Miranda

The teaching experiment with Miranda was based on an equipartitioning LT (EPLT; Confrey et al., 2014) and reflective abstraction prompts (Simon et al., 2018). The struggle that surfaced for Miranda was sustained attention to tasks, and she resisted repeating tasks with varying number sets or contexts, a key aspect of reflective abstraction. She also wanted to figure things out without my support and did not want me to ask questions that would direct her thinking.

In a moment of insight, Miranda would enthusiastically rush through a task using sound reasoning but confuse the role of specific digits in some way and not quite "close the loop" on the idea. If she learned her solution was not right, Miranda would declare she was too confused and bored to continue. For example, when Miranda had an insight that sharing a objects among b persons results in a/b of an object per person, she said enthusiastically, "Mind blown!" I asked what would happen if four people shared three things. She tried to figure it out mentally but had a hard time keeping the numbers straight. When I pressed her to notate her thinking, she declared writing and drawing were boring, and only wanted to do it in her head. I tried to ask her questions to provide structure. However, Miranda then said she was too confused and bored, and she would not re-engage in the task meaningfully until the next session.

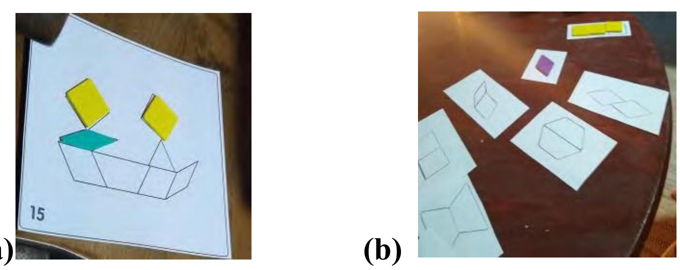
Miranda's effort to make sense of the math was productive in one sense—she would have moments of insight that were temporarily intriguing enough for her to pursue. However, it was unproductive in the sense that the process was not efficient. It was a struggle to find enough variety to sustain Miranda's motivation. Through trial-and-error, I landed on the approach of explicitly summarizing her activity from the previous session and explaining the conceptual idea at the heart of the task. I gave her feedback on where her thinking had gone astray. At this point, Miranda would solve one or two more related tasks and then express a desire to move on.

Eva

Eva had high levels of anxiety and often refused to participate during math class. Her activity in counting, arithmetic, and spatial reasoning was consistent with what is typically seen in children 4-6 years old (Clements & Sarama, 2021). I selected the shape composition LT as the focus for the teaching experiments (Clements & Sarama, 2021). The struggle that emerged for Eva was engaging with any challenge. She would look at a task and within seconds decide she could not do it. Then, she would jump up and run to the yard or play with her dogs.

Eva's first task is shown Figure 1a. Eva appeared to recognize the outlines for two squares because she quickly found these shapes and placed them at the top of the picture. Next, she used trial-and-error to find the right shape and orientation for the rhombus. Then, her attention moved to the connected shapes along the bottom, and she abandoned the task and would not return.

Because Eva would not engage with challenges, I chose to provide a very graduated increase in difficulty with extensive, explicit feedback that I viewed as eliminating struggle. Over the next few sessions, I provided outlines that gradually increased the quantity, combination, and orientation of shapes and the proportion of shared sides (see Figure 1b). I also provided Eva with extensive positive feedback. Each time she filled in a picture, I explicitly pointed out a mathematical feature of her activity: "Nice work, Eva! I noticed that … you saw that this large shape was made of two smaller shapes. Maybe you noticed this outline has three sides? Oh, you did! Great! And I saw you solve a problem—you turned this one to make it fit just right!" This approach led to fewer instances of giving up and gradual progress in the shape composition LT.



(a) **(b)**

Figure 1: (a) abandoned shape composition task; (b) scaffolded shape composition tasks

Macey

Macey's sessions focused on the EPLT (Confrey et al., 2014). Macey quickly took on each task, working until she felt she had achieved a satisfactory solution. However, her progress was slow, and we spent many sessions repeating variants of tasks. Macey struggled to make connections, see relationships, and construct new mathematical understanding. I used reflection prompts to guide her attention toward new ideas. However, prompts such as "What do you notice?" were typically too general. She seemed look for any feature she could describe, not one related to the mathematical ideas. For example, I asked Macey to share a whole "French fry" among an increasing number of sharers. She partitioned the whole each time, named the size of the share, and taped it to a piece of paper. When I asked her what she noticed, she said the denominator counted by ones and the numerators all stayed one. Then, I asked a more focused question, "Do you notice anything about the size of the share and the denominator?" She did not have an answer, so I became more directive: "Look at this denominator and this share. Now look at the next denominator and this share. Do you notice something? No? Can you compare this share to the one before? Which is a bigger sized share?"

I intentionally set up situations to be perturbations of her current conceptions to help her re-construct ideas. However, these situations were unproductive. Typically, Macey would look at the representations for a few seconds and then move some manipulatives or pencils or papers around, sit back in her chair, and look at something else in the area. It did not appear to be avoidance; my intuition tells me it was so inaccessible that she lost her place, maybe forgot what was expected, and was waiting for guidance. While I cannot be sure my interpretation is correct, I feel confident characterizing this as unproductive struggle. Faced with unproductive struggle, I often switched to explicit instruction to see if it supported progress. I would directly point out a relationship we had been exploring and explain the idea I wanted Macey to see. Then I would ask her to point to features of the representations I was describing and ask her to restate what I had just explained. My hope was that by guiding her attention explicitly, and with enough repetition of the idea, Macey would come to internalize the idea rather than remain lost.

Discussion

My purpose for the teaching experiments was to describe conditions under which constructivist approaches supplemented by reflection prompts and explicit guidance supported students' learning and productive struggle. During the sessions, I frequently experienced struggle in deciding on the "right" course of action to support student learning. I wanted to provide opportunities to construct understanding through sense-making but, concerned we were not making progress, I felt I began to rely heavily on explicit guidance. However, during the retrospective analysis, I saw the increased support and explicitness was intentional and individualized, not a refutation of the conjecture or abandonment of the commitment to positioning students as active learners. In each case, the students actively engaged in the mathematical tasks without premature guidance (DeCaro & Rittle-Johnson, 2012). Explicitness was inserted within a constructivist framework to support attention to features relevant to the underlying mathematical ideas and connections.

Once I saw the pattern of purposeful, individualized explicitness, I also realized struggle was productive for all three students. Until then, my notion of productive struggle had nuances of ableist thinking. Ableism describes practices and attitudes that compare individuals to a standard of "normal" resulting in practices serving "standard" people (Stop Ableism, 2021). I had resisted "too much" explicitness because I viewed productive struggle narrowly as *expending effort to make sense of mathematics*. Viewed in this way, it established a standard reflecting cognitive strengths and needs of typically-achieving students. I had de-valued other forms of struggle such as with attention, anxiety, abstraction, or combinations of those.

These observations have two implications. First, disciplinary commitments can be re-framed as a commitment to recognizing and navigating complexity. Following on this, another implication is a more inclusive view of struggle would recognize the struggle that surfaces from cognitive difference. We can support students' productive engagement by intentionally and purposefully planning for struggle in multiple forms—sense-making, attending, processing, remembering, reasoning, etc. From this perspective, a more inclusive definition of productive struggle is *expending effort that leads to greater levels of engagement with mathematical sense-making*. This small change in syntax and vocabulary may have large effects semantically, and those effects may better serve our goal of inclusivity.

References

Alfieri, L., Brooks, P.J., Aldrich, N.J., & Tenenbaum, H.R. (2011). Does discovery-based instruction enhance learning? Journal of Educational Psychology, 103(1) 1-18.

Clements, D. H., & Sarama, J. (2021). *Learning and teaching early math: The learning trajectories approach* (3rd ed.). Routledge.

Confrey, J. & Lachance, A. (2000). Transformative teaching experiments through conjecture-driven research design. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 231-265). Lawrence Erlbaum Associates.

Confrey, J., Maloney, A. P., Nguyen, K. H., & Rupp, A. A. (2014). Equipartitioning, a foundation for rational number reasoning: Elucidation of a learning trajectory. In A. P. Maloney, J. Confrey, & K. H. Nguyen (Eds.), *Learning over time: learning trajectories in mathematics education* (pp. 61–96). Information Age Publishing.

DeCaro, M. S., & Rittle-Johnson, B. (2012). Exploring mathematics problems prepares children to learn from instruction. *Journal of Experimental Child Psychology*, *113*(4), 552-568.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Information Age Publishing.

Hunt, J., & Tzur, R. (2017). Where is difference? Processes of mathematical remediation through a constructivist lens. *Journal of Mathematical Behavior, 48*, 62-76. doi: 10.1016/j.jmathb.2017.06.007

National Mathematics Advisory Panel (NMAP). (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.

Simon, M. A. (2019). Analyzing qualitative data in mathematics education. In K. R. Leatham, (Ed.), *Designing, conducting, and publishing quality research in mathematics education* (pp. 111-122). Springer.

Simon, M. A., Placa, N., Avitzur, A. & Kara, M. (2018). Promoting a concept of fraction-as-measure: A study of the Learning Through Activity research program. J*ournal of Mathematical Behavior*, *52*, 122-133. https://doi.org/10.1016/j.jmathb.2018.03.004

Steffe, L. P., Thompson, P. W., & von Glaserfeld, E. (2000). Teaching experiment methodology: Underlying principles and essential elements. In A. E. Kelly, & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 267-306). Lawrence Erlbaum Associates.

Stop Ableism. (2021). http://www.stopableism.org/p/what-is-ableism.html

Woodward, J. (2004). Mathematics education in the United States: Past to present. *Journal of learning disabilities*, *37*(1), 16-31.

Xin, Y. P., & Tzur, R. (2016). Cross-disciplinary thematic special series: Special education and mathematics education. *Learning Disability Quarterly*, *39*(4), 196-198. https://doi.org/10.1177%2F0731948716669816