Impact of Uncertainty from Renewables on Dynamic State Estimation of Power Networks

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***Abstract*—Due to the widespread installation of stochastic and inertia-less renewable generation the operating point of the power grid changes more rapidly and thus tracking system state variables is becoming more crucial. In this regard, dynamic state estimation (DSE) can play a key role in future power systems since it can provide an accurate estimate of state variables in realtime. However, the consideration of an appropriate power system model is critical to effectively capture renewable uncertainty in the DSE of power systems. In the current literature of power systems DSE usually a linearized or nonlinear ordinary differential equation model of power systems is used which cannot capture the uncertainties associated with renewables. These uncertainties can only be taken into account via the nonlinear differential-algebraic (NL-DAE) model of power systems. In this paper, we present an estimator for the complete NL-DAE representation of the power system which can provide robust state estimation in the presence of uncertainties from renewables, and investigate the impact of renewables on state estimation performance.**

***Keywords*—H_∞ stability, state estimation, power system nonlinear DAE, phasor measurement units**

I. Introduction

CURRENT electrical power systems are rapidly transitioning from fossil fuel power generation (gas and coal-based power plants) to solar and wind-based renewable energy resources (RERs) mainly because of the global decarbonization policies, decrease in the cost of RERs technologies and rapidly increasing energy demand. As the share of these converter-based technologies increases in the power grid, it creates three main challenges, flexibility, adequacy and stability. The solution to these challenges can be found in the applications of dynamic state estimation (DSE) [1]. In particular, DSE can provide estimates of all the states of power system using just a few measurements from phasor measurement units (PMUs). And since PMUs can provide synchronized measurements at a very high sampling rate (60 samples/s), DSE can be performed in realtime and in a synchronized fashion [2].

In the past two decades extensive research has been carried out to effectually capture the dynamic states (i.e., generator rotor angle, frequency, transient voltages) of the conventional, synchronous generator-dominated power system using PMU measurements. In literature, DSE is mainly performed using two methodologies 1) stochastic estimators 2) deterministic observers. Stochastic estimators (e.g., Kalman filters and their derivatives) are recursive and they mainly exploit the statistical properties of the disturbances to minimize their impact on the state estimation. Inherently, Kalman filters are not able to handle nonlinearities and power system is a highly nonlinear system, hence extended Kalman filter (EKF) has been proposed in [3]. EKF linearizes the nonlinearities in the system around a certain equilibrium point and then treats the system as a linear system. To avoid linearization, unscented Kalman filter (UKF) has been proposed in [4]. In [5] researchers have proposed extended particle filter (EPF) for DSE. The EPF is implemented on a fourth-order nonlinear synchronous machine model with non-Gaussian noises and has shown enhanced performance in estimating rotor angles and frequencies as compared to UKF and EKF. A comprehensive study to analyze the different pros and cons of various stochastic estimators is also presented in [6], [7].

On the other hand, deterministic observers are designed using Luenberger criteria [8]. For example, in [9] researchers have proposed an observer for a simplified single machine nonlinear model of a power system. In [10] a multiplier-based deterministic observer design has been proposed for a multi-machine power system. A thorough summary of all the techniques used to perform DSE in power systems can be seen in the following recent survey paper [11].

Despite all these advancements, the uncertainties from RERs have not been taken into account in power systems DSE. The is because in most of the DSE literature, a simplified ordinary differential equation (ODE) model of a power system is used, mainly because of the complexity of performing DSE for the complete NL-DAE representation of power system. And since in the ODE model the dynamics associated with load and renewables are eliminated, hence their effect on DSE cannot be studied [12]. Moreover, currently most of the DSE literature is based on stochastic estimators and as discussed

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previously these estimators require statistical properties (such as Gaussian distribution) of the disturbances to work properly. And since the disturbances associated with renewables are random and do not follow any probabilistic distribution, hence these estimators cannot be applied to handle uncertainties from RERs.

To that end, in this paper we investigate the impact of renewables uncertainty on DSE for NL-DAE model of power systems through an observer that can provide accurate state estimation for renewables-heavy power systems.

The remainder of the paper is structured as follows. Section II describes the NL-DAE model of power systems. Section III presents the theory of the proposed observer design. Simulation studies are presented in Section IV and conclusion is presented in Section V

II. Nonlinear Multi-machine DAE model of Power Systems

We consider a graphical representation of a power system having a total of $\mathcal{N} = \mathcal{G} \cup \mathcal{L} \cup \mathcal{R}$ buses with $\mathcal{G}$ denoting generator buses $\mathcal{L}$ denoting load buses and $\mathcal{R}$ representing buses connected with renewables. The total number of transmission lines/edges are represented using $\mathcal{E}$.

We are representing synchronous generator with a standard two-axis fourth order model having total of four states given as [13]: Rotor angle $\boldsymbol{\delta} = \{\delta_i\}_{i\in\mathcal{G}}$, rotor speed $\boldsymbol{\omega} = \{\omega_i\}_{i\in\mathcal{G}}$, transient voltages along q-axis $\boldsymbol{E}'_q = \{E'_{qi}\}_{i\in\mathcal{G}}$ and transient voltages along d-axis $\boldsymbol{E}'_d = \{E'_{di}\}_{i\in\mathcal{G}}$. All these dynamic states of the generator can be lumped into one vector $\boldsymbol{x}_d = \left[\boldsymbol{\delta}^\top \; \boldsymbol{\omega}^\top \; \boldsymbol{E}'^\top_q \; \boldsymbol{E}'^\top_d\right]^\top \in \mathbb{R}^{n_d}$. Field voltage and mechanical torque are considered as inputs to the synchronous generator and is represented by vector $\boldsymbol{u} = \left[\boldsymbol{T}_M^\top \quad \boldsymbol{E}_{fd}^\top\right]^\top \in \mathbb{R}^{n_u}$.

To capture the topological effects of power system the relationship of the states of the generator with the rest of the power network need to be considered in the model. To that end, the equations for the power generated by the synchronous generator and the models describing power flow/balance equations are considered as algebraic constraints in the model. Hence the algebraic states of the power system are considered as: Total active and reactive power supplied by the generator $\boldsymbol{P}_G = \{P_{Gi}\}_{i\in\mathcal{G}}$, $\boldsymbol{Q}_G = \{Q_{Gi}\}_{i\in\mathcal{G}}$, and buses terminal voltages $\boldsymbol{v} = \{v_i\}_{i\in\mathcal{N}}$ and angels $\boldsymbol{\theta} = \{\theta_i\}_{i\in\mathcal{N}}$. We can combine all these algebraic variables in one vector $\boldsymbol{x}_a = \left[\boldsymbol{P}_G^\top \; \boldsymbol{Q}_G^\top \; \boldsymbol{v}^\top \; \boldsymbol{\theta}^\top\right]^\top \in \mathbb{R}^{n_a}$. Moreover, let us lump the total active and reactive load $\boldsymbol{P}_L = \{P_{Li}\}_{i\in\mathcal{L}}$, $\boldsymbol{Q}_L = \{Q_{Li}\}_{i\in\mathcal{L}}$ and power supplied by renewables $\boldsymbol{P}_R = \{P_{Ri}\}_{i\in\mathcal{R}}$, $\boldsymbol{Q}_R = \{Q_{Ri}\}_{i\in\mathcal{R}}$ in a vector $\boldsymbol{q} = \left[\boldsymbol{P}_R^\top \; \boldsymbol{Q}_R^\top \; \boldsymbol{P}_L^\top \; \boldsymbol{Q}_L^\top\right]^\top \in \mathbb{R}^{n_q}$. Then the NL-DAE representation of power system can be written as follows [13]:

$$\dot{\boldsymbol{x}}_d = \boldsymbol{A}_d\boldsymbol{x}_d + \boldsymbol{F}_d\boldsymbol{f}_d\left(\boldsymbol{x}_d, \boldsymbol{x}_a\right) + \boldsymbol{B}_d\boldsymbol{u} \tag{1a}$$
$$0 = \boldsymbol{A}_a\boldsymbol{x}_a + \boldsymbol{F}_a\boldsymbol{f}_a\left(\boldsymbol{x}_d, \boldsymbol{x}_a\right) + \boldsymbol{B}_a\boldsymbol{q} \tag{1b}$$

where $\boldsymbol{A}_a \in \mathbb{R}^{n_a\times n_a}$, $\boldsymbol{B}_d \in \mathbb{R}^{n_u\times n_d}$, $\boldsymbol{B}_a \in \mathbb{R}^{n_q\times n_a}$, $\boldsymbol{A}_d \in \mathbb{R}^{n_d\times n_d}$, $\boldsymbol{F}_d \in \mathbb{R}^{n_{fd}\times n_d}$ and $\boldsymbol{F}_a \in \mathbb{R}^{n_{fa}\times n_a}$ are all constant matrices and there overall structure is omitted for brevity. The vector valued functions $\boldsymbol{f}_d : \mathbb{R}^{n_d} \times \mathbb{R}^{n_a} \to \mathbb{R}^{n_{fd}}$ and $\boldsymbol{f}_a : \mathbb{R}^{n_d} \times \mathbb{R}^{n_a} \to \mathbb{R}^{n_{fa}}$ group together all the nonlinearities associated with the NL-DAE model of a power system. Notice that in this work RERs are considered as a negative load and they are injecting power into the electrical network.

As we are performing DSE using PMU measurements, so let us define $\boldsymbol{y} \in \mathbb{R}^p$ as the measurement received from PMUs, $\boldsymbol{x} = \left[\boldsymbol{x}_d^\top \; \boldsymbol{x}_a^\top\right]^\top \in \mathbb{R}^n$ as the overall state vector then from Eq. (1) the NL-DAE model of power systems with PMUs measurements can be written as follows:

$$\boldsymbol{E}\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{F}\boldsymbol{f}\left(\boldsymbol{x}\right) + \boldsymbol{B}_u\boldsymbol{u} + \boldsymbol{B}_q\boldsymbol{q} \tag{2a}$$
$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \tag{2b}$$

where matrix $\boldsymbol{E}$ encodes the algebraic equations with rows of zeros and matrix $\boldsymbol{C}$ maps system states $\boldsymbol{x}$ to what typically PMUs measure (i.e., voltage and current phasors). The overall structure of $\boldsymbol{C}$ can be obtained from [14].

III. Estimator Design

Herein, we showcase a simple estimator design for (2) under uncertainty. First, we focus on modeling renewables (and uncertainty associated with them) as well as bounding the nonlinear transients in the system dynamics.

A. Modeling Renewables and Bounding Nonlinearities

In Eq. (2) all the nonlinearities associated with NL-DAE model of a power system are lumped in $\boldsymbol{f}(.)$. To capture this nonlinear effect of a power system in a better way in the DSE we are assuming that $\boldsymbol{f}(.)$ is Lipschitz bounded. This assumption is realistic and holds in the case of electrical power systems as there are indeed upper and lower bounds on all the states of the power system [9].

To calculate the appropriate Lipschitz bound for $\boldsymbol{f}(.)$ with respect to its variables $\boldsymbol{x}$, we use the method presented in [15]. This method can be summarized as follows: Initially, appropriate bounds for all the states in $\boldsymbol{x}_d$ and $\boldsymbol{x}_a$ are determined (notice that these bounds on the states can be determined through operator knowledge of the grid, for example the upper and lower bounds of voltages can be chosen as $\pm 5\%$ of the steady-state values vice versa). Then to approximate the Lipschitz matrix from the defined bounds, Halton sequence is used to produce evenly distributed points inside the bounds and finally, the maximum value for the Jacobian norm of $\boldsymbol{f}(.)$ is determined to estimate the Lipschitz matrix for $\boldsymbol{f}(.)$.

As for uncertainty from renewables and loads, notice that the vector $\boldsymbol{q}$ in Eq. (2) encapsulate $\boldsymbol{P}_R, \boldsymbol{Q}_R, \boldsymbol{P}_L$ and $\boldsymbol{Q}_L$ and all these quantities are varying with respect to time and fluctuating. Albeit load demands and power generated by renewables follow certain patterns (e.g., overall load demand of residential house holds, seasonal patterns) and the grid operators record and publish hour and minute head prediction for both these quantities (see California independent system operator (CAISO) [16] daily minute and hour-ahead predictions for load demand and renewable productions) the prediction may not be accurate, specifically high fidelity estimates for the renewables are difficult to obtain. Accordingly one can write $\boldsymbol{q} = \tilde{\boldsymbol{q}} + \boldsymbol{q}_w$, where $\tilde{\boldsymbol{q}}$ is the known or the predicted values of loads and renewables and $\boldsymbol{q}_w$ lumps all the fluctuations/disturbances. The objective of the proposed observer is to provide accurate estimates of all the states of the power system in the presence of unknown disturbance vector $\boldsymbol{q}_w$. To that end we can rewrite the NL-DAE model (2) as follows:

$$\boldsymbol{E}\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{F}\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{B}_u\boldsymbol{u} + \boldsymbol{B}_q\tilde{\boldsymbol{q}} + \boldsymbol{B}_q\boldsymbol{q}_w \tag{3a}$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}. \tag{3b}$$

The overall power system model (3) is governed by two sets of equations, differential equations (1a) and algebraic equations (1b), the total states that we want to estimate are $\{\boldsymbol{\delta}\ \boldsymbol{\omega}\ \boldsymbol{E}'_q\ \boldsymbol{E}'_d\ \boldsymbol{P}_G\ \boldsymbol{Q}_G\ \boldsymbol{v}\ \boldsymbol{\theta}\}$, none of the states are assumed to be available locally (unless PMU is connected on that bus for the overall observability requirement of the system) all need to be estimated using PMUs measurements. PMUs need to be deployed in a way such that the whole system is observable.

B. Estimator Design for NL-DAE Power System

In this work we are designing Luenberger type observer for NL-DAE model depicted in (3). The overall observer design is mainly based on Lyapunov stability criteria and we use H_∞ notion to achieve a robust performance of the observer under unknown renewable disturbances. In state estimation theory H_∞ notion was first proposed in [17] to devise a robust state observer for a linear system with unknown random disturbances. The basic concept in H_∞ based observer design is that the disturbances are considered as random unknown quantities and then in the observer design a particular H_∞ performance is achieved for the error dynamics. In H_∞ based observer design the observer always make sure that the norm of the error dynamics remains less than a constant time the norm of the disturbances; such that $\|\boldsymbol{e}\|_{L_2}^2 < \gamma\|\boldsymbol{w}\|_{L_2}^2$, where $\boldsymbol{e}$ is the error between estimated and original states, γ denotes the performance level and $\boldsymbol{w}$ lumps all the disturbances. While performing state estimation we try to minimize γ so that robust performance from the observer can be ensured.

With that in mind the observer dynamics for the power system model presented in (3) can be expressed as follows:

$$\boldsymbol{E}\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{F}\boldsymbol{f}(\hat{\boldsymbol{x}}) + \boldsymbol{L}(\boldsymbol{y} - \hat{\boldsymbol{y}}) + \boldsymbol{B}_u\boldsymbol{u} + \boldsymbol{B}_q\tilde{\boldsymbol{q}} \tag{4a}$$

$$\hat{\boldsymbol{y}} = \boldsymbol{C}\hat{\boldsymbol{x}} \tag{4b}$$

where $\hat{\boldsymbol{x}}$ are the estimated states, $\hat{\boldsymbol{y}}$ are the estimated outputs and $\boldsymbol{L}$ is the Luenberger gain matrix. Even though if observer starts from different initial conditions, with the help of provided PMUs measurements $\boldsymbol{y}$, the matrix $\boldsymbol{L}$ guarantees the convergence of estimated states $\hat{\boldsymbol{x}}$ to the original states $\boldsymbol{x}$ as $t \longrightarrow \infty$. Notice that from Eq. (4) we can see that the disturbances from renewables $\boldsymbol{q}_w$ are unknown to the observer. The observer is only aware of the steady state or predicted values of loads and renewables denoted by $\tilde{\boldsymbol{q}}$.

Designing appropriate gain matrix $\boldsymbol{L}$ in the observer dynamics (4) is the primary goal and it should satisfy the following three main objectives:(1) The observer gain $\boldsymbol{L}$ needs to be designed in a way such that robust performance from the observer under unknown disturbances can be achieved, (2) the gain matrix $\boldsymbol{L}$ should ensure quick convergence of $\boldsymbol{x}$ to $\hat{\boldsymbol{x}}$ and, (3) the size of the gain matrix $\boldsymbol{L}$ should be of reasonable magnitude because high gain observers are undesirable as they increase the sensitivity of the system to the disturbances/noise. To that end, in this work we propose a systematic way by posing the calculation of observer gain $\boldsymbol{L}$ as convex linear matrix inequality (LMI) based optimization problem given as follows:

$$\begin{aligned}\mathbf{P_1}\quad &\underset{\epsilon,\kappa,\gamma,\boldsymbol{X},\boldsymbol{R},\boldsymbol{Y}}{\text{minimize}} && c_1\kappa + c_2\gamma + c_3\|\boldsymbol{R}\|_2\\ &\text{subject to} && \text{LMI (5)},\ \boldsymbol{X}\succ 0,\ \epsilon>0, \gamma>0, \kappa>0,\\ &&& \kappa\boldsymbol{I} - \boldsymbol{E}^\top\boldsymbol{X}\boldsymbol{E}\succ 0\end{aligned}$$

where c_1, c_2 and c_3 are weighting constants and LMI (5) is as follows:

$$\boldsymbol{\Upsilon} = \begin{bmatrix}\boldsymbol{\Omega} & * & *\\ (\boldsymbol{X}\boldsymbol{E} + \boldsymbol{E}^{\perp\top}\boldsymbol{Y})^\top\boldsymbol{F} & -\epsilon\boldsymbol{I} & *\\ \boldsymbol{B}_{\boldsymbol{w}}^\top(\boldsymbol{X}\boldsymbol{E} + \boldsymbol{E}^{\perp\top}\boldsymbol{Y}) - \boldsymbol{D}_{\boldsymbol{w}}^\top\boldsymbol{R} & \boldsymbol{O} & -\gamma\boldsymbol{I}\end{bmatrix} \prec 0 \tag{5}$$

$\boldsymbol{\Omega}$ is given as:

$$\boldsymbol{\Omega} = \boldsymbol{A}^\top(\boldsymbol{X}\boldsymbol{E} + \boldsymbol{E}^{\perp\top}\boldsymbol{Y}) + (\boldsymbol{X}\boldsymbol{E} + \boldsymbol{E}^{\perp\top}\boldsymbol{Y})^\top\boldsymbol{A} - \boldsymbol{C}^\top\boldsymbol{R} - \boldsymbol{R}^\top\boldsymbol{C} + \epsilon\boldsymbol{G}^\top\boldsymbol{G} + \boldsymbol{\Gamma}^\top\boldsymbol{\Gamma}.$$

After solving $\mathbf{P_1}$ the observer gain matrix can be retrieved as $\boldsymbol{L} = \boldsymbol{P}^{-\top}\boldsymbol{R}^\top$. Notice that the $*$ in the LMI (5) indicate that value at $\boldsymbol{\Upsilon}_{12} = \boldsymbol{\Upsilon}_{21}^\top$ and vice versa.

For brevity the proof of $\mathbf{P_1}$ is omitted. After solving $\mathbf{P_1}$ the computed observer gain matrix $\boldsymbol{L}$ ensures that the error between original states $\boldsymbol{x}$ and estimated states $\tilde{\boldsymbol{x}}$ (e.g., $\boldsymbol{e} = \boldsymbol{x} - \hat{\boldsymbol{x}}$) converges asymptotically to zero. In $\mathbf{P_1}$ minimizing γ make sure that the impact of the disturbance $\boldsymbol{q}_w$ can be minimized on the performance of the state estimation. Minimizing κ guarantees quick convergence of estimated states $\hat{\boldsymbol{x}}$ to true states $\boldsymbol{x}$. Also, as $\boldsymbol{L} = \boldsymbol{P}^{-\top}\boldsymbol{R}^\top$ thus minimizing $\|\boldsymbol{R}\|_2$ in $\mathbf{P_1}$ provides observer gain matrix $\boldsymbol{L}$ of reasonable magnitude.

To that end in the following section we present simulation studies to showcase the performance of the proposed observer under various severity of unknown renewable disturbances.

IV. Case Studies

The presented observer is tested on IEEE-14 bus system. All the simulations are carried out on MATLAB 2021a running on windows 10 64bit with 64Gb RAM and 11 Gen Intel core-i9-11980HK processor. Both the NL-DAE and observer models are simulated using MATLAB DAEs solver `ode15i`. To ensure full observability of IEEE-14 bus system four PMUs are placed at Buses $2, 6, 7$ and 9. The power base is selected as 100 MVA while all the synchronous machine parameters are extracted from power system toolbox [13]. All the states of the observer are initialized randomly having 7% maximum deviation from steady state values except for the generator frequency whose value is kept the same as synchronous frequency ($2\pi 60$ rad/s). To calculate the observer gain matrix $\boldsymbol{L}$ optimization problem $\mathbf{P_1}$ is solved in YALMIP [18] interfaced with MOSEK [19] as a solver. The calculated observer gain matrix is then fed to observer dynamics (4) to perform state estimation. The steady-state values and the initial conditions of the power system are calculated using power flow solution, which is obtained using function `runpf` in MATPOWER [20].

The simulation studies are carried out as follows: At first the power system operates with total active and reactive load of $P_{L_0} = 2.49$ pu and $Q_{L_0} = 0.725$ pu respectively and with renewable power generation of $P_{R0} = -0.6113$ pu. Then immediately after $t > 0$ the power generated by RERs experiences a step disturbance.

Their new values can be written as: $P_{R_D} = P_{R_0} + \Delta P_{R_0}$. In this work different severity of step disturbances for renewables have been simulated ranging from 3% to 35% such that $\Delta P_{R_0} \in \{0.03P_{R_0}, 0.15P_{R_0}, 0.35P_{R_0}\}$. Moreover to mimic realistic uncertainties from renewables we also assume that there exists noise, such that $P_{R_D} = P_{R_0} + \Delta P_{R_0} + g_R(t)$, where $g_R(t)$ is Gaussian noise with zero mean. Similar to the step disturbance random amount of Gaussian noise has been added and variance of the Gaussian noise has been changed, such that; variance of $g_R(t) \in \{0.01\Delta P_{R_0}, 0.05\Delta P_{R_0}, 0.08\Delta P_{R_0}\}$. Notice that in all the case studies the observer is not aware of the disturbances from renewables, this can be validated by looking at the structure of the proposed observer presented in (4), we can clearly see that the observer has only access to the $\tilde{\boldsymbol{q}}$ which contain the steady-state or predicted values of renewables.

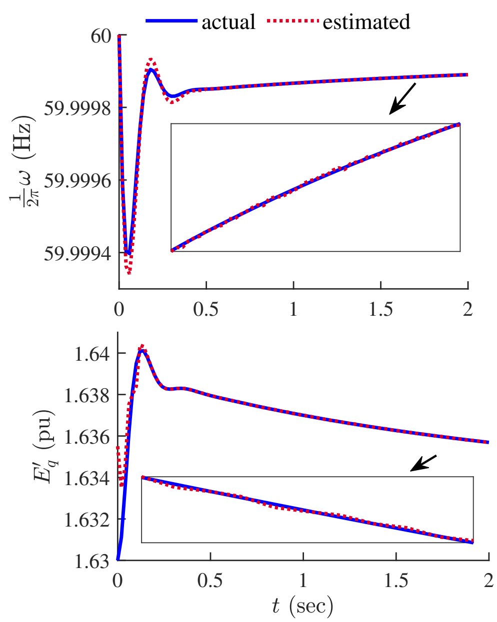

Figure 1. Estimation results for Generator-2 frequency and transient voltage with 3% disturbance in power supplied by renewables.

The estimation results are presented in Figs. 1, 2 and 3. For 3% disturbance in renewable power generation, we can see from Fig. 1 that although the observer started from different initial conditions and is completely unaware of the disturbances it provides accurate state estimation. This can also be validated from the estimation error norm depicted in Fig. 4. We can see that observer is driving the error between

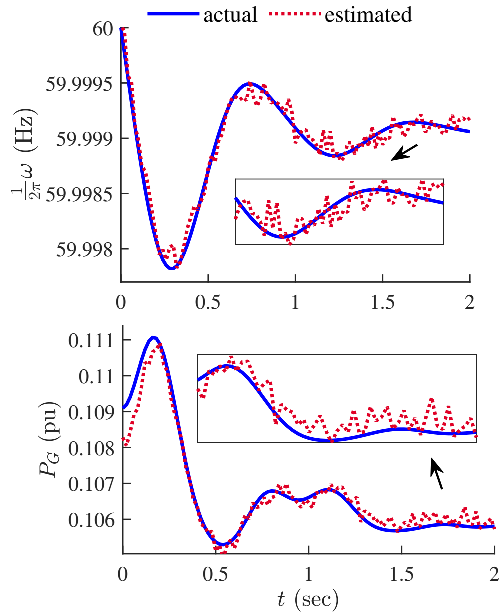

Figure 2. Estimation results for Generator-2 real power generation and frequency with 15% disturbance in power supplied by renewables.

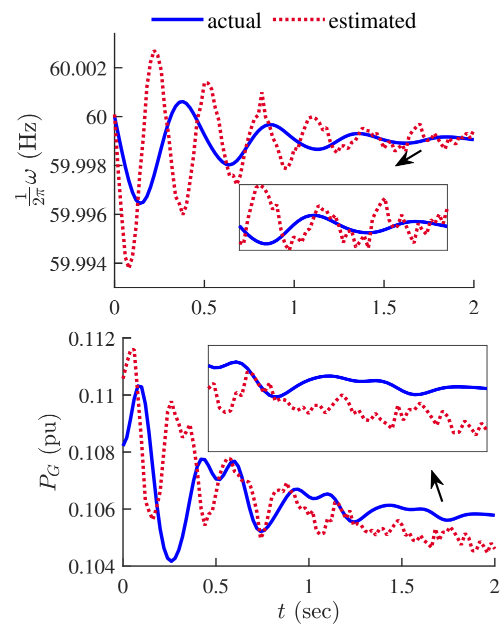

Figure 3. Estimation results for Generator-2 real power generation and frequency with 35% disturbance in power supplied by renewables.

true and estimated values of the states variables asymptotically near zero. Notice that this is mainly because of the robust H_∞ stability notion which is utilized in our observer design. As discussed in Section II in H_∞ based design the observer try to keep the norm of the error $\boldsymbol{e}$ less then a constant time the norm of the disturbance and and by looking at the error norm given in Fig. 4 we can advocate that this criterion is indeed satisfied. For 3% renewable disturbances we get $\|\boldsymbol{q}_w\|_{L_2} = 0.1013$ while after solving $\mathbf{P_1}$ we obtained $\gamma = 0.98$ hence $\gamma\|\boldsymbol{q}_w\|_{L_2} = 0.0993$. And we can clearly see from Fig. 4 that after $1.5s$ value of the $\|\boldsymbol{e}\|_{L_2}$ is less then 0.05

thus the observer is providing accurate estimation results.

However, we can see from Figs. 2 and 3 as we increase the amount of disturbances from renewables the estimation results are getting poorer. This is because for 15% and 35% disturbances we obtained value for the performance level $\gamma = 0.792$ and 0.7013 and for $\|\boldsymbol{q}_w\|_{L_2} = 0.6025$ and 1.381 we obtain the value for $\gamma\|\boldsymbol{q}_w\|_{L_2}$ for both case studies as: 0.4773 and 0.9685. From Fig. 4 we can see that after $1s$ although the value of $\|\boldsymbol{e}\|_2$ is less than 0.4773, 0.9685 and thus satisfying the H_∞ stability definition, there exist too much wiggle room for the observer to provide poor estimates. Because here the H_∞ criteria is not strict enough and the observer only has to keep the value of $\|\boldsymbol{e}\|_2$ less than 0.4773 and 0.9685 respectively and thus can provide poor estimates. This is indeed a drawback of using H_∞ criteria and H_∞ stability-based observers can only provide good estimation results when the magnitude of unknown disturbances is small.

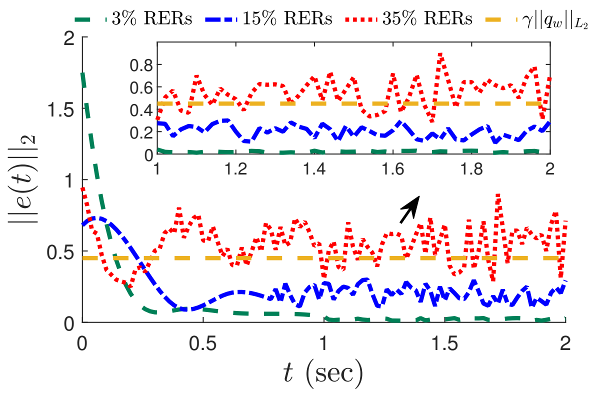

Figure 4. Estimation error norm for all the scenarios.

V. Conclusion and Future research work

This technical note presents a novel observer which can provide robust state estimation for the NL-DAE representation of power systems subject to unknown renewable uncertainties. The overall concept of the observer design is presented as a simple convex linear optimization problem and thus can be easily solved using many commercial optimization solvers. The proposed observer does not require any statistical properties of the disturbances and it considers the uncertainties associated with renewables as unknown bounded signals. The performance of the observer has been tested under various severity of load and renewable disturbances. Simulation studies show that even though only steady-state or predicted values of renewables are available to the observer it can still track the original states and drive the estimation error norm asymptotically to zero. Future research work will focus on the inclusion of the actual converter-based model of renewable energy resources in NL-DAE representation of power systems.

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