Embedding properties of network realizations of dissipative reduced order models

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Abstract

Realizations of reduced order models of passive SISO or MIMO LTI problems can be transformed to tridiagonal and block-tridiagonal forms, respectively, via different modifications of the Lanczos algorithm. Generally, such realizations can be interpreted as ladder resistor-capacitor-inductor (RCL) networks. They gave rise to network syntheses in the first half of the 20th century that was at the base of modern electronics design and consecutively to MOR that tremendously impacted many areas of engineering (electrical, mechanical, aerospace, etc.) by enabling efficient compression of the underlining dynamical systems. In his seminal 1950s works Krein realized that in addition to their compressing properties, network realizations can be used to embed the data back into the state space of the underlying continuum problems.

In more recent works of the authors Krein's ideas gave rise to so-called finite-difference Gaussian quadrature rules (FDGQR), allowing to approximately map the ROM state-space representation to its full order continuum counterpart on a judicially chosen grid. Thus, the state variables can be accessed directly from the transfer function without solving the full problem and even explicit knowledge of the PDE coefficients in the interior, i.e., the FDGQR directly "learns" the problem from its transfer function. This embedding property found applications in PDE solvers, inverse problems and unsupervised machine learning.

Here we show a generalization of this approach to dissipative PDE problems, e.g., electromagnetic and acoustic wave propagation in lossy dispersive media. Potential applications include solution of inverse scattering problems in dispersive media, such as seismic exploration, radars and sonars.

To fix the idea, we consider a passive irreducible SISO ROM

$$f_n(s)=\sum_{j=1}^{n}\frac{y_i}{s+\sigma_j}, \tag{62}$$

assuming that all complex terms in (62) come in conjugate pairs.

We will seek ladder realization of (62) as

$$\begin{array}{llll} r_j u_j & + & v_j - v_{j-1} & = -s\hat{h}_j u_j, \\ u_{j+1}-u_j & + & \hat{r}_j v_j & = -s h_j v_j, \end{array} \tag{63}$$

for $j=0,\ldots,n$ with boundary conditions

$$u_{n+1}=0,\quad v_1=-1,$$

and $4n$ real parameters $h_i, \hat{h}_i, r_i$ and $\hat{r}_i$, $i=1,\ldots,n$, that can be considered, respectively, as the equivalent discrete inductances, capacitors and also primary and dual conductors. Alternatively, they can be viewed as respectively masses, spring stiffness, primary and dual dampers of a mechanical string. Reordering variables would bring (63) into tridiagonal form, so from the spectral measure given by (62) the coefficients of (63) can be obtained via a non-symmetric Lanczos algorithm written in J-symmetric form and $f_n(s)$ can be equivalently computed as

$$f_n(s)=u_1.$$

The cases considered in the original FDGQR correspond to either (i) real y, θ or (ii) real y and imaginary θ. Both cases are covered by the Stieltjes theorem, that yields in case (i) real positive h, $\hat{h}$ and trivial r, $\hat{r}$, and in case (ii) real positive h,r and trivial $\hat{h}$,$\hat{r}$. This result allowed us a simple interpretation of (62) as the staggered finite-difference approximation of the underlying PDE problem [2]. For PDEs in more than one variables (including topologically rich data-manifolds), a finite-difference interpretation is obtained via a MIMO extensions in block form, e.g., [4, 3].

The main difficulty of extending this approach to general passive problems is that the Stieltjes theory is no longer applicable. Moreover, the tridiagonal realization of a passive ROM transfer function (62) via the ladder network (63) cannot always be obtained in port-Hamiltonian form, i.e., the equivalent primary and dual conductors may change sign [1].

Embedding of the Stieltjes problems, e.g., the case (i) was done by mapping h and $\hat{h}$ into values of acoustic (or electromagnetic) impedance at grid cells, that required a special coordinate stretching (known as travel time coordinate transform) for continuous problems. Likewise, to circumvent possible non-positivity of conductors for the non-Stieltjes case, we introduce an additional complex s-dependent coordinate stretching, vanishing as $s \to \infty$ [1]. This stretching applied in the discrete setting induces a diagonal factorization, removes oscillating coefficients, and leads to an accurate embedding for moderate variations of the coefficients of the continuum problems, i.e., it maps discrete coefficients onto the values of their continuum counterparts.

Not only does this embedding yields an approximate linear algebraic algorithm for the solution of the inverse problems for dissipative PDEs, it also leads to new insight into the properties of their ROM realizations. We will also discuss another approach to embedding, based on Krein-Nudelman theory [5], that results in special data-driven adaptive grids.

References

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