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The London Millennium Footbridge Revisited: Emergent Instability Without Synchronization

By Igor Belykh, Mateusz Bocian, Alan Champneys, Kevin Daley, Russell Jeter, John H.G. Macdonald, and Allan McRobie

The pedestrian-induced instability of London's Millennium Bridge is widely held up as the canonical example of synchronization in complex networks [9]. The popular explanation maintained that once the number of pedestrians reached a certain threshold, the pedestrians could supposedly synchronize their footsteps with each other at the bridge's natural frequency. The result was the onset of dangerous sideways oscillations.

Multiple engineering analyses and publications have debated this original interpretation [4, 7, 8]. Nevertheless, the belief that a textbook example of coupled pedestrian synchronization caused the Millennium Bridge instability remains part of numerous presentations in print, film, and radio [6].

We propose an alternative theory by arguing that any synchronization in the timing of pedestrian footsteps is a consequence—not a cause—of the instability [2]; this result is consistent with observations on 30 bridges, including the Brooklyn Bridge and Golden Gate Bridge. We show that unsynchronized pedestrians produce negative damping — a positive feedback effect wherein energy is transferred from pedestrians who are trying not to fall due to perturbations that are caused by bridge motion. Prior studies found negative damping empirically from measurements on London's Millennium Bridge itself [5], though the researchers believed that synchronization caused this effect. In contrast, we show that negative damping is more fundamental and can occur without synchronization.

To quantify the effective total negative damping, we use a mathematical model that assumes that possible coordination between pedestrians transpires solely because of sensory stimuli from the moving bridge. We parsimoniously assume that walking is fundamentally a process in which the stance leg acts as a rigid strut, thus causing the body's center of mass to behave like an inverted pendulum in the frontal plane during each footstep. The step ends when the other leg meets the ground and—ignoring the brief double-stance phase that occurs in realistic gaits—the pedestrian switches to an inverted pendulum on that other leg, which prevents them from falling over. We consider a single lateral vibration mode for the bridge, which is forced by the motion of N pedestrians who walk perpendicularly to this vibration. We assume that the displacement of the lateral bridge mode $x(t)$ is governed by a second-order equation of motion:

$$M\ddot{x}+C\dot{x}+Kx=\sum_{i=1}^{N}\tilde{H}^{(i)}(x,y^{(i)}).$$

M, C, and K are respectively the mass, damping, and stiffness coefficients of the bridge mode, and $y^{(i)}(t)$ is the lateral displacement of the ith pedestrian's center of

See **Footbridge Revisited** *on page 2*

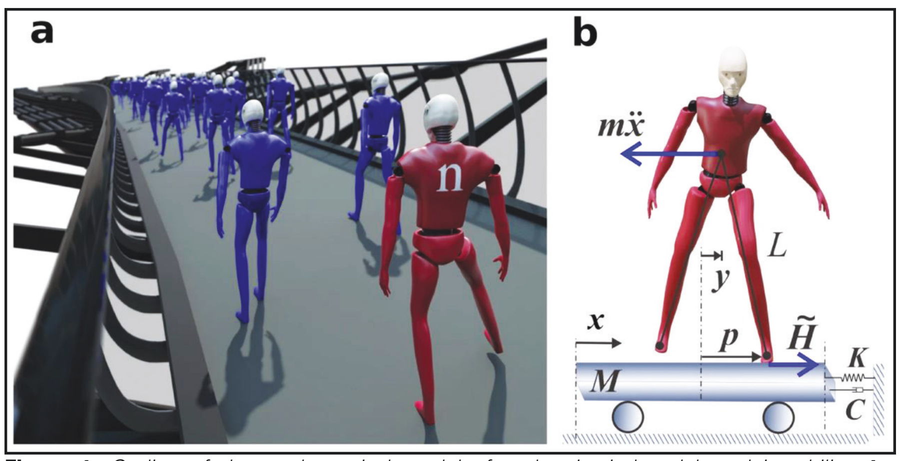

Figure 1. *Outline of the mathematical model of pedestrian-induced lateral instability.* **1a.** *Pedestrians are added sequentially at fixed time increments. The addition of the nth pedestrian $(n=N_{\text{crit}})$ causes the overall damping coefficient to become negative, meaning that the amplitude of motion increases rather than diminishes.* **1b.** *Inverted pendulum model of bridge mode and pedestrian lateral motion. Figure courtesy of [2].*

Biological Near-symmetries Explain the Similarity and Diversity of Life

By Matthew R. Francis

Despite the wide variety of shapes that organisms exhibit, their forms are not random. Instead, much of life possesses relatively clear structural patterns; humans and other mammals are bilaterally symmetric, sea stars have five-fold symmetry, and flowers display a wide variety of symmetrical patterns.

However, none of these biological symmetries are exact in a mathematical sense. The *near*-symmetries in living things are nevertheless real — to the extent that evolutionary, developmental, and environmental pressures must play a part. Yet when symmetries are not mathematically precise, mathematicians must find new ways to express them and determine their meanings.

Mathematical biologists Punit Gandhi (Virginia Commonwealth University), Veronica Ciocanel (Duke University), Adriana Dawes (Ohio State University), and Karl Niklas (Cornell University) teamed up to develop novel processes for identifying biological symmetries to glean a deeper understanding of their origins. They presented their research during a session that was moderated by Niklas at the 2022 American Association for the Advancement of Science (AAAS) Annual Meeting,[1] which took place virtually in February. "If [an organism] has bilateral symmetry but you try to flip its image, it will simply not align in the way that mathematics would quantify it," Ciocanel said. "We were inspired by the idea that biology will never have that kind of perfect symmetry."

To tackle this problem, the researchers examined multiple species of flowering plants, algae, and animals with embryos that are easy to study. They used algorithms that minimize human choice—since people miss subtle patterns and are prone to bias when identifying symmetries—to analyze both computer-generated images and real-life photos. To quantify near-symmetries in organisms, the team employed a measure called transformation information (TI). "One way to interpret TI is by how much information is lost when assuming that the object is exactly symmetric under the transformation," Gandhi said.

Although researchers have developed other methods for the quantification of inexact symmetries, these approaches generally require that one pick a particular type of transformation (e.g., rotation by a fixed angle) or select invariant points or axes before proceeding. By contrast, TI allows scientists to compare multiple potential symmetries and algorithmically determine invariants. "I think that TI is a really nice hands-off quantification of symmetries in an unbiased way," Dawes said. "It requires less time, there's less opportunity for bias (intentional or not) to be introduced into the measure, and it also gives you a way of then saying, 'How different is this flower from that flower?'"

In more general terms, TI helps extract symmetry information from biological systems where—for instance—a single plant might have numerous non-identical flowers, or several specimens from one species might exhibit variations in the closeness with which they hew to a particular symmetry. For these cases, TI can in principle identify the underlying symmetry separately from random variation. "Most daisies are going to have rotational symmetry," Dawes said. "You've got your round center and you've got your white petals around the outside. We identify a daisy by that rotational symmetry and the color properties" (see Figure 1).

While daisies may deviate from this Platonic ideal, Dawes argued that investigating representative specimens can directly inspire the identification of general symmetries. "If you want to know the common symmetry features among all daisies, it would make sense to do TI on a bunch of different samples and look at the common features in the TI function," she said.

See **Near-symmetries** *on page 4*

[1] https://aaas.confex.com/aaas/2022/meetingapp.cgi

Figure 1. *A daisy is a prime example of a flower with apparent rotational and reflection near-symmetries. The challenge is recognizing the symmetries that are real properties of the species and identifying their levels of robustness between plants. Public domain image.*

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Footbridge Revisited

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mass relative to the bridge (see Figure 1, on page 1). The forcing term $\tilde{H}^{(i)}$ is the lateral component of the ith pedestrian's foot force on the bridge. According to Newton's second law of motion, the lateral component of the center of mass for a pedestrian of mass m obeys the equation

$$m\ddot{y}^{(i)} + m\ddot{x} = -\tilde{H}^{(i)}(x, y^{(i)}),$$
$$i = 1, \ldots N.$$

$\tilde{H}^{(i)}$ is a piecewise smooth function with abrupt changes at foot transitions that are associated with the pedestrian's gait. We utilized three variants of $\tilde{H}^{(i)}$ that correspond to three different gait control strategies and yield three different models of pedestrian gait adaptation [2, 3, 8], one of which has a strong propensity for synchronization.

Using a multiple-scale asymptotic analysis, we derived a general expression that quantifies the average contribution to the bridge damping from the interaction force of a single pedestrian over one gait cycle. This quantity σ has three components: (i) A coefficient of lateral bridge velocity-dependent component of pedestrian foot force on the bridge (σ_1), which ignores gait timing adjustment; (ii) a coefficient of lateral bridge velocity-dependent component of force due to the adjustment of pedestrian lateral gait timing (σ_2); and (iii) a coefficient of lateral bridge velocity-dependent component of force due to adjustment of the forward gait (σ_3).

One should numerically calculate the expressions $\sigma_{1,2,3}$ as time averages (integrals) of partial derivatives of $\tilde{H}^{(i)}$, which are associated with the pedestrian's lateral and forward gaits. These expressions are evaluated individually for each pedestrian i and depend on the pedestrian's stride frequency ω_i, in addition to the bridge's vibration frequency Ω.

The terms σ_2 and σ_3 depend on the timing of pedestrian stepping behavior in response to bridge motion. Yet in all of our simulations, we found that σ_1 plays the most important role in triggering large-amplitude vibrations. This effect is counterintuitive because in the absence of phase synchrony between the bridge and pedestrian, one may imagine that the lateral foot force on the bridge will average to zero. However, this is not the case; Figure 2 provides an explanation.

We calculate the total effective damping coefficient c_T as

$$c_T = c_0 + N\bar{\sigma}(\bar{\omega}, \Omega) \equiv c_0 + \sum_{i=1}^{N}\left(\sigma_1^{(i)}(\omega_i, \Omega) + \sigma_2^{(i)}(\omega_i, \Omega) + \sigma_3^{(i)}(\omega_i, \Omega)\right),$$

where c_0 is the coefficient of the bridge's inherent damping and $\bar{\omega}$ represents the mean pedestrian stride frequency. We found that $\bar{\sigma}$ is negative over a large range of pedestrian and bridge frequency ratios. The overall modal damping c_T hence becomes negative when the number of pedestrians exceeds a critical value:

$$N = N_{\text{crit}} = -c_0 / \bar{\sigma}.$$

As a result, negative damping causes bridge vibrations to grow. In all of our simulations, the occurrence of negative damping and onset of bridge instability always precede the emergence of increased coherence among pedestrian footstep timings — even for the model that is highly prone to synchronization.

In a key scientific conclusion of our work, we argue that negative damping due to pedestrian attempts to maintain balance is likely the essential cause of most cases of lateral bridge instability. Indeed, our simulations revealed that increased coherence in the timing of pedestrian footsteps is part of a secondary nonlinear adjustment to the amplitude of vibration after initiation of the instability. This secondary effect typically produces saturation of the vibration amplitude, but it can further exacerbate instability in extreme cases.

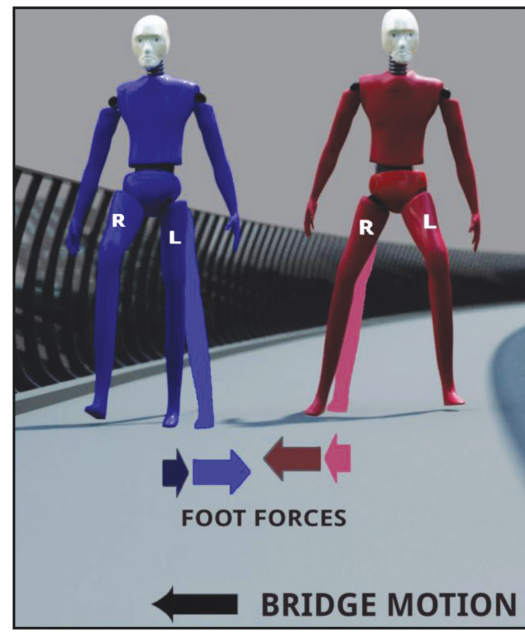

Figure 2. *Two identical pedestrians with equal and opposite gaits simultaneously place their stance feet on the bridge. If the bridge is still, the lateral force from each pedestrian is equal and opposite so that no net lateral force exists on the bridge. But if the bridge moves to the left, the blue figure's leg decreases its angle to the vertical within the frontal plane during the step, whereas the red figure's leg angle increases. The magnitude of the red figure's lateral foot force therefore increases during this bridge motion, whereas that of the blue figure decreases. On average, a change in resultant force thus occurs in the direction of the bridge's motion. This mechanism was first identified by Chris Barker [1]. Figure courtesy of [2].*

We achieved these findings via asymptotic analysis, which is applicable to a wide class of foot force models. Moreover, we conducted a comprehensive review of the literature on real bridges that have experienced large-amplitude lateral pedestrian-induced vibrations. Based on this review, it is clear that any direct evidence of synchronization is scant at best. In contrast, our theory is fully consistent with all known observations.

Our findings indicate that large amplitude oscillations can occur for a wide range of bridge frequencies. Therefore, bridge engineering techniques that try to avoid the problem by ensuring that bridge frequency is not close to typical pedestrian stride frequencies (often referred to as frequency tuning) are potentially dangerous.

For a wide range of systems in nature and society, our work argues more generally that this macro-scale instability (in this case, the bridge motion) may emerge from micro-scale behavior (in this case, of many individual pedestrians) without any obvious causal synchrony. It also points to other examples in economic cycles and the tuning of remarkably sensitive hearing organs in mammals and insects.

This article is based on [2] and Igor Belykh's minisymposium presentation at the 2021 SIAM Conference on Applications of Dynamical Systems,[1] which took place virtually last year. It is dedicated to the memory of John Macdonald, who passed away unexpectedly as the proofs were being prepared. He was an inspiration and will be sorely missed.

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[1] https://www.siam.org/conferences/cm/conference/ds21

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Igor Belykh is a professor of applied mathematics at Georgia State University. Mateusz Bocian represents the Bridge Engineering Group at Wrocław University of Science and Technology. Alan Champneys is a professor of applied nonlinear mathematics at the University of Bristol and a past chair of the SIAM Activity Group on Dynamical Systems. Kevin Daley is a Ph.D. student in applied mathematics and bioinformatics at Georgia State University. Russell Jeter is Director of Analytics at Motus Nova, LLC. John H.G. Macdonald was a professor of structural dynamics at the University of Bristol. Allan McRobie is a professor of structural engineering at the University of Cambridge.

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How Can I Help Ukraine? And Why Should Applied Mathematicians Care?

By Jeff Sachs

LETTER TO THE EDITOR

As a SIAM member with deep connections to Ukraine, I am heartbroken, horrified, and furious at the atrocities being committed there. News media have largely focused on the military conflict and its casualties and refugees, along with the geopolitics and economic consequences. But we should also remember the cost to the scientific and mathematical communities. The immediate consequences are clear and include casualties, refugees, and impairment of communication, nutrition, shelter, and medical infrastructure; the additional impact of delaying research—in fields like energy production, computer science, and medicine, for example—are also very real, even if difficult to quantify.

Hundreds of clinical trials for the development of novel drugs had been underway in Ukraine. These include more than 100 trials for therapies against cancer. Even if they do not become direct military casualties, these cancer patients no longer have the hope that those trials provided. The resulting suffering and mortality are multiplied by the impediment to development and approval of the therapies. Given the hundreds of thousands of people who suffer from the diseases for which trials are delayed, thousands of additional deaths—"excess mortality," in epidemiological terms—are likely.

Other direct impacts on our scientific and mathematical communities arise through the rift that conflict creates and the dissolution of collaborations. The Massachusetts Institute of Technology has announced that it is ending a decade-long collaboration with the Skolkovo Institute of Science and Technology (Skoltech), one of Russia's leading institutes. CERN (the European Council for Nuclear Research) has also substantially cut back its interaction with Russian organizations. The many examples of this type of disunion will isolate some of our dear and talented colleagues at all stages of their careers, regardless of their position on the conflict. It also clearly impacts both Ukrainians and Russians who study abroad — whether through loss of funding or access to research materials, the stress of concern for loved ones, or directly through the conflict's trauma.

It is not lost on any of us that our work relationships can humanize those on the other side of geopolitical boundaries. Perhaps the resulting empathy and communication can help prevent catastrophes like the one that is currently unfolding. To that end, it is probably best to avoid pre-judging colleagues based on their nationality or the language that they speak. A poignant reminder of this is an open letter[1] that clearly rejects the invasion, which was courageously signed by nearly 7,000 Russian scientists and mathematicians in spite of the potential resulting peril for themselves and their families.

For those who wish to contribute (monetarily or otherwise) to alleviate the suffering of those in the path of conflict, relevant information can be found in my associated LinkedIn article.[2] And in case anyone's heart needs softening on these matters, consider taking two minutes to listen to the Saturday Night Live performance by Ukrainian chorus "Dumka"[3] — the only exception to a comedic opening other than after September 11, 2001.

This letter represents the author's personal opinion, not that of his employer, SIAM, or its Board of Trustees.

Jeff Sachs is a distinguished scientist at Merck & Co., Inc. He has been a member of SIAM for over 35 years and enthusiastically applies ordinary and partial differential equations, statistics, machine learning, and signal processing to many areas of science and technology.

[1] https://www.science-diplomacy.eu/russian-scientists-publish-open-letter-condemning-the-war-with-ukraine

[2] https://www.linkedin.com/pulse/what-can-i-do-ukraine-why-should-anything-jeff-sachs

[3] https://www.tvinsider.com/1033961/saturday-night-live-ukrainian-chorus-dumka-cold-open

Preparing Virginia's Students for New Post-secondary Pathways in Data Science

By Tina Mazzacane, Deborah Crawford, Lisa Bussian, Aanand Vasudevan, and Padmanabhan Seshaiyer

Inspired by a groundbreaking 2018 study entitled *Catalyzing Change in High School Mathematics* from the National Council of Teachers of Mathematics [1], the Commonwealth of Virginia is considering a redesign of its *Mathematics Standards of Learning* (SOL) to promote deeper learning. A diverse group of mathematicians, mathematics educators from higher education, and K-12 leaders across Virginia convened to share ideas that inspired critical conversations about the way in which K-12 mathematics education prepares *all* students for an evolving postsecondary landscape.

From 2019 to 2021, business and industry stakeholders discussed possible ways to modernize mathematics education in Virginia public schools. Conversations addressed (i) *What* essential content should be included in the Mathematics SOL, (ii) *where* content across grade levels could better focus on college and career readiness, and (iii) *how* the introduction of additional course options for students might create distinct mathematics pathways. A common emerging thread was the need for mathematics content that focused on *data science* and would better equip students for postsecondary options. The proposed Data Science SOL—meant to support a locally designed, high-school-level data science course—were presented to the Virginia Board of Education in November 2021 and are currently under review. These prospective standards mark the start of a modernization of the Virginia Mathematics SOL, which are scheduled for revision in 2023.

While higher education institutions around the world have been steadily incorporating some version of data science into a wide range of undergraduate curricula (from business to healthcare and even the social sciences), K-12 education lags behind in terms of both pedagogical changes and overall implementation. In most of the U.S., K-12 mathematical standards have not evolved significantly in the last 100 years, and school systems continue to run their mathematics courses as disparate blocks. In contrast, countries that have shown more success in science, technology, engineering, and mathematics education tend to teach three continuous years of integrated math wherein concepts of algebra, geometry, probability, statistics, and data analysis are presented in unison. This strategy helps students develop deeper conceptual knowledge and make stronger connections with the material.

By virtue of design, data science combines elements of mathematics, statistics, and computer science to solve real-world problems (see Figure 1). Therefore, introducing students to data science at an early age allows them to incorporate inter- and intradisciplinary approaches in the problem-solving processes. A good data science curriculum should include foundational competencies like statistical and computational thinking, mathematics, communication skills, and ethics. While mathematics, statistics, and computer science tend to share certain similarities with data science—they all aim to extract knowledge from data, for example—significant differences exist in the modeling processes, size of the data, type of problem, practitioners' backgrounds, and language in question.

The Virginia Department of Education (VDOE) has taken a timely, proactive approach to address the need for data science programming. As part of the modernizing mathematics initiative during the 2023 revisions to Virginia's SOL, the VDOE will support school divisions in piloting a flagship data science course across the state. Participants will investigate the data science cycle via a project-based learning method that allows student choice in learning experiences (see Figure 2, on page 6). The proposed course focuses on five units: data and society, data and ethics, data and communication, data modeling, and data and computing. Students will utilize open-source technology tools to identify and explore problems that involve relational database concepts and data-intensive computing in order to ultimately find solutions. They will also engage in a data science problem-solving structure to interact with large data sets and formulate problems; collect, clean, and visualize data; create data-based models; and effectively communicate data-formulated solutions.

In addition, the course intends to build data literacy through "data talks" — low-threshold, high-ceiling activities that engage and motivate students in data-related topics like correlation versus causation, bias, and appropriate visualizations; a similar technique known as "number talks" builds computational fluency. Short, targeted lessons called "mini bytes" can either offer timely instruction to the whole class or serve as differentiation or scaffolding approaches for individuals on an as-needed basis. Community and business connections permit students to investigate local and real-world problems and establish mentored relationships with data science professionals.

Development of the proposed data science standards involved three phases of collaboration among a diverse group of K-12 educators, industry representatives, and higher education professionals in Virginia. The phase 1 team created the course standards based on the data science cycle (see Figure 2, on page 6). The phase 2 team then used feedback from the first phase as well as universal backwards design principles to identify ideas and concepts for the Data Science SOL, along with the resulting knowledge and skills. Finally, the phase 3 team generated draft unit guides, rubrics, and project templates that outlined the way in which students demonstrate evidence of learning; these materials include a self-assessment and a built-in feedback cycle from teachers, mentors, and peers.

CANVAS—Virginia's statewide learning management system—will deliver the course's professional learning resources during initial pilot training. A virtual version of the data science course with a dedicated virtual instructor will also be piloted

See **Pathways in Data Science** on page 6

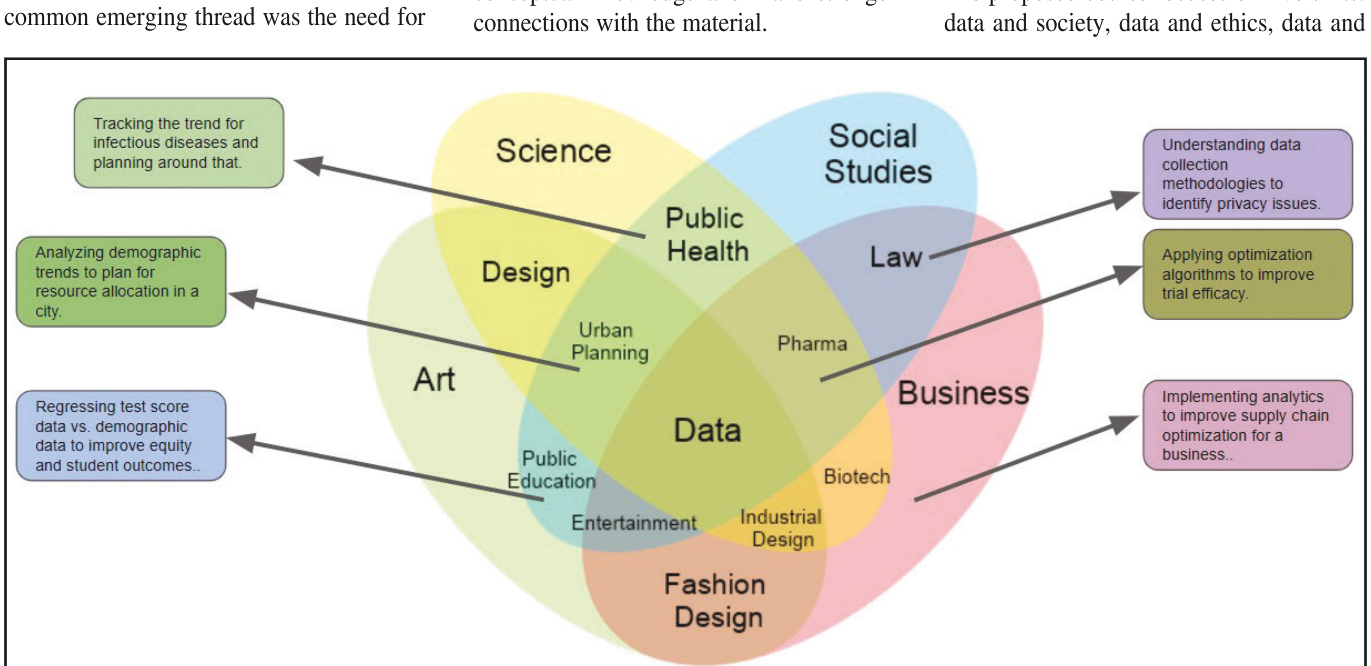

Figure 1. *The multifaceted role of data science. Figure courtesy of the Virginia Department of Education.*

Near-symmetries

Continued from page 1

Perfect and Imperfect Symmetries

G.V. Vstovsky first introduced TI in 1997 to apply basic concepts of information theory to broken symmetries in physics-based applications [4]. Approximate symmetries play important roles across scientific fields, from frustrated lattices (solids that do not form perfect crystal structures) to computer vision problems.

To quantify asymmetry, one must first define a function $\mu : D \rightarrow \mathbb{R}^+$ that maps points on the object D to positive real values. For a two-dimensional (2D) object,

$$\mathrm{TI} = \frac{1}{|\tilde{D}|}\int_{\tilde{D}} \mu(x) \ln\left[\frac{\mu(x)}{T_a \mu(x)}\right] dA.$$

T_a is a given symmetry transformation and $\tilde{D}$ is the region of overlap between the original domain D and transformed domain, while

$$|\tilde{D}| = \int_{\tilde{D}} dA.$$

If a symmetry transformation is mathematically exact, its TI is zero because the transformation will map the domain onto a perfect copy. For real biological systems, near-symmetries correspond to minimum (albeit typically nonzero) TI values. The group tested their technique on computer-generated patterns from a reaction-diffusion model that was developed by Alan Turing [3]. Such models are useful for this kind of test because they contain exact symmetries and do not require complicated pre-processing procedures; the center of the image is also the center of the pattern, and the algorithm does not need to detect backgrounds or edges.

The AAAS presenters then examined bay leaves and desmids (a type of green algae) for bilateral symmetry, along with several flowers for rotational symmetry (see Figure 2). These common organisms have basic symmetries that one can identify by sight; *Pachypodium* blossoms—which have a distinctive pinwheel curl to their petals that breaks reflection symmetry but exhibits an approximate five-fold rotational symmetry—are another example [2]. The TI algorithm identified the center of various flowers as well as appropriate near-symmetries.[2]

However, flat images of three-dimensional (3D) objects create their usual set of problems. "Projection issues are definitely something we worried about," Gandhi said. For instance, he and his collaborators compared two photos of the same flower that were taken from both a face-on and flattened view. "We could actually see differences in the symmetries just by the process of how we image it."

Dawes elaborated on this concept. "[This is] where a little bit of user discrimination comes into play," she said. "Karl [Niklas] and I went through pictures and decided which ones were appropriate for the analysis because of differences in viewpoints that weren't capturing the symmetry characteristics we were interested in." She pointed out that one could use TI in three dimensions in principle, though doing so raises its own set of challenges. "How do you do a meaningful comparison of TI with 3D data compared to 2D data?" Dawes asked. Of course, there is the question of how to obtain usable 3D images in the first place.

Endless Forms Most Wonderful

Because the group also examined development within single cells and embryos, Ciocanel's AAAS talk addressed the challenge of symmetry that breaks as cells divide and develop. "We were interested in taking a spherical cell embryo and understanding how the symmetry of that system would break through rules that are somewhat conserved across model organisms," she said.

Similar to how physicists look for broken symmetries to explain different particle properties, mathematical biologists can seek out meaningful underlying principles beneath near-symmetries. Such asymmetries might arise from external influences—like light, gravity, environmental chemistry, and so forth—or from developmental or genetic factors. "One of the things that we looked into with these models of cell polarity was symmetry breaking, driven by different initial conditions: different protein localizations around the boundary of the cell," Ciocanel said. "We found that the measure can capture those things really well but also shows robustness."

Dawes presented a TI-driven principal component analysis for *C. elegans*—a type of nematode worm—as it grows within an egg. "When you watch the worm, you can see that it's undergoing these tremendous changes in body shape," she said. "But it's all occurring within the constraint of the egg shell. What happens is that the TI really picks up on that ellipsoidal constraint from the egg shell."

These biological examples pertain to the question of *why* many organisms have near symmetry at all, as opposed to perfect forms or complete amorphousness (in fact, exact symmetry can trigger the "uncanny valley" response because we are psychologically primed to expect imperfections). Gandhi and his colleagues suspect that there are reasons for variations in basic symmetry; otherwise, these patterns would be undesirable from an evolutionary standpoint. The data back up this supposition. "There's evidence [of] some advantage to having slight asymmetries in terms of resilience," he said.

In the concluding paragraph of *On the Origin of Species*, Charles Darwin waxed eloquent about "endless forms most beautiful and most wonderful" and indicated that diversity of form evolved from simpler ancestors in the distant past [1]. Near-symmetries may help explain why—despite the huge diversity of life—organisms have symmetries in the first place.

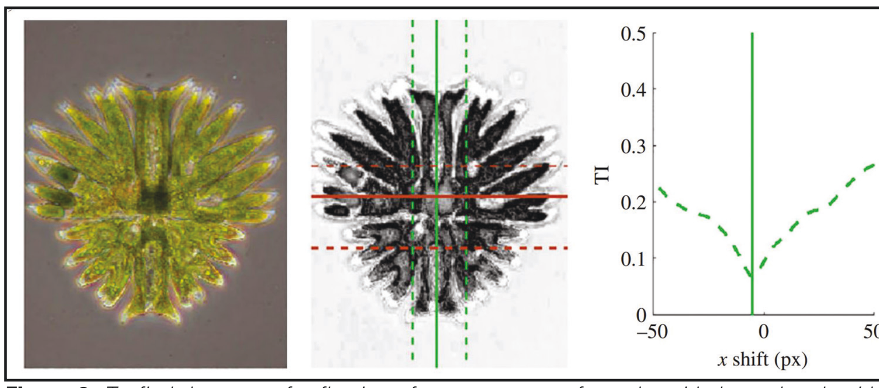

Figure 2. *To find the axes of reflection of near-symmetry for a desmid algae, the algorithm adjusts the position of the axes until the transformation information (TI) function reaches a minimum value. Figure adapted from [2].*

[2] A sample transformation information (TI) algorithm for MATLAB is available at https://github.com/PunitGandhi/TransInfo.

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Matthew R. Francis is a physicist, science writer, public speaker, educator, and frequent wearer of jaunty hats. His website is BowlerHatScience.org.

Looking Ahead to the 2022 SIAM Annual Meeting

By Edmond Chow and Sunčica Čanić

The 2022 SIAM Annual Meeting[1] (AN22) will take place in a hybrid format from July 11-15 in Pittsburgh, Pa. On a broad scale, SIAM's Annual Meeting aims to bring together researchers who are working in many different areas of industrial, applied, and computational mathematics. Yet the meeting has a focused aspect as well — the minisymposia that are organized in various application areas can make the Annual Meeting feel like a smaller version of your favorite SIAM conference.

Along with the dozens of minisymposia that participants put together at every Annual Meeting, AN22 will also feature tracks of minisymposia that are sponsored by the SIAM Activity Groups on Linear Algebra[2] and Mathematical Aspects of Materials Science.[3] Furthermore, AN22 will be held jointly with three other SIAM conferences: the SIAM Conference on Applied Mathematics Education,[4] the SIAM Conference on the Life Sciences,[5] and the SIAM Conference on Mathematics of Planet Earth.[6] Each of these meetings will present a joint invited lecture at AN22.

[1] https://www.siam.org/conferences/cm/conference/an22
[2] https://www.siam.org/membership/activity-groups/detail/linear-algebra
[3] https://www.siam.org/membership/activity-groups/detail/mathematical-aspects-of-materials-science
[4] https://www.siam.org/conferences/cm/conference/ed22
[5] https://www.siam.org/conferences/cm/conference/ls22
[6] https://www.siam.org/conferences/cm/conference/mpe22

Every year, SIAM bestows many awards and hosts several prize lectures during the Annual Meeting, including the John von Neumann Prize, W.T. and Idalia Reid Prize, AWM-SIAM Sonia Kovalevsky Lecture, SIAM Prize for Distinguished Service to the Profession, and various student and paper prizes. Anne Greenbaum (University of Washington) will deliver the 2022 Sonia Kovalevsky Lecture[7] and speak about two of her favorite problems, while Leah Edelstein-Keshet (University of British Columbia) will deliver the John von Neumann Prize Lecture[8] — SIAM's highest honor and flagship lecture. Edelstein-Keshet is recognized for her far-reaching contributions to mathematical biology, specifically her work on cellular biophysics and her influential textbook: *Mathematical Models in Biology*. AN22 likewise boasts the I.E. Block Community Lecture, which is free and open to the public. This year's speaker is Kristin Lauter[9] (Meta AI Research), who will discuss artificial intelligence and cryptography.[10] The meeting is also full of other special events, including the Workshop Celebrating Diversity and the Association for Women in Mathematics Workshop.

[7] https://sinews.siam.org/Details-Page/anne-greenbaum-named-awm-siam-sonia-kovalevsky-lecturer
[8] https://sinews.siam.org/Details-Page/leah-edelstein-keshet-is-the-2022-siam-john-von-neumann-prize-lecturer
[9] https://sinews.siam.org/Details-Page/2022-siam-block-community-lecture-presented-by-dr-kristin-lauter
[10] See page 12 of this issue of *SIAM News* for an article by Kristin Lauter on this topic.

Moreover, the SIAM Annual Meeting is a great place for students to network and build connections. The "Student Days" program includes special sessions that appeal directly to students, such as tutorials that offer accessible introductions to active research areas in applied mathematics.

Themes for AN22 include probabilistic, statistical, and stochastic algorithms; computational physics and physics-inspired methods; machine learning theory and applications in science and engineering; methods for partial differential equations, optimization, control, and uncertainty; high-performance and large-scale computing; and scientific computing, particularly in the area of fluid dynamics. The Organizing Committee has selected invited presentations to cover these and other topics. More information about the presentations—together with minisymposia, workshops, and panel discussions—is available online.[11]

The Annual Meeting is a place for SIAM business as well. The SIAM Board of Trustees and Council—the finance and policy leaders of the organization—will come together at AN22 to discuss and vote on SIAM's most important decisions. The yearly Business Meeting—which includes presentations from the chair of the Board along with SIAM's president, executive director, and chief operating officer—is open to all conference attendees who wish to learn about SIAM's financial health and priorities.

After more than two years of fully virtual conferences, AN22 will be one of the first SIAM meetings since March 2020 to have an in-person component. All participants, whether in-person or remote, will have full access to every session. Recordings of the sessions will also be available to conference attendees. The virtual platform for the hybrid meeting is powered by Pathable and is different from the platform that SIAM used for strictly virtual conferences in 2021. In addition to providing easy access to all sessions, the new platform is more mobile friendly, displays agendas in users' time zones, allows attendees to interactively customize their agendas with their calendar apps, and makes it easy to set up private meetings. Remote participants will find an intuitive and attractive web interface that they can browse, while in-person users can utilize the mobile app to organize their daily schedules.

[11] https://www.siam.org/conferences/cm/program/program-and-abstracts/an22-program-abstracts

Whatever your preferred style of attendance may be, we hope that you will join us in July for AN22.[12] We look forward to seeing you online or in Pittsburgh!

Edmond Chow and Sunčica Čanić are co-chairs of the Organizing Committee for the 2022 SIAM Annual Meeting. Edmond Chow is an associate professor in the School of Computational Science and Engineering at Georgia Institute of Technology. He is a SIAM Fellow. Sunčica Čanić is a professor in the Department of Mathematics at the University of California, Berkeley. She is a SIAM Fellow and a Fellow of the American Mathematical Society.

[12] https://www.siam.org/conferences/cm/registration/an22-registration

Reflecting on the 2021 Gene Golub SIAM Summer School in South Africa

By Bubacarr Bah

The African Institute for Mathematical Sciences (AIMS) South Africa[1] hosted the 11th Gene Golub SIAM Summer School[2] (G2S3) from July 19-30, 2021. The event—which was originally scheduled for 2020 but got postponed due to the ongoing COVID-19 pandemic—took place mostly virtually, which allowed for the inclusion of additional attendees from African institutions. The theme of the 2021 school was "Theory and Practice of Deep Learning," with a focus on the theory, implementation, and application of neural networks. Course material exposed students to math-based concepts from functional and harmonic analysis and optimization theory, which they used to explore the mathematical underpinnings of deep learning. Participants also experimented with multiple deep learning applications, such as computer vision and forecasting, and listened to additional lectures by industry practitioners who work on real-life problems.

[1] https://aims.ac.za
[2] https://sites.google.com/aims.ac.za/g2s3

Because the 2021 school was the first iteration of G2S3 to take place in Africa, it garnered a lot of interest. A total of 46 attendees represented 26 nationalities from 17 different countries around the world. About 50 percent of participants were African, and nearly all of these students were located in Africa; this was thus the first school of its kind to have such significant participation from Africa itself. Most participants were graduate students in either Ph.D. or M.Sc. programs, and roughly 31 percent were female. Figure 1 provides a more detailed breakdown of attendance.

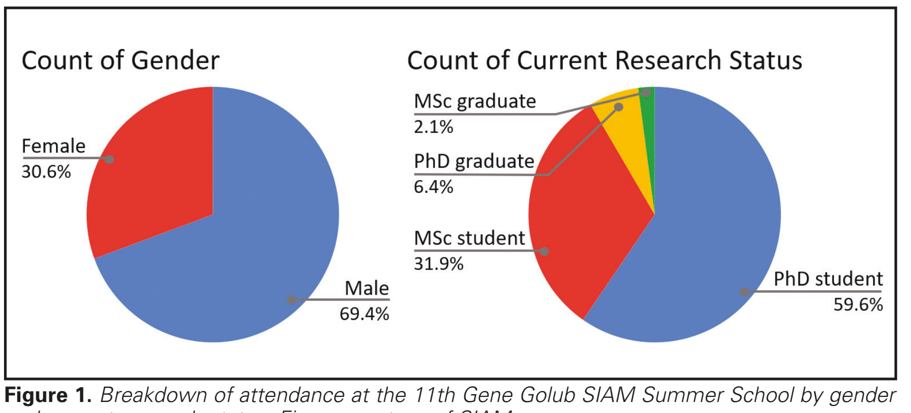
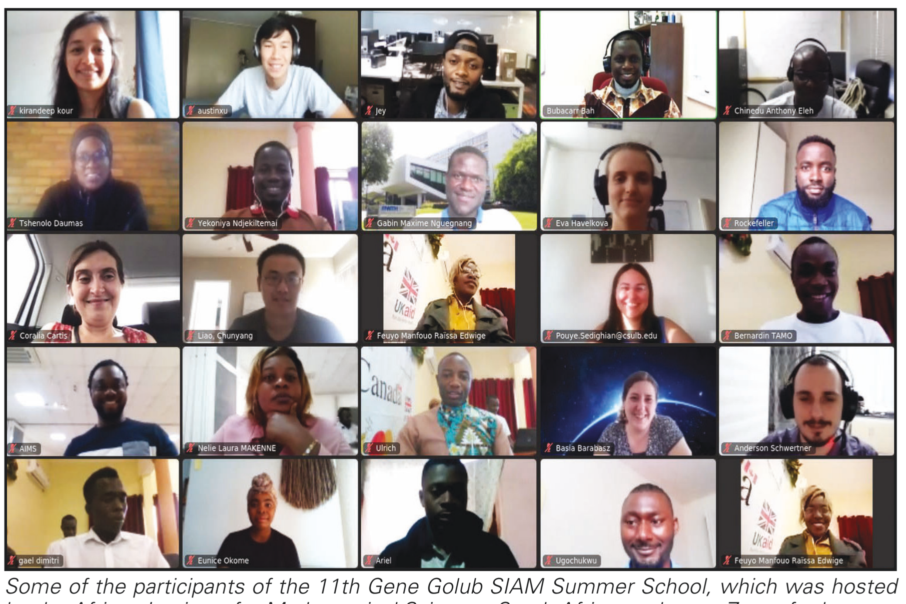


Figure 1. *Breakdown of attendance at the 11th Gene Golub SIAM Summer School by gender and current research status. Figure courtesy of SIAM.*

The school was held in a hybrid format due to COVID-19. Although most attendees participated virtually, several in-person cohorts did work together at the AIMS centres in Rwanda and Cameroon. The lectures and tutorials took place on Zoom and the social sessions transpired on Wonder. Daniel Nickelsen (AIMS South Africa) served as the moderator.



Some of the participants of the 11th Gene Golub SIAM Summer School, which was hosted by the African Institute for Mathematical Sciences South Africa, gather on Zoom for lectures and tutorials. Photo courtesy of Daniel Nickelsen.

Barry Green, the director of AIMS South Africa, delivered the opening remarks that marked the beginning of G2S3 activities. As with any virtual event, time zone differences added a logistical complication. All lectures were scheduled for the evening hours in Africa and Europe, which corresponded to the morning and early afternoon in North America. Lecturers included Bubacarr Bah (AIMS South Africa), Coralia Cartis (University of Oxford), Gitta Kutyniok (Ludwig Maximilian University of Munich), Kasso Okoudjou (Tufts University), and Jared Tanner (University of Oxford).

The Organizing Committee—which consisted of Bah, Cartis, and Okoudjou—arranged tutorials in the afternoon for cohorts in Africa and Europe that were then repeated for the North American cohort at a later time. A social event for all cohorts in the form of a coffee/tea break took place between the two main lectures every day. Students, tutors, and lecturers were invited to bring their coffee, tea, or beverage of choice and spend some time interacting with each other on Wonder. Despite modest participation (likely due to fatigue from excessive screen time), the virtual gatherings nonetheless generated a sense of international interaction between participants. In contrast, the high-quality lectures, tutorials, demonstrations, practicals, and exercises were extremely interactive. The students greatly appreciated and benefited from these learning opportunities and were thankful for the chance to connect with fellow peers around the world.

At the school's conclusion, we solicited feedback from attendees to gauge their levels of satisfaction with both the program and implementation. The response was overwhelmingly positive. "It was very useful to have such a wide range of depths of neural networks, which definitely appealed to the diverse backgrounds of the students," one attendee said. Another praised the format of the tutorials, adding that there was "enough [information] to represent the breadth of the field, but not so much that students would get lost in the material." A third student reflected on the structure of the school as a whole. "The thought put

See **SIAM Summer School** *on page 8*

Alumni Panel Series Offers a Glimpse into Life as an Early-Career Mathematician

University of North Carolina at Chapel Hill SIAM Student Chapter Organizes Inaugural Career Event

By Katherine Slyman

On the fourth floor of Phillips Hall at the University of North Carolina (UNC) at Chapel Hill, mathematics graduate students have various iterations of the same conversation within their respective cohorts. *Where will my degree take me in life? What am I going to do with my doctorate in mathematics? What kind of career should I pursue?* These types of questions are constantly present in the minds of all Ph.D. students, regardless of year or subject. Most early-stage mathematicians have broad ideas about their career paths when they start their programs, but they tend to develop more interests—as well as some doubts—as the programs proceed.

In short, students generally crave insight into a prospective career before actually committing to that path. For example, they might wonder how their academic degrees will earn them jobs in business, industry, or government. Suddenly, they find that they are finishing their dissertations and still do not know what comes next. We sought to break this cycle and provide mathematics graduate students with firsthand accounts of different career possibilities.

To do so, the UNC Chapel Hill SIAM Student Chapter began hosting the UNC Math Alumni Career Panel Series.[1] Katherine Slyman and Katherine Daftari—the chapter's co-presidents—organized the first-ever panel, which was inspired by the aforementioned questions and general graduate student interest in utilizing the knowledge and stories of previous students who had recently entered postdoctoral life (much more recently than some of the faculty with whom they work). The three-day event, which took place in November 2021, offered all mathematics graduate students the opportunity to learn from the experiences of a diverse group of successful graduates in teaching positions, postdoctoral appointments, and industry jobs. It also accorded networking and mentoring relationships that could manifest into future job openings; this aspect was especially helpful because an academic setting is not always conducive for fostering the connections and skills that are necessary in nonacademic careers.

[1] https://math.unc.edu/2021/11/unc-math-alumni-career-panel-series

During the University of North Carolina (UNC) Math Alumni Career Panel Series—hosted by the UNC Chapel Hill SIAM Student Chapter—a diverse group of successful graduates in teaching positions, postdoctoral appointments, and industry jobs described their experiences and career-related decisions during graduate school. Figure courtesy of Katherine Daftari.

The Panelists

To identify our pool of speakers, we utilized departmental records to view the first jobs of alumni who had completed their Ph.D.s between two and seven years ago. We then contacted these candidates, explained the goals of our panel, and offered key information about their prospective presentations. We broke the panel categories into teaching, postdoctoral, and industry positions. The teaching section consisted of graduates who received tenure-track offers after completing their Ph.D.s. The postdoctoral category encompassed government positions, academic postdoctoral positions, and even a National Science Foundation postdoctoral award winner. Finally, the industry group comprised alumni who currently hold nonacademic appointments. Some of the participants included an associate professor and chair of mathematics at Agnes Scott College, a postdoctoral fellow at Johns Hopkins University, an Oak Ridge Institute for Science and Education Fellow at the Environmental Protection Agency, and a machine learning technical trainer at Google.

The Panel Series

Over the course of three days, the invited speakers—separated by position type—delivered short presentations (no longer than 30 minutes each) that described the experiences and career-related decisions at UNC that led to their current positions. We encouraged them to share relevant details, such as how they prepared for employment while in graduate school, how they found their current jobs, what the application process was like, and what their current day-to-day work entails. Through the targeted presentations and Q&A sessions that followed, we created a friendly and informal environment wherein graduate students could openly converse with successful UNC alumni without faculty oversight. Our speakers provided early-career graduate students with valuable exposure to various types of available career opportunities; we hope that their presentations will motivate our students to think carefully about their futures and maximize their remaining time in the UNC program.

Due to COVID-19 and travel restrictions, we gathered all interested graduate students together in one classroom and conducted Zoom sessions with the alumni participants. We also had a few local speakers who attended in person and interacted with students face to face. Dinner was provided to everyone, and each night ended with a survey that gauged the most helpful aspects of the event.

Conclusions

This panel series received an overwhelmingly positive response from all of the involved students. Alumni speakers shared advice about navigating career prospects as a graduate mathematics student and provided helpful tips to ensure success as a job applicant. Because the event was planned by and intended for graduate students, attendees were free to ask whatever questions they had without fear of judgment from faculty or advisors. This openness fostered an inclusive and comfortable environment for everyone that added to the series' success. We intend to hold this event again in the future to expose more graduate students to the variety of career possibilities that are available upon graduation.

Katherine Slyman is an applied mathematics doctoral candidate at the University of North Carolina (UNC) at Chapel Hill. She studies dynamical systems with applications to climate science and currently serves as co-president of the UNC Chapel Hill SIAM Student Chapter.

Pathways in Data Science

Continued from page 3

via Virtual Virginia.[1] The iterative curricular design process incorporated feedback from national and local stakeholders, and the next phase—following the Board of Education's first review—will incorporate public feedback to further refine the standards and make the course accessible to all Virginia students.

[1] https://www.virtualvirginia.org

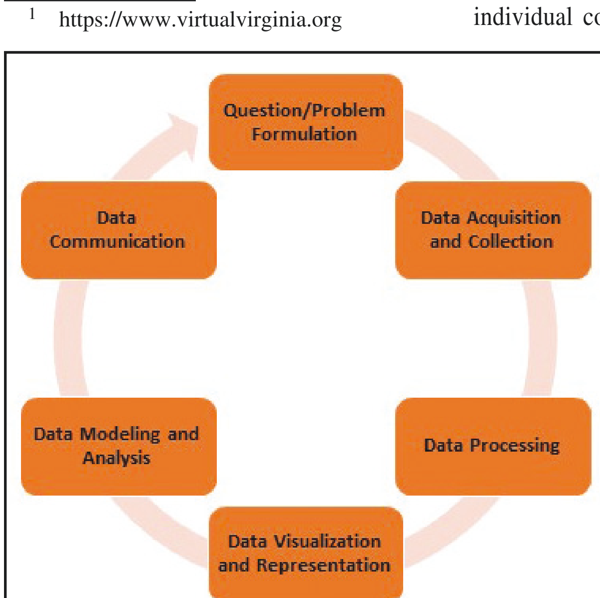


Figure 2. *The data science cycle. Figure courtesy of the Virginia Department of Education.*

The data science standards seek to ensure that data is collected, processed, analyzed, interpreted, visualized, and shared with a variety of stakeholders, and that information about the student experience is compiled. The experience is project-based in that students collaborate with each other and their community to investigate questions of interest through data visualization, modeling, prediction, analysis, and communication. Student teams are structurally similar to teams in the workforce, as every individual contributes their own assets to each project. These efforts are essential to the creation of a strong and *diverse* undergraduate and graduate workforce.

A data science skillset is valuable, easily transferable across disciplines, and fills a critical need in most workplace settings [2]. Students who study and practice data science are more likely to see themselves in different data science roles based on their unique strengths in areas like statistics, programming, and visualization. Talitha Washington, director of the Atlanta University Center Consortium's Data Science Initiative,[2] recognizes the value of such experience in the context of future careers. "Data science is a rapidly emerging skill that is needed to solve problems across all subjects," she said. "Opportunities for students to broaden their abilities to read, interpret, and communicate information from data do not replace mathematics. Rather, they expand students' knowledge for interacting in our data-driven world."

The variety of skills and application options in the field of data science underscores the value of a diverse team, which can employ an asset-based approach and build student agency. This tactic will also help data science education move beyond a deficit model that is driven by accountability and problems in practice, and towards an asset model that is driven by a positive approach that examines students' strengths, interests, and cultures.

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[1] National Council of Teachers of Mathematics. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Reston, VA: NCTM.

[2] Seshaiyer, P. (2021). Novel frameworks for upskilling the mathematics education workforce. In D. Burghes & J. Hunter (Eds.), *Mathematics education for sustainable economic growth and job creation* (pp. 90-107). New York, NY: Routledge.

[2] https://aucenter.edu/category/data-science

Tina Mazzacane is the K-12 Mathematics Coordinator at the Virginia Department of Education's Office of STEM and Innovation. She leads the Modernizing Mathematics in Virginia initiative. Deborah Crawford is the K-12 supervisor for mathematics and world language at Frederick County Public Schools. She leads the Data Science Pathways initiative for the Modernizing Mathematics in Virginia initiative. Lisa Bussian is a mathematics teacher and chair of the math department at Patriot High School in Nokesville, Va., where she teaches Advanced Algebra II and AP Computer Science Principles. She is a member of the Modernizing Mathematics in Virginia Pathways Committee. Aanand Vasudevan is a mathematics teacher at South Lakes High School in Reston, Va., where he teaches geometry and IB Mathematics Applications and Interpretations. He is a member of the Modernizing Mathematics in Virginia Pathways Committee. Padmanabhan Seshaiyer is a professor of mathematical sciences at George Mason University. He co-leads the Data Science and Mathematical Modeling Pathways initiatives for the Modernizing Mathematics in Virginia initiative. Seshaiyer also serves as chair of the SIAM Diversity Advisory Committee and vice chair of the U.S. National Academies Commission on Mathematics Instruction.

Sofia Kovalevskaya: Mathematician and Writer

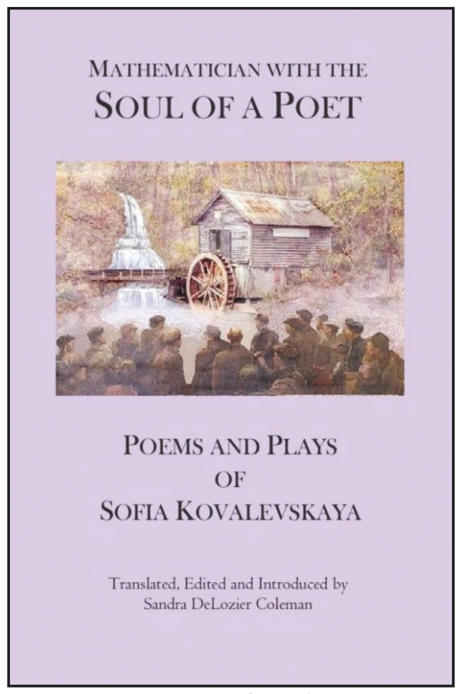
Mathematician with the Soul of a Poet: Poems and Plays of Sofia Kovalevskaya. *Translated, edited, and introduced by Sandra DeLozier Coleman. Bohannon Hall Press, November 2021. 252 pages, $19.95.*

BOOK REVIEW
By Ernest Davis

Many people consider Sofia Kovalevskaya (1850-1891) the greatest female scientist before Marie Curie and the greatest female mathematician before Emmy Noether. She was the first woman to receive a doctorate in mathematics and become a full university professor (with the exception of a handful of predecessors in Italy a century earlier). She also made a number of important original contributions to mathematics. For example, the Cauchy-Kovalevskaya theorem establishes existence and uniqueness for a broad class of systems of partial differential equations (PDEs); Augustin-Louis Cauchy proved a special case in 1842 and Kovalevskaya proved the general case in 1875. Her discovery of the "Kovalevskaya top"—one of very few cases of three-dimensional rigid body motion with integrable equations—earned her the Prix Bordin from the French Academy of Sciences in 1888.

In 19th-century Europe—and especially in Russia—ambitious, gifted women faced enormous obstacles at every stage of their lives. Both Kovalevskaya's father and governess refused to let her learn algebra. Instead, she taught herself in secret — in part from books and in part, incredibly, from her father's old math class notes that were recycled as wallpaper in her bedroom. In order to escape her father's rigid control and be allowed to enroll in a university, Kovalevskaya entered into a "fictitious marriage" with fellow student Vladimir Kovalevsky. She moved to Germany and, through great efforts, was able to take classes in math and physics at the University of Heidelberg. She wanted to study at the University of Berlin, but the school would not even permit her to audit classes. However, mathematician Karl Weierstrass met Kovalevskaya and was so enormously impressed with her talents that he tutored her privately. Weierstrass considered her to be the most talented of his many students and grew very attached to her; they remained lifelong friends.

In 1874, Kovalevskaya completed her thesis, which consisted of three papers: her aforementioned work on PDEs, a paper on the dynamics of Saturn's rings, and a paper about elliptic integrals. Weierstrass successfully persuaded the University of Berlin to issue Kovalevskaya a doctorate even though she never attended classes or defended her thesis. For many years afterward, however, she was unable to obtain any kind of university position — no institution in Germany or Russia would consider such a thing. Finally, in part due to the efforts of Gösta Mittag-Leffler (who was also a student of Weierstrass), Kovalevskaya received an appointment as *privatdocent* at Stockholm University in 1883; she was later promoted to ordinary (full) professor in 1889.



Mathematician with the Soul of a Poet: Poems and Plays of Sofia Kovalevskaya. Translated, edited, and introduced by Sandra DeLozier Coleman. Courtesy of Bohannon Hall Press.

Kovalevskaya led an altogether extraordinary life in her 41 years. She was a socialist, a feminist, and a radical. One illustrative instance occurred in early 1871 during the aftermath of the Franco-Prussian War. At the time, Paris was ruled by the Commune and under attack from the national army. Kovalevskaya and her husband traveled from Berlin and somehow smuggled themselves past the nearby German lines and *into* the besieged city to help her beloved elder sister, Anna, who was deeply involved with the Commune.

Kovalevskaya was also a gifted writer. Her best-known work, *A Russian Childhood*, is a personal memoir that is evocative, perceptive, moving, and extremely entertaining. Her account of novelist Fyodor Dostoevsky's unsuccessful courtship of her sister Anna is a masterpiece of narrative and vivid characterization. While I have not read her semi-autobiographical novel, *Nihilist Girl*, it is said to be remarkable.

Translator Sandra DeLozier Coleman's engagement with Kovalevskaya is itself a fascinating story that Coleman recounts in great detail. In addition to her 30-year career as a mathematics professor, Coleman is a poet, writer, and artist. In 1996, she served as a book reviews editor for *AMATYC Review* (a publication of the American Mathematical Association of Two-Year Colleges) and penned a review of three then-recent biographies of Kovalevskaya. Coleman became obsessed—the word is not too strong—with Kovalevskaya and avowed to translate her writings, despite the fact that she did not even know the Cyrillic alphabet at the time. In the 25 years since—and assisted by several Russian-speaking colleagues and friends—Coleman has pursued this goal with immense energy and dedication.

Her new book, *Mathematician with the Soul of a Poet*, is the fruit of these long labors. The text includes translations of nine of Kovalevskaya's poems and a pair of plays that were not previously translated into English, along with extensive commentary and biographical analysis. It also contains a translation from German of a short poem by Weierstrass — a toast that he gave at his own 70th birthday party.

Kovalevskaya's poems are a mixed bag. They are mostly written in rhyme and meter, though one is in free verse and another is a prose poem. Thematically, they include semi-humorous descriptions of her married life (one is even written from her husband's point of view);[1] personal, introspective musings; and a strange fantasy of a young girl who daydreams about becoming a martyr. Coleman carries out the translations with extreme care to convey not only the meaning but also the meter, rhyme scheme, and music of the original works. Doing so is often very challenging. For instance, Kovalevskaya's 32-line poem called "If You in Life" alternates between 12-syllable and 10-syllable lines and sticks to two rhymes in an ABAB scheme for its entire duration. The translation faithfully follows this overly constrained form.

Kovalevskaya's two plays—*How It Was* and *How It Might Have Been*—are "parallel dramas" that form a pair called *The Struggle for Happiness*. They were written collaboratively by Kovalevskaya and her close friend Anne Charlotte Edgren-Leffler, who was an actress, writer, and sister of Mittag-Leffler. The two plays feature the same characters and opening situation,

See **Sofia Kovalevskaya** *on page 10*

[1] Kovalevskaya's relationship with her husband was complicated. Their marriage was originally planned as a purely fictitious union so that Kovalevskaya could leave her father's house, and she and her husband lived apart while she was a student. Later, however, they lived together and in fact had a daughter.

Cosine Addition Formula and Perpetual Motion

MATHEMATICAL CURIOSITIES
By Mark Levi

Here is a simple physical interpretation of the familiar identity

$$\cos(\beta-\alpha)=\cos\beta\cdot\cos\alpha + \sin\beta\cdot\sin\alpha. \quad (1)$$

Consider a right triangle OAB with a hypotenuse of length 1, along with the constant force field $\mathbf{F}$ of unit vectors that form angle β with OA (see Figure 1). We can think of $\mathbf{F}$ as the constant gravitational field that is viewed in an inconveniently rotated frame.

The work done by $\mathbf{F}$ around any closed path vanishes. Indeed, otherwise we could build a perpetual motion machine by moving a particle around a closed path that is oriented so that the work done by $\mathbf{F}$ is positive, thus getting free energy. For the closed path ΔOAB in particular, we have

$$\text{work}_{OB}=\text{work}_{OA}+\text{work}_{AB}. \quad (2)$$

This equation is the identity (1) in disguise.

Indeed, since the work is given by the force's component in the direction of displacement multiplied by displacement length, we have $\text{work}_{OB}=1\cdot\cos(\beta-\alpha)$, $\text{work}_{OA}=\cos\beta\cos\alpha$, and $\text{work}_{AB}=\sin\beta\sin\alpha$. Therefore, (2) amounts to (1) as claimed.

Each of the three terms in (1) acquires physical meaning, just like in (2).

Depending on one's standards of rigor, the aforementioned expression might be considered a proof. I did not strive for full rigor, trying to keep this piece short.

Mark Levi (levi@math.psu.edu) is a professor of mathematics at the Pennsylvania State University.

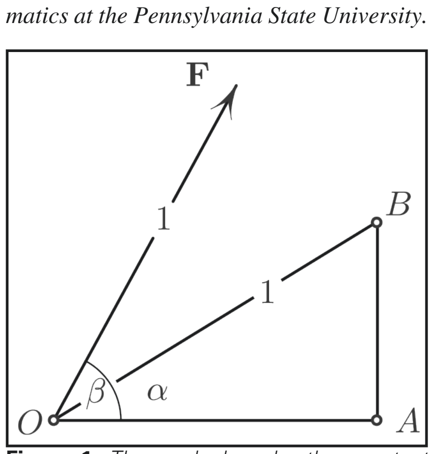


Figure 1. *The work done by the constant force* **F** *along the path* OA *equals the work done along the path* OAB*. Figure courtesy of Mark Levi.*

Artificial Intelligence: Ethics Versus Public Policy

By Moshe Y. Vardi

The computing field is currently experiencing an image crisis. In 2017, *Wall Street Journal* columnist Peggy Noonan described Silicon Valley executives as "moral Martians who operate on some weird new postmodern ethical wavelength" [4]. Niall Ferguson—a historian at Stanford University's Hoover Institution—defined cyberspace as "cyberia, a dark and lawless realm where malevolent actors range" [7]. The following year, Salesforce CEO Marc Benioff declared that a "crisis of trust" affects data privacy and cybersecurity.[1]

Many people view this situation as a crisis of ethics. In October 2018, *The New York Times* reported that "[s]ome think chief ethics officers could help technology companies navigate political and social questions" [6]. Numerous academic institutions are hurriedly launching new courses on computing, ethics, and society. Others are taking broader initiatives and integrating ethics across their computing curricula. The ongoing narrative implies that (i) a deficit of ethics ails the modern technology community and (ii) an injection of ethics is the remedy.

The prospect of increasingly powerful artificial intelligence (AI), which has marched from milestone to milestone over the past decade, is of specific concern. In recent years, many challenging problems—such as machine vision and natural language processing—have proven amenable to machine learning (ML) in general and deep learning in particular. Growing concerns about AI have only intensified the ethics narrative. For example, the Vatican's Rome Call for AI Ethics[2] has found support with a myriad of organizations, including tech companies. Multiple tech companies are also involved with the Partnership on AI,[3] which was established "to study and formulate best practices on AI technologies, to advance the public's understanding of AI, and to serve as an open platform for discussion and engagement about AI and its influences on people and society" [5]. Facebook (now Meta) has donated millions of U.S. dollars to establish a new Institute for Ethics in Artificial Intelligence[4] at the Technical University of Munich, since "ensuring the responsible and thoughtful use of AI is foundational to everything we do" [1]. Google announced that it "is committed to making progress in the responsible development of AI."[5]

ETHICS IN MATHEMATICS

Nevertheless, the problem with present-day computing lies not with AI technology per se, but with its current use in the computing industry. AI is the fundamental technology behind "surveillance capitalism," which Shoshana Zuboff defines as an economic system that centers on the commodification of personal data with the core purpose of profit-making [9]. Under the mantra of "information wants to be free," several tech companies have become advertising companies and perfected the technology behind micro-targeted advertising, which matches ads with individual preferences. Zuboff argued eloquently about the societal risk of surveillance capitalism. "We can have democracy, or we can have a surveillance society," she wrote in a 2021 article in *The New York Times*, "but we cannot have both" [10]. Internet companies effectively harvest the grains of information that we share, then use them to construct heaps of data about us. Similarly, the grains of influence that internet companies provide yield a mound of influence of which we are unaware, as evidenced by the Cambridge Analytica scandal [2]. ML enables these outcomes by mapping user profiles to advertisements. AI also moderates content for social media users with a primary goal of maximizing engagement and—as a consequence—advertising revenues.

The AI-ethics narrative thus leaves me deeply skeptical. It is not that I am against ethics; instead, I am dubious of the diagnosis and remedy. For example, consider the Ford Model T: the first mass-produced and mass-consumed automobile. The Model T went into production in 1908 and jump-started the automobile age. But with the automobile came automobile crashes, which now kill more than 1,000,000 people worldwide each year.[6] Nevertheless, the fatality rate has been decreasing over the past 100 years with improved road and automobile safety, licensed drivers, drunk driving laws, and the like. The solution to automobile crashes is not ethics training for drivers; it is public policy, which makes transportation safety a public priority. I share this ethics skepticism with Dutch philosopher Ben Wagner, who wrote that "[m]uch of the debate about ethics seems to provide an easy alternative to government regulation" [8].

At the same time, I do believe that surveillance capitalism—while perfectly legal and enormously profitable—is unethical. For example, the Association for Computing Machinery's (ACM) Code of Ethics and Professional Conduct[7] starts with "[c]omputing professionals' actions change the world. To act responsibly, they should reflect upon the wider impacts of their work, consistently supporting the public good." It would be extremely difficult to argue that surveillance capitalism supports the public good. The strain between a legal, profitable, and arguably unethical business model on one hand, and a façade of ethical behavior on the other hand, creates unsustainable tension within some tech companies. In December 2020, computer scientist Timnit Gebru found herself at the center of a public controversy that stemmed from her abrupt and contentious departure from Google as technical co-lead of the Ethical Artificial Intelligence Team; higher management had requested that she either withdraw an as-yet-unpublished paper that detailed multiple risks and biases of large language models, or remove the names of all Google co-authors.[8] During the aftermath of Gebru's dismissal, Google fired Margaret Mitchell — another top researcher on its AI ethics team.[9] In response to these firings, the ACM Conference for Fairness, Accountability, and Transparency suspended its sponsorship relationship with Google, stating briefly that "having Google as a sponsor for the 2021 conference would not be in the best interests of the community" [3].

The biggest problem that computing faces today is not that AI technology is unethical—though machine bias is a serious issue—but that large and powerful corporations use AI technology to support a business model that is arguably unethical.

See **Public Policy** *on page 11*

[1] https://gzconsulting.org/2018/06/04/salesforce-there-is-a-crisis-of-trust-concerning-data-privacy-and-cybersecurity
[2] https://www.romecall.org/the-call
[3] https://partnershiponai.org
[4] https://ieai.mcts.tum.de
[5] https://ai.google/responsibilities/responsible-ai-practices
[6] https://www.who.int/news-room/fact-sheets/detail/road-traffic-injuries
[7] https://www.acm.org/code-of-ethics
[8] https://www.washingtonpost.com/technology/2020/12/23/google-timnit-gebru-ai-ethics
[9] https://www.bbc.com/news/technology-56135817

SIAM Summer School

Continued from page 5

into the careful planning—accommodating various time zones and attaching practicals to each lecture—was an incredible amount of work," the participant said. "Yet it appeared seamless."

In summation, the 2021 school—the first G2S3 installment to be hosted by an African institution—commenced successfully in a hybrid format, with some in-person participation at the AIMS centres in Rwanda and Cameroon. It exposed participants to the state of the art of the mathematics of deep learning.

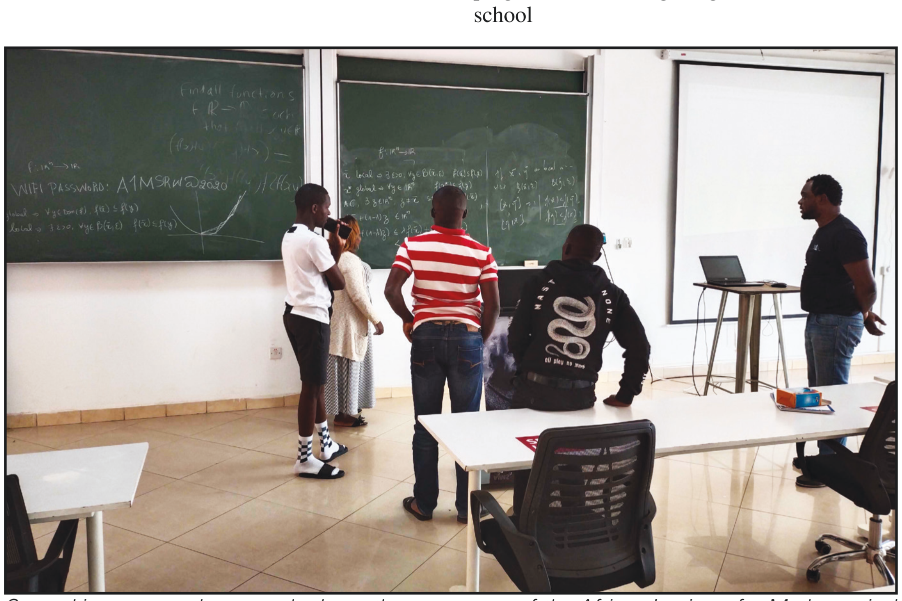
Acknowledgments: We would like to extend a special thanks to our sponsors: SIAM, the BMBF (German Federal Ministry of Education and Research), and the Alexander von Humboldt Foundation. We are also grateful for the numerous tutors who interacted with participants throughout the duration of the school.

The 2022 Gene Golub SIAM Summer School on Financial Analytics: Networks, Learning, and High Performance Computing will take place at the Gran Sasso Science Institute in L'Aquila, Italy, from August 1-12, 2022. The application portal to participate is now available online;[3] all applications are due by April 15th.

Interested in organizing a future school? Letters of intent that propose topics and organizers for the 2024 iteration of G2S3 are due by January 31, 2023. Visit the G2S3 website to learn more.

Bubacarr Bah is a senior researcher—designated as the German Research Chair of Mathematics with a specialization in data science—at the African Institute for Mathematical Sciences South Africa and a senior lecturer (assistant professor) in Stellenbosch University's Division of Applied Mathematics.

[3] https://www.siam.org/students-education/programs-initiatives/gene-golub-siam-summer-school



Several in-person cohorts worked together at centres of the African Institute for Mathematical Sciences in Rwanda and Cameroon during the 11th Gene Golub SIAM Summer School, which took place in July 2021. Photo courtesy of Yae Olatoundji Gaba.

The Rational Krylov Toolbox

By Stefan Güttel

Rational Krylov methods have become an indispensable tool in the field of scientific computing. Originally invented for the solution of large, sparse eigenvalue problems, these methods have seen an increasing number of other uses over the last two decades. Example applications include model order reduction, matrix function approximation, matrix equations, nonlinear eigenvalue problems, and nonlinear rational least squares fitting.

Here I aim to provide a short introduction to rational Krylov methods, using the MATLAB Rational Krylov Toolbox (RKToolbox) for demonstration.[1] The RKToolbox is freely available online and features an extensive collection of examples that users can explore and modify. The following text contains pointers to select RKToolbox examples, which are printed in `typewriter font`. Readers can also utilize these examples for teaching purposes, as they might serve as good starting points for computational coursework projects.

Rational Krylov: A Brief Introduction

A rational Krylov space is a natural generalization of the more widely known (standard) Krylov space. The latter is defined as

$$\mathcal{K}_m = \operatorname{span}\{b, Ab, \ldots, A^{m-1}b\},$$

where A is a square matrix and b is a nonzero vector of compatible size. We can represent every element in $\mathcal{K}_m$ as a *polynomial* $p(A)b$. On the other hand, a rational Krylov space is a linear space that is spanned by *rational functions* $r(A)b$. For illustrative purposes, assume that we have three distinct complex numbers ξ_1, ξ_2, and ξ_3 — all of which are different from A's eigenvalues. We can then define the four-dimensional *rational Krylov space* as

$$\mathcal{Q}_4 = \operatorname{span}\{b, (A-\xi_1 I)^{-1}b, (A-\xi_2 I)^{-1}b, (A-\xi_3 I)^{-1}b\} \quad (1)$$

(in artificial cases, $\mathcal{Q}_4$ can have a dimension that is less than four, but we assume that this does not happen here). In order to work with $\mathcal{Q}_4$ numerically, we compute an orthonormal basis with a method that is very similar to the standard Arnoldi algorithm. This method computes a matrix decomposition of the form

$$A\underbrace{[v_1,v_2,v_3,v_4]}_{V}\underbrace{\begin{bmatrix}\times&\times&\times\\ \times&\times&\times\\ &\times&\times\\ &&\times\end{bmatrix}}_{K} = \underbrace{[v_1,v_2,v_3,v_4]}_{V}\underbrace{\begin{bmatrix}\times&\times&\times\\ \times&\times&\times\\ &\times&\times\\ &&\times\end{bmatrix}}_{H}, \quad (2)$$

where the columns of V form an orthonormal basis of $\mathcal{Q}_4$ and the rectangular four-by-three matrices H and K are of upper Hessenberg form (i.e., all entries below the first subdiagonal are 0).

The computation of (2) requires the solution of three shifted linear systems with A. This requirement is typically the computational bottleneck of rational Krylov methods, but there is potential for coarse-grain parallelism if we solve these systems simultaneously; more information is available in the RKToolbox `example_parallel.html`.[2]

SOFTWARE AND PROGRAMMING

Eigenvalue Computations

Decompositions of the form (2) play a central role in eigenvalue computations [4]. It turns out that eigenvalues of appropriate submatrices of H and K—so-called Ritz values—may actually approximate some of A's eigenvalues quite well. This capability is completely analogous to the way in which we use the standard Arnoldi and Lanczos algorithms to approximate eigenvalues of large matrices. If A is a symmetric matrix, we even have a good theoretical understanding of which eigenvalues of A are well approximated by Ritz values [2]. Figure 1 illustrates a neat electrostatic interpretation of this concept. The RKToolbox provides examples for the numerical solution of linear and nonlinear eigenvalue problems at `example_eigenproblems.html`[3] and `example_nlep.html`.[4]

[1] http://rktoolbox.org

[2] http://guettel.com/rktoolbox/examples/html/example_parallel.html

[3] http://guettel.com/rktoolbox/examples/html/example_eigenproblems.html

[4] http://guettel.com/rktoolbox/examples/html/example_nlep.html

```
x = rkfun;                            % RKFUN representing f(x)=x
chebyshev   = rkfun('cheby', 8);      % Chebyshev polynomial
butterworth = 1./(1 + x.^16);
chebyshev1  = 1./(1 + 0.1*cheby.^2);
chebyshev2  = 1./(1 + 1./(0.1*cheby(1./x).^2));
elliptic    = rkfun('step');
```

Figure 2. *Example of rational filter construction.*

RKFUN Calculus

A useful feature of the decomposition in (2) is that matrices H and K encode rational functions, or so-called RKFUNs. The RKToolbox uses this matrix representation to enable an array of numerical operations with rational functions. The object-oriented MATLAB implementation of RKFUNs was inspired by the Chebfun package [3]. In addition to root and pole finding, it is possible to add, multiply, and concatenate RKFUNs — as well as convert them into partial fraction, quotient, and continued fraction forms. We can implement all of these operations with established numerical linear algebra routines. For example, the conversion to continued fraction form utilizes the nonsymmetric Lanczos tridiagonalization process.

RKToolbox demonstrations of RKFUN calculus include `example_rkfun.html`,[5] `example_feast.html`,[6] and `example_filter.html`.[7] Figure 2 is an example of rational filter construction.

As Figure 3 illustrates, we can easily plot the filters with a command such as `ezplot(butterworth, [0,2])`. The elliptic filter—also known as the Cauer filter—is based on Zolotarev's equioscillating rational functions, which are implemented

See **Rational Krylov** *on page 11*

[5] http://guettel.com/rktoolbox/examples/html/example_rkfun.html

[6] http://guettel.com/rktoolbox/examples/html/example_feast.html

[7] http://guettel.com/rktoolbox/examples/html/example_filter.html

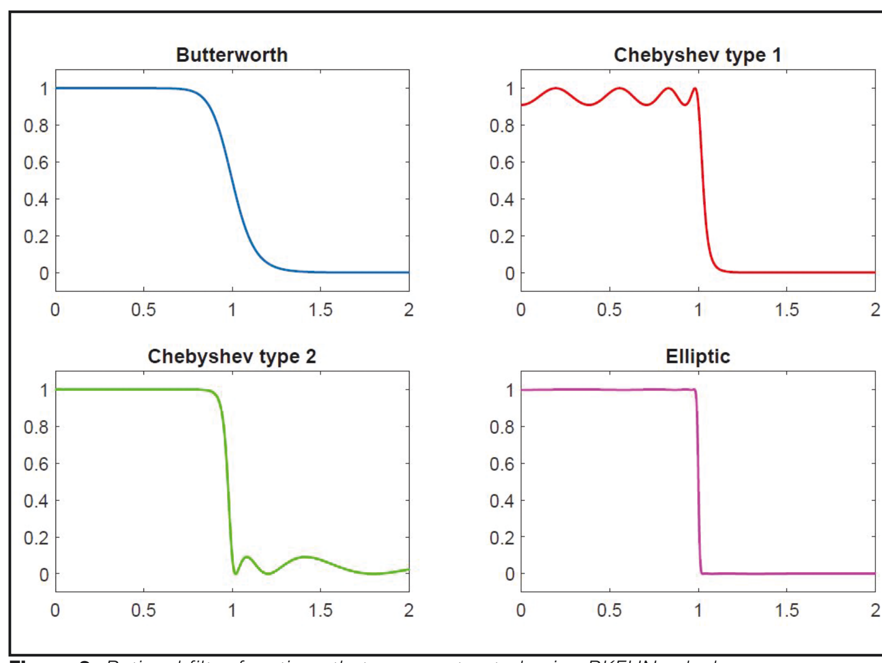


Figure 3. *Rational filter functions that are constructed using RKFUN calculus.*

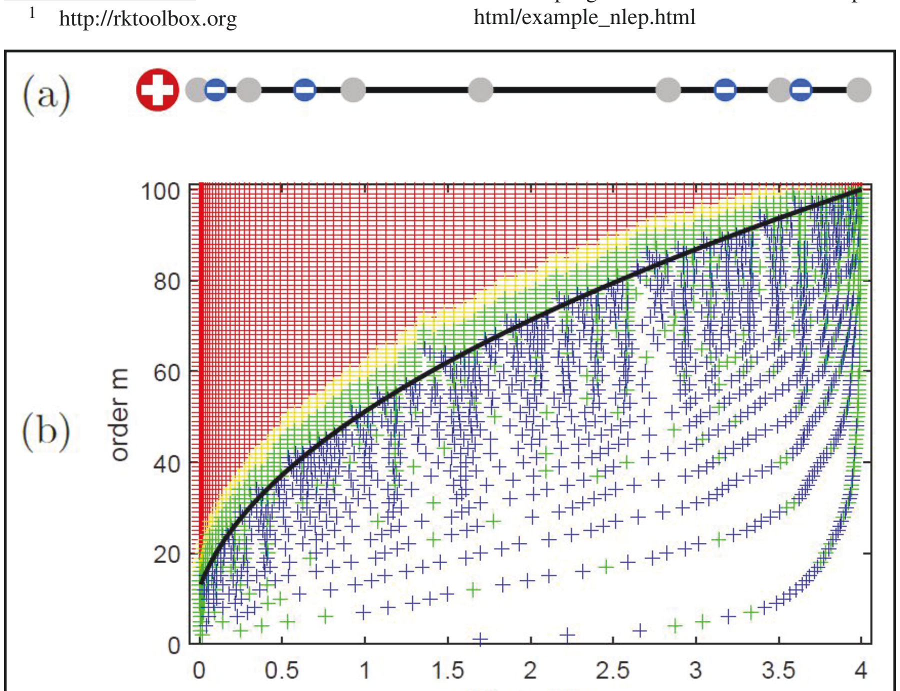


Figure 1. *Electrostatic interpretation of rational Ritz values.* **1a.** *Imagine that the eigenvalues of* A *(gray disks) are placed on a wire (black line). The* m*th-order Ritz values (blue disks) will then tend to distribute like electrons on the wire, satisfying the following properties: (i) Every two Ritz values are separated by at least one eigenvalue, (ii) the Ritz values repel each other, and (iii) they are attracted by an external field that is produced by the poles (red disk on the left); one can think of these poles as positive charges. Ritz values hence tend to find eigenvalues that are near the poles and in regions with few other eigenvalues.* **1b.** *Ritz values (colored "plus" signs) of order* $m=1,2,\ldots,100$ *for the tridiagonal matrix* `A = gallery('tridiag', 100)` *in MATLAB. All eigenvalues of* A *lie in the interval* $[0,4]$ *and all poles of the rational Krylov space are located at* 0. *The colors indicate a Ritz value's distance to a closest eigenvalue of* A *and range from blue (not very close) to green and yellow and ultimately to red (extremely close). The region of converged Ritz values, which is depicted by the black curve, can be characterized analytically.*

Sofia Kovalevskaya

Continued from page 7

but the characters make different decisions at critical points that lead to opposite outcomes. *How It Was* is a tragedy that concludes with unhappy marriages, financial ruin, and a suicide, but *How It Might Have Been* has a happy ending in all respects. Coleman speculates that Kovalevskaya may have come to this idea through her interest in Henri Poincaré's analysis of differential equations' sensitivity to small changes in the initial conditions.

The book's title, *Mathematician with the Soul of a Poet*, is derived from a passage in a letter by Kovalevskaya:

> One of the foremost mathematicians of our century says very justly that it is impossible to be a mathematician without also being a poet in spirit. It goes without saying that to understand the truth of this statement, one must repudiate the old prejudice by which poets are supposed to fabricate what does not exist, and that imagination is the same as "making things up." It seems to me that the poet must see what others do not see, must see more deeply than other people. And the mathematician must do the same.

Many researchers suppose that the mathematician she quotes was Weierstrass, and that Weierstrass was speaking not just of mathematicians in general but of Kovalevskaya specifically.

Judging by her literary works, I would say that Kovalevskaya had more the soul of a gifted writer of prose than the soul of a poet. I do not know why the former would be considered of lesser prestige. Moreover, she seems to have had an affinity for other great writers of prose; she remained a close friend of Dostoevsky until his death and visited George Eliot at her house a number of times — she even wrote an insightful reminiscence in tribute after Eliot's passing. Yet whether poet or prosist, Kovalevskaya's mind and soul were certainly extraordinary, and Coleman's labor of love sheds a new and interesting light on the esteemed mathematician.

Ernest Davis is a professor of computer science at New York University's Courant Institute of Mathematical Sciences.

Mathematician Sofia Kovalevskaya, 1850-1891. Public domain image.

Public Policy

Continued from page 8

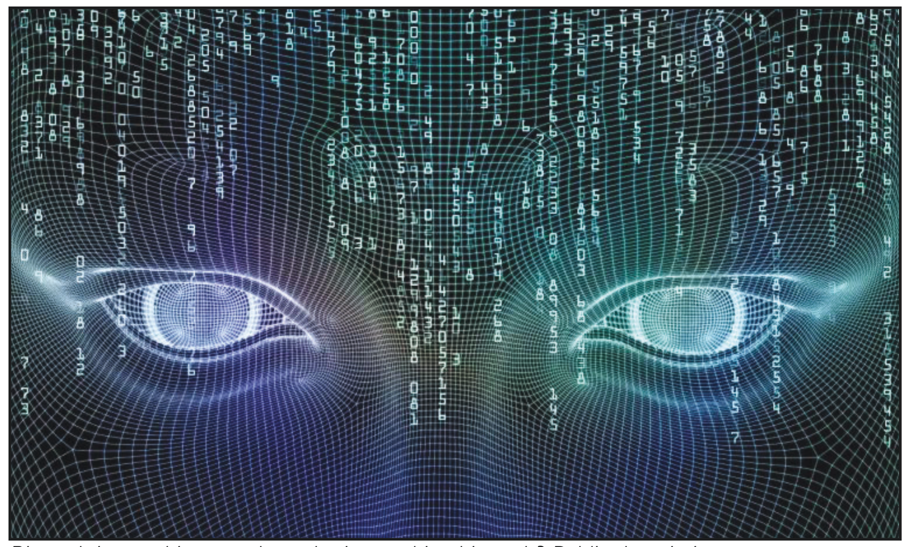
The computing community must address its relationship with surveillance capitalism corporations. For example, the ACM's A.M. Turing Award[10]—the highest award in computing—is now accompanied by a prize of $1 million that is supported by Google. Yet relationships with tech companies are not the only quandary. We must also consider the way in which society views officers and technical leaders within these companies. Holding community members accountable for the decisions of the institutions that they lead raises serious questions. The time has come for difficult and nuanced conversations about responsible computing, ethics, corporate behavior, and professional responsibility.

That being said, it is unreasonable to expect for-profit corporations to avoid profitable and legal business models. Ethics cannot be the remedy for surveillance capitalism. If society finds the surveillance business model offensive, the remedy should be public policy—in the form of laws and regulations—rather than an ethics outrage. Of course, we cannot divorce public policy from ethics. For instance, we ban human organ trading because we find it ethically repugnant, but the ban is enforced via public policy rather than ethical debate.

The information technology (IT) industry has successfully lobbied for decades against attempts to legislate/regulate IT public policy under the mantra "regulation stifles innovation." Of course regulation may stifle innovation. In fact, the whole point of regulation is to stifle certain types of innovation — the kind that public policy wishes to stifle. At the same time, regulation also *encourages* innovation. Automobile regulation, for example, undoubtedly increased automobile safety and fuel efficiency. Regulation can be a blunt instrument and must be wielded carefully; otherwise, it may discourage innovation in unpredictable ways. Public policy is hard, but it is better than anarchy.

Do we need ethics? Of course! But the current situation is a crisis of public policy, not a crisis of ethics.

[10] https://amturing.acm.org



Big tech is watching you, but who is watching big tech? Public domain image.

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Moshe Y. Vardi is a university professor and the Karen Ostrum George Distinguished Service Professor in Computational Engineering at Rice University, where he leads an initiative on technology, culture, and society.

Rational Krylov

Continued from page 10

in the RKToolbox's `gallery` (a collection of special rational functions).

Model Order Reduction

Model order reduction is one of the key drivers of research on rational Krylov methods. *Interpolatory Methods for Model Reduction*[8] [1]—particularly chapters three and five therein—describes the way in which rational Krylov spaces serve as natural projection spaces for large-scale, linear time-invariant (LTI) dynamical systems. The iterated rational Krylov algorithm (IRKA) is of particular interest for the optimal interpolatory reduction of LTI systems in the $\mathcal{H}_2$ norm. The RKToolbox contains several model order reduction problems, such as `example_frequency.html`,[9] `example_iss.html`,[10] and `example_cdplayer.html`.[11]

Here I have provided a brief introduction to rational Krylov methods and referenced a number of applications, like eigenvalue problems and model order reduction.

Krylov-based model order reduction is very closely linked with rational approximation. In a follow-up article to appear in the next issue of SIAM News, the author will discuss the RKFIT method for nonlinear rational approximation — a core algorithm in the RKToolbox.

All figures are courtesy of the author.

[8] https://go.siam.org/ZYyIyM

[9] http://guettel.com/rktoolbox/examples/html/example_frequency.html

[10] http://guettel.com/rktoolbox/examples/html/example_iss.html

[11] http://guettel.com/rktoolbox/examples/html/example_cdplayer.html

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Stefan Güttel is a professor of applied mathematics at the University of Manchester and a Fellow of the Alan Turing Institute. He received SIAM's 2021 James H. Wilkinson Prize in Numerical Analysis and Scientific Computing.

Celebrating the Launch of the Mathematics for Action Toolkit

Mathematics is extremely relevant to increasing our planet's resilience and sustainability and better understanding the challenges it faces. To that end, the global mathematics community recently collaborated to produce a toolkit titled *Mathematics for Action: Supporting Science-Based Decision Making*. As an outgrowth of a worldwide effort called Mathematics of Planet Earth, the toolkit is published by UNESCO and closely connected to the United Nations Member States' 2030 Agenda for Sustainable Development. This agenda includes 17 Sustainable Development Goals (SDGs) that represent an urgent call to action for all countries to collectively tackle societal and planetary challenges.

The policy-oriented, open access toolkit comprises a collection of two-page briefs that pertain to different SDGs and seek to inform decision-makers, scientific and technical advisors, scientific attachés, parliamentarians, and diplomats about relevant applications of mathematics and statistics. It also publicizes mathematics' relevance to global challenges, explores mathematical modeling's ability to address "what-if" scenarios, and reveals innovative opportunities for science-based decision-making.

The toolkit was produced by the following international consortium of experts under the management of Centre de Recherches Mathématiques (CRM): African Institute for Mathematical Sciences (AIMS), African Mathematical Union (AMU), CRM, UNESCO Cat II Centre CIMPA, European Mathematical Society (EMS), Institut des Sciences Mathématiques et de leurs Interactions (INSMI/CNRS), Institut de Valorisation des Données (IVADO), International Commission on Mathematical Instruction (ICMI), International Mathematical Union (IMU), and International Science Council (ISC). The 32 authors come from the five UNESCO regions. A multinational team led by Christiane Rousseau of CRM edited the document, with scientific editing and design by Barbara Cozzens of Whistling Thorn Strategies.

The toolkit was launched virtually during an online webinar on March 14, 2022, which is the International Day of Mathematics. It is now freely available on UNESCO's website.[1]

[1] https://unesdoc.unesco.org/ark:/48223/pf0000380883.locale

Private Artificial Intelligence: Machine Learning on Encrypted Data

By Kristin Lauter

Artificial intelligence (AI) refers to the science of utilizing data to formulate mathematical models that predict outcomes with high assurance. Such predictions can be used to make decisions automatically or give recommendations with high confidence. Training mathematical models to make predictions based on data is called machine learning (ML) in the computer science community. Tremendous progress in ML over the last two decades has led to impressive advances in computer vision, natural language processing, robotics, and other areas. These applications are revolutionizing society and people's overall way of life. For example, smart phones and smart devices currently employ countless AI-based services in homes, businesses, medicine, and finance.

However, these services are not without risk. Large data sets—which often contain personal information—are needed to train powerful models. These models then enable decisions that may significantly impact human lives. The emerging field of *responsible AI* intends to put so-called guard rails in place by developing processes and mathematical and statistical solutions. Humans must remain part of this process to reason about policies and ethics, and to consider proper data use and the ways in which bias may be embedded in existing data. Responsible AI systems aim to ensure fairness, accountability, transparency, and ethics, as well as privacy, security, safety, robustness, and inclusion.

Here I focus on the privacy problem in AI and the intersection of AI with the field of cryptography. One way to protect privacy is to lock down sensitive information by encrypting data before it is used for training or prediction. However, traditional encryption schemes do not allow for any computation on encrypted data. We therefore need a new kind of encryption that maintains the data's structure so that meaningful computation is possible. Homomorphic encryption allows us to switch the order of encryption and computation so that we get the same result if we first encrypt and then compute, or if we first compute and then encrypt (see Figure 1).

A solution for a homomorphic encryption scheme that can process any circuit was first proposed in 2009 [3]. Since then, cryptography researchers have worked to find solutions that are both practical and based on well-known hard math problems. In 2012, the "ML confidential" paper introduced the first solution for training ML models on encrypted data [5]. Four years later, the CryptoNets paper [4] demonstrated the evaluation of deep neural network predictions on encrypted data — a cornerstone of private AI. *Private AI* uses homomorphic encryption to protect the privacy of data while performing ML tasks, both learning classification models and making valuable predictions based on such models.

The security of homomorphic encryption is based on the mathematics of lattice-based cryptography. A lattice can be thought of as a discrete linear subspace of Euclidean space, with the operations of vector addition, scalar multiplication, and inner product; its dimension n is the number of basis vectors. New hard lattice problems were proposed from 2004 to 2010, but some were related to older problems that had already been studied for several decades: the approximate Shortest Vector Problem (SVP) and Bounded Distance Decoding. The best-known algorithms for attacking the SVP are called lattice basis reduction algorithms, which have a more than 30-year history. This history starts with the Lenstra-Lenstra-Lovász algorithm [9], which runs in polynomial time but finds an exponentially bad approximation to the shortest vector. More recent improvements include BKZ 2.0 and exponential algorithms like sieving and enumeration. The Technical University of Darmstadt's Hard Lattice Challenge[1] is publicly available online for anyone who wants to try to attack and solve hard lattice problems of large size to break the current record.

Cryptographic applications of lattice-based cryptography were first proposed by Jeffrey Hoffstein, Jill Pipher, and Joseph Silverman in 1996 and led to the launch of NTRU Cryptosystems [6]; compare this to the age of other public key cryptosystems such as RSA (1975) or Elliptic Curve Cryptography ECC (1985). Because lattice-based cryptography is a relatively new form of encryption, standardization and regulation are needed to build public trust in the security of the schemes. In 2017, we launched HomomorphicEncryption.org:[2] a consortium of experts from industry, government, and academia to standardize homomorphic encryption schemes and security parameters for widespread use. The community approved the first Homomorphic Encryption Standard in November 2018 [1].

Homomorphic encryption scheme parameters are set so that the best-known attacks take exponential time (exponential in the dimension n of the lattice, meaning roughly 2^n time). Lattice-based schemes have the advantage that there are no known polynomial time quantum attacks, which means that they are good candidates for post-quantum cryptography (PQC) in the ongoing five-year PQC competition.[3] Nowadays, all available major homomorphic encryption libraries from around the world implement schemes that are based on the hardness of lattice problems. These libraries include Microsoft SEAL (Simple Encrypted Arithmetic Library)[4]—which became publicly available in 2015—as well as other publicly available libraries like IBM's HElib,[5] Duality Technologies' PALISADE,[6] and Seoul National University's HEEAN.[7]

Next, I will discuss several demos of private AI in action. We developed these as the Microsoft SEAL team to demonstrate a range of practical private analytics services in the cloud. The first demo is an encrypted fitness app: a cloud service that processes all workout, fitness, and location data in the cloud in encrypted form. The app displays summary statistics on a phone after locally decrypting the results of the analysis.

The second demo is an encrypted weather prediction service that takes an encrypted ZIP code and returns encrypted information about the weather at the location in question, which is then decrypted and displayed on the phone. The cloud service never learns the user's location or the specifics of the weather data that was returned.

Finally, the third demo shows a private medical diagnosis application. The patient uploads an encrypted version of a chest X-ray image to the cloud service. The medical condition is diagnosed by running image recognition algorithms on the encrypted image in the cloud; the diagnosis is returned in encrypted form to the doctor or patient (see Figure 2).

Other examples of private AI applications include sentiment analysis in text, cat/dog image classification, heart attack risk analysis based on personal health data [2], neural net image recognition of handwritten digits [4], flowering time determination based on a flower's genome, human action recognition in encrypted video [7], and pneumonia mortality risk analysis via intelligible models.

All of these processes operate on encrypted data in the cloud to make predictions and return encrypted results in a matter of seconds. Many applications were inspired by collaborations with researchers in medicine, genomics, bioinformatics, and ML. The annual iDASH workshops,[8] which are funded by the National Institutes of Health, attract teams from around the world to submit solutions to the Secure Genome Analysis Competition. Participating teams represent organizations such as Microsoft, IBM, startup companies, and academic institutions from various countries around the world.

Homomorphic encryption has the potential to effectively protect the privacy of sensitive data in ML. It is interesting to consider how protecting privacy in AI through encryption intersects with other goals of responsible AI. If the data is encrypted, for example, it is not possible to inspect it for bias. However, encryption makes it harder for a malicious adversary to manipulate the AI models to produce undesirable outcomes.

For more detailed information about private AI and encryption, please see my article in the proceedings of the 2019 International Congress on Industrial and Applied Mathematics [8].

Kristin Lauter will deliver the I.E. Block Community Lecture[9] at the 2022 SIAM Annual Meeting,[10] which will take place in Pittsburgh, Pa., in a hybrid format from July 11-15, 2022. She will speak about artificial intelligence and cryptography.

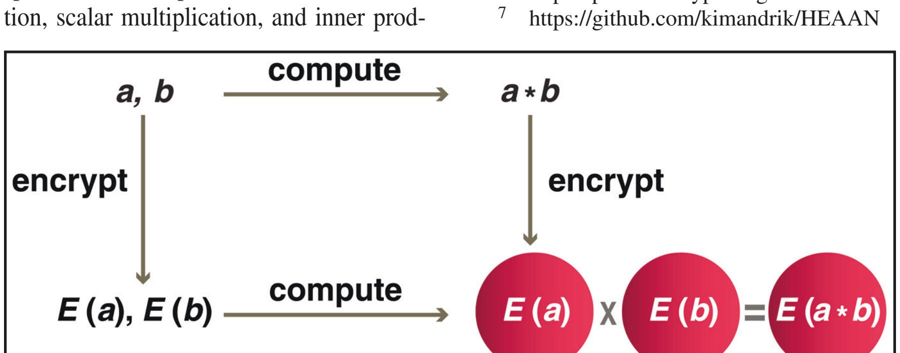

Figure 1. *Diagram of homomorphic encryption. If one begins with two pieces of data a and b, the outcome is the same when following the arrows in either direction: across and then down (compute, then encrypt) or down and then across (encrypt, then compute). Figure courtesy of SIAM.*

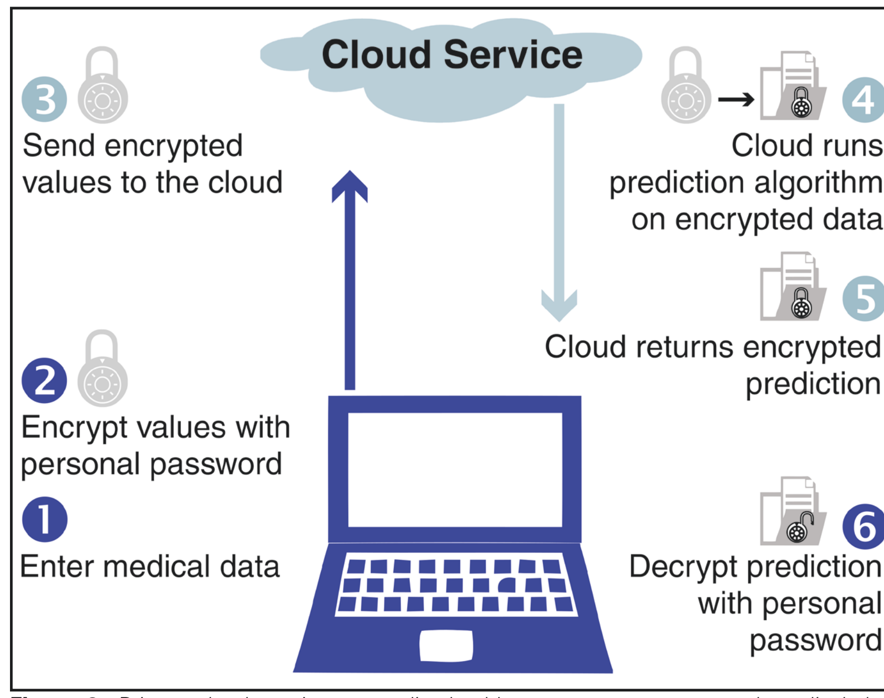

Figure 2. *Private cloud service to predict health outcomes on encrypted medical data. Figure courtesy of SIAM.*

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Kristin Lauter is the Director of West Coast Research Science for Meta AI Research (FAIR). She served as president of the Association for Women in Mathematics from 2015-2017. Lauter was formerly a Partner Research Manager at Microsoft Research, where her group developed Microsoft SEAL: an open source library for homomorphic encryption.

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