# Comparing Student Strategies in Vector Unknown and the Magic Carpet Ride Task

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*We present findings from a study analyzing and comparing the strategies participants deployed in playing the game Vector Unknown and completing the Magic Carpet Ride task. Both the game and task are designed to give students an introduction to basic concepts about vectors needed for success in linear algebra. We found that participants used a diverse array of strategies, tending to favor algebraic approaches to the Magic Carpet Ride task. We also found that participants tended to try the same strategies in both tasks, but did not usually follow through with the same strategy in both contexts. These findings have implications for instructors considering using one or both tasks in their linear algebra class.*

*Keywords:* linear algebra, game-based learning, inquiry-oriented instruction

Game-based learning (GBL) has proven to be a popular approach in STEM education and STEM education research (Klopfer & Thompson, 2020). However, much of the research into GBL in mathematics education has been focused on K-12 and especially K-8 education (Byun & Joung, 2018). One of the few games developed specifically for undergraduate mathematics instruction is Vector Unknown (VU; Mauntel et al., 2021), an adaptation of the Magic Carpet Ride (MCR) task from the Inquiry-Oriented Linear Algebra (IOLA) curriculum (Wawro, Zandieh, et al., 2013). VU, like the MCR task, is designed to give players an introduction to basic concepts about vectors needed for success in linear algebra. While the goals of both tasks overlap, the differences between them may lead to differences in the kinds of thinking students engage in when playing VU or solving the MCR task. This could be important to instructors deciding how they might use either or both in their own instruction. To begin to explore these differences, we present findings from a qualitative interview study with the following research questions:

RQ 1: What strategies do students deploy in solving the Magic Carpet Ride task and in playing levels in the Vector Unknown game?

RQ 2: What patterns are apparent in the use of strategies across tasks?

## Context and Background

### Literature Review

The idea that games and puzzles are environments where people engage in mathematical thinking is not new. Puzzles like Sudoku and games like Chess and Go have been the objects of study for mathematicians over the years (Silva, 2011). Moreover, the use of video games for the teaching and learning of STEM topics has been a popular application of GBL given their computational nature, their ability to simulate complex situations, and the active engagement they demand (Klopfer & Thompson, 2020). Vector Unknown is one of the few video games that have specifically been developed for undergraduate linear algebra (Mauntel et al., 2021).

Drawing from K-12 literature on GBL in mathematics education, there are clear indications of potential positive outcomes for student learning. In their meta-analysis on GBL research in K-12 math education, Byun and Joung (2018) computed an average effect size of d = 0.37 from quantitative studies, indicating a small-to-moderate-sized positive effect on math learning outcomes. Even so, some quantitative (Beserra et al., 2014) and mixed method studies (Ke, 2008) comparing games to similar non-game active learning opportunities have found that games may not always offer additional advantage over other active learning activities in terms of learning outcomes. While the VU game (Mauntel et al., 2021, Mauntel et al., 2020, Mauntel et al., 2019) and the MCR task (Wawro et al., 2012; Wawro, Rasmussen, et al., 2013) have each been the subject of several publications, no work thus far has compared student thinking in these tasks.

**The Tasks**

To properly contextualize the remainder of this paper, we present a brief summary of the two tasks being compared in this study below.

**Magic Carpet Ride.** The MCR task used in this study comes from the IOLA curriculum (Wawro et al., 2012; Wawro, Zandieh, et al., 2013) and is designed as a "day one" task as part of a larger "Magic Carpet Ride" unit that introduces concepts related to vectors, span, and linear (in)dependence. The day one MCR task asks students to determine if they can reach Old Man Gauss's cabin at the point (107, 64) using two forms of transportation represented by vectors <3, 1> and <1, 2>. The next task in the unit asks students to consider whether there are "some locations that [Old Man Gauss] can hide and you cannot reach him with these two modes of transportation." This task was used as a follow-up in some of the interviews as time allowed.

**Vector Unknown.** In VU, each level randomly generates a goal position (represented by a basket) and two pairs of vectors that are scalar multiples of each other (so one possible set of vectors is <-3, 2>, <-9, 6>, <1, 3>, and <2, 6>). Players then use any two of those vectors and integer scalars to get a rabbit from the origin to the goal. In this study, the current first three levels were used for the interviews. All participants played Levels One and Two, which work the same except for the Predictive Path feature. In Level One, as players choose their vectors and scalars, a Predictive Path line shows them where the rabbit will move when they hit "Go;" this feature is absent from Level Two. Level Three includes the Predictive Path and has an added component of a player first needing to get to three keys on the map and *then* go to the goal position. Completing Level Three usually requires the player to move from a location other than the origin after gathering some or all of the keys. See Figure 1 for an illustration of the game.

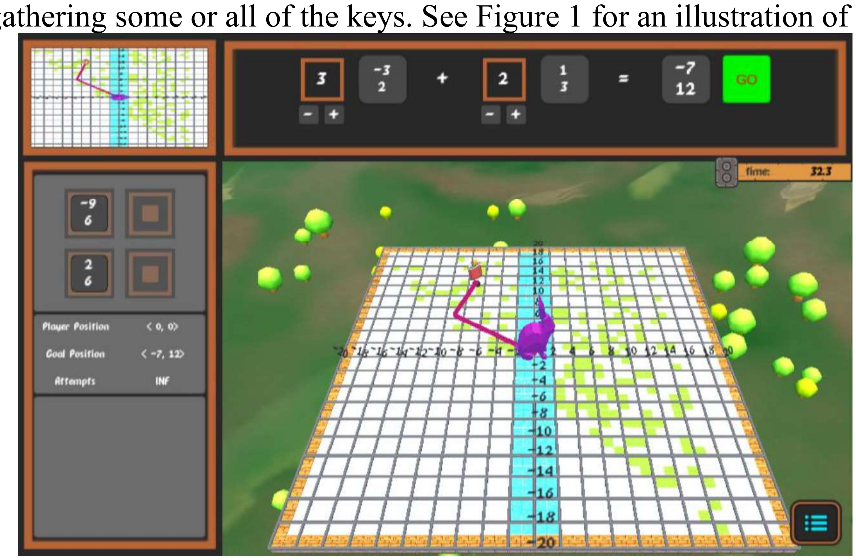


*Figure 1. Gameplay of Vector Unknown.*

## Conceptual Framework

In answering our research questions, "strategy" needed to be operationalized. Because the purpose of this research was to compare two specific tasks, and because both of those tasks have published material to draw from, we used past work to develop a conceptual framework to operationalize strategy. This framework was based primarily on one developed with VU (Mauntel et al., 2021), supplemented by student sample work from the MCR task (Wawro et al., 2012; Wawro, Zandieh, et al., 2013), and further modified during the analysis process. Figures 2, 3, and 4 outline the conceptual framework.

| Strategy | Mauntel et al. Description | Adaptations / Notes |
| --- | --- | --- |
| Guess and Check | Player presses buttons while attending to how the vector equation changes or to how the geometric Predictive Path changes. | Name changed from "Button Pushing" to "Guess and Check," to also include trying random scalars in the MCR task. |
| Quadrant-based Reasoning | Player chooses a vector to match the signs/quadrant of the goal position, references the direction of the Predicted Path or a quadrant on the graph to make sense of the direction of a vector, or understands vectors as slopes. | Slope-based strategies are the most common way this appears in the MCR task, as both the goal and given vectors have the same signs/quadrant. |
| Focus on one Coordinate | Player reduces the aim of the goal to one coordinate and attempts to reach that one coordinate. | In MCR, this occurs when the participant focuses on the North or East direction, one at a time. |
| Focus on one Vector | Player focuses on getting as close to the goal as possible with one vector and then utilizes another vector to reach the goal and/or alternates between the two. | In MCR, this occurs when the participant focuses on one of the modes of transport at a time. |

*Figure 2. Conceptual framework, part 1.*

In past work with VU, Mauntel et al. (2021) used an iterative approach which sorted the strategies players used into four categories: Button-Pushing, Quadrant-based Reasoning, Focus on one Coordinate, and Focus on one Vector (see Figure 2). Additionally, each strategy participants used was classified as either Numeric or Geometric (see Figure 3), depending on whether the participant was relying on the numeric data (like the vector equation) or visual data (such as the predictive path) to solve the problem.

| Strategy Type | Descriptors for Strategy Type |
| --- | --- |
| Numeric | Using arithmetic to solve or check a possible solution<br>Referring to the vector equation (VU) or numeric values of vectors/goal |
| Algebraic | Setting up a system of equations<br>Creating an equation for a line<br>Creating symbols for unknowns |
| Geometric | Interpreting graphical information<br>Drawing vectors or lines on a graph<br>Using the Predictive Path (VU) |

*Figure 3. Conceptual framework, part 2.*

Our review of student sample work for the MCR task (from Wawro et al., 2012 and Wawro, Zandieh, et al., 2013) lead to three adjustments to the framework. As a minor change, we renamed "Button-Pushing" to "Guess and Check." More substantially, we noticed that algebraic solution strategies – strategies involving written equations with unknowns or variables – were more prominent in the MCR student sample work than in the VU research. To address this, we first added a fifth strategy: "System of Equations," (see Figure 4), to specifically categorize the use of a system of linear equations. Second, Algebraic strategies were separated out from Numeric and Geometric as a third category for any strategy where the participant employed an algebraic equation or expression to attempt to solve the problem (see Figure 3).

| **Strategy** | **Description** | **Basis for Addition** |
|---|---|---|
| System of Equations | Player creates a system of equations with two unknowns and then solves it to solve the problem. | Review of MCR student sample work (Wawro, Zandieh, et al., 2013). |
| Linearly Independent Vector Selection | Player chooses two vectors based on which pairs of vectors are scalar multiples of each other. | Strategy observed during analysis that did not fit into existing conceptual framework. |

*Figure 4. Conceptual framework, part 3 (new strategies).*

Finally, the conceptual framework was further revised during the analysis process. In particular, an additional novel solution strategy for VU of "Linearly Independent Vector Selection" was noted. This strategy will be described and explored in the findings section.

## Methods

### Data Collection

The participants for this study were five students recruited from a third-semester calculus course at a large public university in the southwestern United States. All five students who indicated interest in the study participated in task-based interviews. For the purposes of this study, no demographic data were collected. As such, all participants will be referred to with the gender-neutral pronoun 'they' and pseudonyms generated from a list of gender-neutral names (Van Fleet & Atwater, 1997). Participants were asked if they had ever taken a college-level linear algebra course, and only one participant (Chris) said they had.

Interviews were conducted via Zoom due to the COVID-19 pandemic. The components of these interviews focused on in the analysis presented here consisted of two approximately 30-minute task portions for each of the MCR and VU tasks. In these interviews, the interviewer primarily described the tasks to be completed and did not typically interrupt the participant's solving process, unless they had not spoken for a long time or were nearing the end of the allotted time. The order of the task portions varied from interview to interview. Figure 5 lists the participants by pseudonym and shows the order they completed the two tasks in.

| **Participant** | Rikaine | Pat | Terry | Auren | Chris |
|---|---|---|---|---|---|
| First Task | VU | VU | MCR | MCR | MCR |
| Second Task | MCR | MCR | VU | VU | VU |

*Figure 5. Participants and task order.*

### Data Analysis

The conceptual framework outlined earlier in this paper was applied as a codebook. The two authors developed the conceptual framework over a sequence of meetings, settling on the approach as described above. Then, the lead author coded the transcript by identifying each time a participant applied or attempted to apply one of the strategies outlined in that framework. Further, a student's strategy was stratified based on whether they were applying Numeric, Algebraic, or Geometric versions of that strategy. Once identified, the strategies used by each participant were collected together and reviewed for accuracy and clarity before identifying which strategies best characterized the participant's solving process for each respective task. For MCR, this meant identifying which strategy was ultimately used to arrive at a solution, if any. For VU, where participants completed multiple levels, this meant identifying which strategy was used most frequently to arrive at solutions.

## Findings

Our findings are oriented around our two research questions. Given space limitations, the first subsection addresses the first research question by summarizing the strategies students deployed in the MCR task and VU. The second subsection focuses on the novel strategy of Linearly Independent Vector Selection, as previous literature (Mauntel et al., 2021; Wawro et al., 2012) showcases detailed examples of the other strategies. Then, the two subsequent subsections address the second research question, by articulating two notable patterns apparent in the strategies used: the relative prevalence of Algebraic, Geometric, and Numeric strategies and the repetition of strategies across the tasks.

### RQ 1 – Overall Distribution of Strategies

To begin to address RQ 1, we use Figure 6 to visually provide a summary of the diverse set of strategies that were used by the sample of participants. Within each box, an A, G, or N represents that the participant used an Algebraic, Geometric, or Numeric instantiation of that strategy, respectively. Bolded letters indicate the strategies that best characterized their performance on that task as defined in the Data Analysis section above.

| **Participant** | Rikaine | | Pat | | Terry | | Auren | | Chris | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Task** | VU | MCR | VU | MCR | MCR | VU | MCR | VU | MCR | VU |
| Guess and Check | G | N | G | | | G | | | | |
| Quadrant-based Reasoning | G, N | **G, N** | | | A, N | **G, N** | | **N, G** | | |
| Focus on one Coordinate | | | **G, N** | N | | | N | | | N, G |
| Focus on one Vector | **G, N** | N | N | | N | | | | | |
| System of Equations | | | | **A** | | | **A** | | **A** | **A** |
| Linearly Independent Vector Selection | N | | N | | | | | N | | N |

*Figure 6. Summary of strategies used by each participant*

### RQ 1 - Linearly Independent Vector Selection

As mentioned in the conceptual framework, a novel strategy for VU was observed in the analysis process. In four of the interviews, participants chose vectors based on the observation that some of the vectors were scalar multiples of each other. This strategy is perhaps best summarized by Chris's explanation [the vectors referred to are in square brackets]:

Chris: So what I'm thinking is that we can, uh, look for. First, identify any same vectors, any vectors that are linearly dependent, and I can already tell that the top right one [<-1, 5>] here and the bottom left one [<-2, 10>] here are linearly dependent, so they're the same vector, and then the also the other ones, the top left [<1, 1>] and bottom right [<-9, -9>] are linearly dependent so they're the same vector so that means that I can just use these top two as those are the only truly independent vectors available to me.

As Chris was the only participant who had taken linear algebra previously, they were the only participant to describe this strategy in terms of linear independence. Other participants typically only noted that pairs of vectors were scalar multiples of each other, such as in this excerpt from Rikaine:

Rikaine: Um and then, since these are, since these point in the same direction, I'm only really considering this one [<-3, -1>]. Just because I can scale it up to <-9, -3> if I need to.

This strategy is only implementable in VU (notice it is only present in VU columns, bottom row of Figure 6), as MCR has only two choices for modes of transport. Another notable aspect of this strategy was that, in two of the four cases where participants made use of this fact, this only occurred after additional interviewer questioning led them to observe that there were always pairs of vectors which were scalar multiples of each other. Thus, only one player (Rikaine) made an observation and selected vectors in this way without prior prompting or prior linear algebra experience.

**RQ 2 - Algebraic, Geometric, and Numeric Strategies**

Across participants, Geometric and Numeric instantiations of strategies were both common for VU (see the VU columns on Figure 6). Often, Geometric thinking was more apparent when the level had the Predictive Path feature. Only Chris used an Algebraic solution in VU. After completing the first level using a Focus on One Coordinate Strategy, and toying around with strategies in the second level, Chris reluctantly decided to pursue an Algebraic solution:

Chris: Ehhh, I wanted to avoid the algebra, but I think I'm going to have to use the algebra [laughs]. Going back to the method, okay setting up a two-by-two matrix.

They made this choice after having already solved the MCR task using a system of equations and an augmented matrix and having previously taken linear algebra at the college level.

On the other hand, MCR was mostly solved with Algebraic strategies, with Rikaine solving it primarily Numerically and Terry being unable to complete the task in the time allotted. This is not to say that Geometric thinking did not appear at all in solving the task – all of the participants drew a graph at some point. Some did so to set up their coordinate system and/or to visualize the problem, while a few did so after interviewer prompting to illustrate their solution. In these cases, however, drawing the graph was not directly linked to any of the solution strategies in our conceptual framework, and participants did not use these graphs as tools throughout the problem solving process.

**RQ 2 - Repetition of Strategies**

Most of the participants would at least attempt the same strategy in both the MCR task and VU. This can be seen by comparing the MCR and VU columns for each participant in Figure 6. For example, in the following excerpts, Pat attempts to apply the Focus on one Coordinate strategy first in VU and then in MCR:

Pat [During VU]: How to get to -7 with my x.…That [mousing over <-1, 3>] will at least get me to negative seven x.

Pat [During MCR]: I wonder if it's better to get to 64 first. So if it's 32 hours by magic carpet, that's the point (32, 64). Which would arrive me at the y, the y, the y value of his house.

In each case, Pat attempts to match one of the coordinates first. However, while they carried this strategy through to completion with VU, they ultimately chose to solve the MCR task with a

System of Equations. The only participant who did not at least attempt the same broad strategy across both environments was Auren. They relatively quickly settled on using a System of Equations on the MCR task, which they completed first. When they played VU, they did not ever consider this strategy, preferring instead to use Quadrant-based Reasoning.

## Discussion

The findings above reflect the different affordances for students that VU and the MCR task offer. The fact that participants tended to try the same strategy for both tasks but did not tend to complete both tasks with the same strategy particularly supports this conclusion. This suggests that the two tasks are different enough that students may decide a strategy that worked in VU is not the best strategy for the MCR task, thus engaging them in different ways of thinking.

Two clear ways VU differs from the MCR task are in the offloading of computation and the limited magnitude of goal positions. Because the vector equation at the top of VU automatically updates whenever you change a scalar, it offloads computational work. The addition of the Predictive Path in some levels offloads even more work, as it allows the player to see both the Numeric value of the result and the path it takes on the coordinate plane to get there. In addition, goal positions have their $x$ and $y$ coordinates each somewhere between -20 and 20. This means that the goal position is relatively small in magnitude, particularly when compared to the goal position of the MCR task, (107, 64). These differences in features may be related to the differences in how students use Algebraic, Numeric, and Geometric thinking across the two tasks. Because the MCR task involves working with larger numbers, and because all computation and graphing is left to the student, students may be more inclined to think algebraically to avoid having to do a lot of computations or draw a precise graph. Conversely, because VU handles many computations for the player, asks the player to work with smaller numbers, and shows the player information on an already-made graph, it can be easier to engage in Numeric and Geometric thinking while solving this task. While each individual task may better support different kinds of student thinking, using them in conjunction with one another may support students using all three of these kinds of thinking.

Another difference is that players have a surplus of vectors to choose from in VU, with the four vectors available consisting of two pairs of vectors that are scalar multiples of each other. Thus, students not only have choice in what vectors they use, they also may be able to make observations about linear independence and dependence through playing levels of the game. We saw this with the strategy of Linearly Independent Vector Selection. In comparison, the first task in the MCR unit only includes two linearly independent vectors. However, the rest of the MCR unit does lead students toward considerations of linear independence and dependence. In addition, simply having students play VU may not automatically lead to students having any insight about linear independence. We can see this in the fact that three of the students who used this strategy only did so after additional interviewer questions. This suggests an important caveat: it is not just the design but also the implementation of the tasks that matters. Instructors who subtly or not-so-subtly prompt students to look closer at the vectors that are available to them may be able to scaffold these kinds of observations for students.

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## References


Beserra, V., Nussbaum, M., Zeni, R., Rodriguez, W., & Wurman, G. (2014). Practising Arithmetic Using Educational Video Games with an Interpersonal Computer. *International Forum of Educational Technology & Society*, *17*(3).

Byun, J., & Joung, E. (2018). Digital game-based learning for K-12 mathematics education: A meta-analysis. *School Science and Mathematics*, *118*(3–4), 113–126. https://doi.org/10.1111/ssm.12271

Ke, F. (2008). A case study of computer gaming for math: Engaged learning from gameplay? *Computers and Education*, *51*(4), 1609–1620. https://doi.org/10.1016/j.compedu.2008.03.003

Klopfer, E., & Thompson, M. (2020). Game-Based Learning in Science, Technology, Engineering, and Mathematics. In J. L. Plass, R. E. Mayer, & B. D. Homer (Eds.), *Handbook of Game-Based Learning* (pp. 387–408). The MIT Press.

Mauntel, M. C., Levine, B., Plaxco, D., & Zandieh, M. (2021). The Characterization and Evolution of Strategies about Vector Equations in the game. *Digital Experiences in Mathematics Education*, *7*, 453–476.

Mauntel, M. C., Levine, B., Plaxco, D., & Zandieh, M. (2020). Get That Basket! Deciphering Student Strategies in the Linear Algebra Game Vector Unknown. In S. S. Karunakaran, Z. Reed, & A. Higgins (Eds.), *Proceedings of the 23rd Annual Conference on Research in Undergraduate Mathematics Education* (pp. 398–406).

Mauntel, M. C., Sipes, J., Zandieh, M., Plaxco, D., & Levine, B. (2019). "Let's See" - Students Play Vector Unknown, An Inquiry-Oriented Linear Algebra Digital Game. In A. Weinberg, D. Moore-Russo, H. Soto, & M. Wawro (Eds.), *Proceedings of the 22nd Annual Conference on Research in Undergraduate Mathematics Education* (pp. 959–965).

Silva, J. N. (2011). On mathematical games. *BSHM Bulletin*, *26*(2), 80–104. https://doi.org/10.1080/17498430.2011.560511

Van Fleet, D. D., & Atwater, L. (1997). Gender neutral names: Don't be so sure! *Sex Roles*, *37*(1–2), 111–123. https://doi.org/10.1023/a:1025696905342

Wawro, M., Rasmussen, C., Zandieh, M., & Larson, C. (2013). Design research within undergraduate mathematics education: An example from introductory linear algebra. In T. Plomp & N. Nieveen (Eds.), *Educational design research - Part B: Illustrative cases* (pp. 905–925). SLO. http://iola.math.vt.edu/media/pubs/Wawro_et_al_2013_DR_chapter.pdf

Wawro, M., Rasmussen, C., Zandieh, M., Sweeney, G. F., & Larson, C. (2012). An Inquiry-Oriented Approach to Span and Linear Independence: The Case of the Magic Carpet Ride Sequence. *Primus*, *22*(8), 577–599. https://doi.org/10.1080/10511970.2012.667516

Wawro, M., Zandieh, M., Rasmussen, C., & Andrews-Larson, C. (2013). *Inquiry oriented linear algebra: Course materials.* http://iola.math.vt.edu