UNDERSTANDING JOINT EXPLORATION: THE EPISTEMIC POSITIONING IN COLLABORATIVE ACTIVITY IN A SECONDARY MATHEMATICS CLASSROOM

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This study examines how joint exploration is established and maintained among students and the teacher in secondary mathematics classrooms. We use the theoretical perspective of positioning to conceptualize joint exploration as involving the negotiation and coordination among participants to position students with epistemic agency and authority. Using a constant comparative method, we use classroom video data of two episodes containing joint exploration and closely analyze the shifts in epistemic positioning within them. We find that shifts in epistemic positioning, especially when students position one another with epistemic authority, help to support continued joint exploration. We also find that the teacher can play an important role in decentering themselves as the epistemic authority.

Keywords: Classroom Discourse, Problem Solving, Instructional Activities and Practices

Mathematics education reform has long called for students to collaboratively engage in the broad array of mathematical practices and reasoning used within the discipline (NGA & CCSSO, 2010; NCTM, 1989). To be authentically engaged in the discipline of mathematics, students should have opportunities to exercise epistemic agency and authority, which focus on their role in taking on the work of knowledge building. Epistemic agency goes beyond the idea of conceptual agency in mathematics, related to developing solution strategies and meaning of concepts (Cobb et al., 2009), by recognizing the roles students play in making decisions about the process by which ideas are constructed (Damsa et al., 2010; Stroupe, 2014). Specifically, this vision involves students making decisions as a part of mathematical inquiry or exploration, such as which mathematical questions and problems are worth pursuing or which approaches to take in investigating them. Within the context of a classroom community, these decisions are often made as joint negotiations between teacher and students (Krist, 2020). As many mathematics classrooms provide little opportunity for students to exercise joint epistemic agency and authority, it is crucial to better understand how teachers and students interact in ways that position students as active participants, particularly during mathematical explorations.

In this paper, we examine *joint exploration* in secondary mathematics classrooms when the teacher is present by analyzing the epistemic positioning of participants. It aims to answer the following research questions: *How are episodes of joint exploration established and maintained in a secondary mathematics classroom? What social and/or epistemic roles do teachers and students play, and how do these roles shift throughout the episodes?*

Exploration as a Form of Mathematical Activity

Exploration, or investigation, is an important aspect of knowledge generation across a wide range of disciplines, with disciplinary distinctions made in the methods for how this exploration is taken up. Exploration is centered around mathematical questions, ideas, or problems that are not sufficiently known and thus represent an intellectual need to be fulfilled (Harel, 2001). As such, exploration involves jointly orienting toward inquiry as a fundamental component of the work (Keifert & Stevens, 2019). Furthermore, processes by which questions, ideas, or problems are framed, clarified, and made investigable are an integral part of mathematical exploration. For example, research has examined students' problem posing in mathematics (Cai et al., 2015; Silver 1994) and the activities involved in problematizing in science learning (Suárez, 2020; Phillips et al., 2018). In addition to activity orienting toward an intellectual need, exploration also involves the investigation and potential closure around whether that intellectual need is fulfilled (Keifert & Stevens, 2019). These activities could include stating what is known and not known, offering suggestions of next steps, monitoring and reporting on the status of the activity to the group, and confirming a solution, among others. Thus, mathematical exploration provides a context for students to actively engage in many types of productive disciplinary work (Engle et al., 2002) for the purpose of fulfilling an intellectual need.

We focus on *joint* exploration, highlighting the instances in which students and the teacher participate collaboratively. This focus helps uncover the interactional and political dynamics involved in establishing and maintaining opportunities for exploration, given that it is not normative for mathematics learning to incorporate it. Thus, we claim that joint exploration likely necessitates re-negotiating the roles and participatory structures of traditional classroom contexts that distribute the social and intellectual authority and agency to students, rather than residing solely with the teacher (Ko & Krist, 2019). To investigate these complex interactions related to epistemic agency and authority, we apply the theoretical lens of positioning.

Positioning Theory as a Theoretical Lens

Positioning theory considers both social and intellectual roles and authority in analyzing interaction. It highlights the interactional nature of activity, which is afforded and constrained by normative possibilities of the authority and responsibilities associated with different roles (Davies & Harré, 1990). From this theoretical lens, participants in activity take up roles or positions, which afford them specific ways of acting and recognition among participants. These positions are flexible and can shift over time. Shifts tend to indicate important moments of activity because they typically involve participants negotiating and coordinating roles.

This theory has been recently used in analysis of discourse in mathematics classrooms in ways that help highlight the role of identity and power in micro-interactions among students and the teacher (Herbel-Eisenmann et al., 2017). For example, research on epistemic positioning (i.e., positions related to knowledge and its creation) in mathematics classrooms has identified two main positions students and the teacher commonly take up during mathematical activity: 1) a *knower* who provides mathematical information and 2) an *actor* who performs an action doing mathematics (DeJarnette & González, 2015; Lo & Ruef, 2020). Within these positions, participants can take on primary or secondary roles depending on whether they provide or request the activity to be completed. In a traditional classroom structure, the teacher is often in a position of authority as a primary knower, viewed by the students as the conduit of disciplinary knowledge. This traditional relationship between teacher and students can be conceived of as an inherent asymmetry in authority over knowledge, as well as bearing the responsibility for controlling the conversation (Mercer & Dawes, 2008).

In particular, we hypothesize that the presence of the teacher could both support and constrain students to be actively involved in joint exploration. The teacher could help facilitate collaborative work, model many disciplinary practices, and position students with epistemic authority to take on intellectual roles in exploration. However, the teacher may also be positioned as a knowledgeable disciplinary expert and authority, such that students could rely on the teacher to fulfill any intellectual need rather than take up exploration. Furthermore, we hypothesize that when a teacher seeks to create opportunities for students to participate in exploration, the inherent asymmetry of the teacher-student dynamic would need to be challenged by positioning students in roles that are associated with intellectual and social authority.

Methods

For this study we analyzed two comparative episodes drawn from a large classroom video dataset (Dyer, 2016). Below we describe the data, episode selection, and analysis of the episodes.

Data and Episode Selection

We use classroom video data from one focal teacher, Mrs. Perry, from a class with grade 10 and 11 students. Mrs. Perry was selected because her lessons contained a large amount of time with students working in small groups and previous research has documented that Mrs. Perry's teaching practice is responsive to student thinking (Dyer & Sherin, 2016). We believed that both of these factors would make it more likely for joint exploration to occur with the teacher present.

Data from Mrs. Perry's classroom included 10 videotaped 100-minute lessons, filmed approximately every week for the final three months of school, which corresponded to about a quarter of her lessons during that time period because her school used block scheduling. Video was collected from three different angles and separate audio was captured for each group of students. We selected two lessons, one from each of the first two months, from different units of instruction on 1) trigonometric functions, and 2) exponential and logarithmic functions.

We selected two episodes of joint exploration, one from each lesson. These episodes were identified by watching video of the two selected lessons and identifying potential instances of joint exploration as a sensitizing concept (Blumer, 1954) in which the teacher was physically present. We defined instances of joint exploration as the collaborative activity of investigating disciplinary questions and ideas among students and/or teachers through interaction. Thus, our analytic criteria specified that instances must have (a) at least two participants contribute substantively, intellectually or socially, to the group's sensemaking activity, and (b) participants seek to construct new knowledge related to a content idea or question to fulfill an intellectual need. We selected two contrasting cases based on the different types of participation from the teacher in each of these episodes as we hypothesized the teachers' participation would have a large influence on how episodes of joint exploration are established and maintained.

Episode Analysis

We used a constant comparative method (Strauss & Corbin, 1990) that involved constructing rich descriptions of the of the epistemic positioning for each of the episodes. We bounded episodes by starting with the teacher approaching the student(s) and ending with the teacher leaving the students to interact with another student group. We created transcripts of each episode, which we used in tandem with the video and audio records in all subsequent analysis. Two authors independently wrote descriptive accounts of the positioning with the aim of highlighting salient aspects of the epistemic positioning that occurred. These descriptions considered: Who is seeking knowledge? Who (or what) is being sought for knowledge? What is an individual's level of certainty about the knowledge? Who confirms the knowledge as certain?

We used the descriptive accounts of positioning to identify portions of the episodes in which the positioning of a participant in interactions shifted. We considered a shift in epistemic positioning to be: (1) an individual changing from expressing certainty to uncertainty (or vice versa) about a piece of information; (2) an individual being newly sought as a source of information or (3) a change in who confirmed information as valid.

Results

We present two contrasting episodes from the same classroom that we identified as instances of joint exploration and report shifts in epistemic positioning that we identified in each episode.

Episode I: Solve a Cosine Equation using a Graph

The first episode involves a group of three students, Ellie, Nick and Theo, and the teacher, Ms. Perry, jointly exploring a task that she provided to the class. This task asked students to solve the equation –15=20cos(30x) for the portion of the function shown in the provided graph (Figure 1, left), where x is measured in degrees. Using the inverse cosine function to solve the equation yields one solution (x » 4.62). Students could then use the graph and the period of the cosine function (12) to find the remaining three solutions (x » –4.62, 7.38, 16.62) by locating the x-values of the points of intersection of the function and the line y = –15 (Figure 1, right).

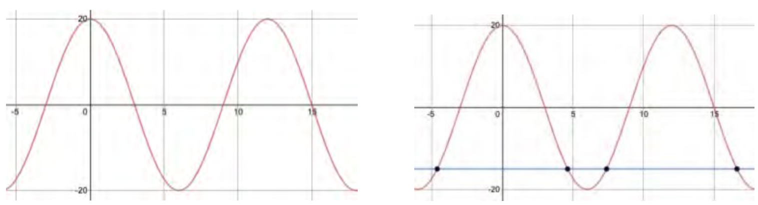

Figure 1: Solve a Cosine Equation Graph Provided (left) and its Solutions (right)

This episode involves joint exploration, as Ellie, Nick, Theo and Mrs. Perry (teacher) work collaboratively to find the remaining solutions. Each participant contributes by posing questions about the task, offering solution strategies, or directing the next steps of the activity. These questions, strategies, and directions are taken up by others to advance the search for remaining solutions. We present three main shifts in epistemic positioning that served to establish and maintain joint exploration below.

Ellie positions Nick as having epistemic authority. This exchange begins the episode of joint exploration. The first shift occurs when Ellie looks to Mrs. Perry to confirm her solution, who is standing up to leave. As Mrs. Perry leaves momentarily, Ellie then turns to Nick to confirm her answer of 4.61.

Ellie: Yes, I don't know if I'm right, is this right? (*moves paper towards Mrs. Perry who stands up to briefly leave group, turns to Nick*) What did you get Nick?
Nick: I got 4.62, 16.62
Ellie: Yesss (*exclaims, holds both fist in the air*). Wait, I just got 4.61. (*Mrs. Perry returns*)
Nick: You're supposed to find all this, just one more. All you have to do is add the period…
Ellie: But how do you make that into like an equation so I can solve it?
Nick: (*Stands up, leaning over Ellie's work across from him*) You don't necessarily, … put it into the equation. What you can do… knowing that this is going to be repeating over and over again … so say you want here… you would add this to 12.

Ellie seeks confirmation for her answer on the task and first looks to Mrs. Perry. When she leaves, Ellie instead turns to Nick to confirm her answer, indicating a shift in Nick's epistemic position. Ellie cheers as Nick gives an equivalent answer to hers, but then questions him when he lists a second answer. Ellie continues to ask Nick questions and he continues to fulfill his role as a source of information, offering an explanation of how to find the other solution.

Mrs. Perry shares epistemic authority. The second shift involves Mrs. Perry sharing epistemic authority with the students in the group by re-positioning herself as one who gives directives without giving mathematical information. By re-positioning herself, she shares her epistemic authority with the students and helps sustain the joint exploration in this exchange. When Mrs. Perry re-enters the conversation, she continues to generally redirect Nick, Theo, and Ellie to the graph, rather than providing a clear next step.

Mrs. Perry: Well, where's the one, what x does that say? (*pointing to Nick's work*)
Nick: 4.61, I got 4.62 because I rounded.
Mrs. Perry: Can you like find that, mark that on the x-axis where that is?
Nick: That would be about, let's assume, here (*writing on paper*)
…
Mrs. Perry: Maybe you should mark that on there, like you were about to, because maybe that will help you to think about, okay, how can I?
Nick: Because I know you can add to the period to get this
Mrs. Perry: Okay so that will get you that one, so you could at least get that one by adding the period and we just have to figure out how to get the other two.

Mrs. Perry's moves to re-position herself to the students include clarifying which x value the students are referring to and directing them to mark that value on the x-axis. Mrs. Perry does affirm Nick's proposed solution of adding the period to the first solution to find a second solution. Then, she summarizes what is left to find, the remaining two solutions, rather than providing more specific epistemic guidance. These moves position the students to continue the exploration independent of Mrs. Perry as the epistemic authority.

Theo and Nick position each other as having epistemic authority. The third shift occurs while Mrs. Perry is still with the group when Theo enters the conversation and begins to work with Nick to find the other solutions. Nick and Theo shift to position each other as sources of information. This shift also maintains the joint exploration. As Nick poses the question of how to find the remaining solutions, in addition to 4.62 and 16.62, Theo offers ideas that Nick takes up.

Nick: How would you solve for that one, though?
Theo: You would add 12
Nick: When you're adding 12, you're just going through an entire period, 6 you're going through half a period
Theo: Yes, which wouldn't work because it's a cosine..I, could you add the three? Possibly?
Nick: A fourth of the period?
Theo: Yeah.
Nick: Let's find out! (*typing into calculator*) No, that's not what I wanted..Ipe!... There we go, that's another one.

When Nick asks, "how would you solve for that one, though?" Theo responds with an answer, although it is restating the strategy Nick was employing, adding one period (12) to the first solution. This continues the joint exploration between Nick and Theo as they search for a way to find the remaining two solutions. Theo proposes a strategy of adding 3, which Nick reframes as "a fourth of the period" and proceeds to test out using a calculator. Nick reports back as he continues to use the calculator and eventually seems to have success. Nick and Theo, together take up the epistemic authority shared with them to find the remaining solutions. Theo's offer of a possible strategy and Nick's uptake of this strategy maintain the exploration of the episode, which ends when Nick finds another solution.

Episode II: "Is there a Natural Logarithm that is equivalent to *e*?"

This episode involves a student, James, posing a problem that he created. James calls for the teacher's'attention while she is at his group and asks whether there is a number whose natural logarithm is equivalent to *e* (it is $e^e \approx 15.15$, whose natural logarithm is *e*), which does not appear to be directly from the homework they discussed immediately before.

We consider this episode to be an instance of joint exploration among Mrs. Perry, James and a third student, Sergey, seated beside James. Each participant contributes intellectually (with information) or socially (with directives) to answer the question posed by James. Further, the participants are all not certain of their proposed answers right away. We present three main shifts in epistemic positioning that served to establish and maintain joint exploration below.

Mrs. Perry's initial release of epistemic authority. Mrs. Perry's initial reaction to James' question involves a change in epistemic positioning that serves to establish joint exploration. This shift involved a release of epistemic authority by Mrs. Perry.

James: Is there, I've just been messing around a little bit. Is there a natural log that is equivalent to *e*? (*looking at calculator, then looks up at Mrs. Perry, resting head on hand*) Like 15 point something?

Mrs. Perry: (*slowly*) Is there a natural log that is equivalent to *e*? (*pauses, steps back,*) You mean like, you take the natural log of something and you get *e*? (*looking at James, who nods*) Is that what you mean? (*pauses, puts hand up to mouth and then brings it down*) Ahhh so... (*leans slightly back briefly writing in the air*)

The interaction begins with James positioning Mrs. Perry as an epistemic authority, asking her this question and looking to her for an answer. Notably, James does have an accurate estimate to answer his question when he poses it. Yet, he is not certain and looking to Mrs. Perry as a source of epistemic authority to confirm his hypothesis of the existence of a natural logarithm that is equivalent to *e*. This is followed by Mrs. Perry restating James' question, pausing, and stepping back to lean backward, away from James and Sergey. Mrs. Perry's words and movement seem to suggest that the answer is unknown to Mrs. Perry, in contrast to being positioned with epistemic authority. By pausing to consider the question, Mrs. Perry creates an opportunity for James and Sergey to take up the intellectual authority within the conversation.

James' re-positioning of Sergey and himself as epistemic authorities. As Mrs. Perry shifts away from being an epistemic authority, James and Sergey position each other as epistemic authorities. In response to James' question, Sergey joins the conversation and affirms the existence of such a value, continuing the exploration.

Sergey: I mean, yeah, cuz it'd be a power.
James: of *e*, right? (Mrs. Perry: yeah)
Sergey: Would it just be 1? (*looking at James*)
James: It would be *e* to the power of *e* (*looking at Mrs. Perry*)…
Sergey: So the log, log base *e*
James: Oh, log of *e* to the *e* (*pauses*) is *e*? (*looking at Sergey, laughs*) Wait a second, is that right? Lemme check. (*picks up calculator*)
Mrs. Perry: Well wait, write it down, write it down. I can't think right. I have to see it.

bending down to table, leaning over student work) So,
James: I'm wondering, so I think we just figured out that (*writing*) log base *e*..It would be ln of *e* to the *e*. Okay. So, what's wait. Does that, does that work?
Mrs. Perry: That seems right. Log (*looking at work*)
James: Let's try that so (*typing into calculator*)
Mrs. Perry: Waaait no (*long pause, bends head all the way forward*)

Sergey supports James in exploring his question. He first affirms that such a value would exist, explaining that "it would be a power [of *e*]". Sergey initially offers 1 as a solution, which James quickly disregards. Sergey is undeterred, however, in contributing information. Sergey reminds James that he is interested in the natural logarithm (base *e*), rather than the common logarithm, and reorients James to this in several instances, which James acknowledges and takes up. Notably, James shifts his gaze from Mrs. Perry to Sergey, indicating a shift from positioning Mrs. Perry as an authority to Sergey as supporting his exploration. James begins to talk through his hypothesis that "log of *e* to the *e* is *e*" and uses the calculator to confirm his proposed solution, re-positioning himself as an epistemic authority. Rather than rely on Mrs. Perry for confirmation, James uses a calculator to confirm his solution.

James' and Sergey's epistemic authority begins to equalize. With Mrs. Perry still present, James and Sergey continue to take up the epistemic authority and their authority relative to each other appears to equalize. This can be seen as they reach an answer they are both satisfied with.

James: *e* to the *e* to the 1. Yeah, fifteen point, yeah, there it is. And then log of that is *e*
Sergey: No (James: Just kidding) natural log of that (*looking at James' work*)
James: (*mumbles*) ln. Yeah, my bad.
Sergey: Yeah
James: Yeah

At first, Sergey joins the conversation with lower epistemic authority than James: he offers 1 as a solution, which James disregards. Here, James acknowledges Sergey's contribution about it being the natural log (or log base e) rather than log and readily accepts by saying "yeah, my bad.". Their mutual affirmations of "yeah" at the end of this episode indicate that they are confident in their solution and acknowledge each other's confirmation as well, indicating a similar status of epistemic authority. At the conclusion of the exploration, Sergey and James arrive at a solution they were both satisfied with, without any affirmation from Mrs. Perry.

Comparison of Two Episodes

In both episodes, exploration is initiated by a student asking, or attempting to ask Mrs. Perry a question and request information. In episode one, this is initiated when Ellie asks Mrs. Perry if her work is correct and in episode two, when James poses his question to Mrs. Perry. Mrs. Perry's responses in both, either briefly leaving, or restating the question, are a shift away from her being positioned as the epistemic authority and coincide with the start of joint exploration.

In both episodes, joint exploration is maintained by two shifts in positioning: (1) Mrs. Perry, the teacher, redistributes authority by re-positioning the students as capable of seeking the answer, and (2) the students (Nick and Theo in Episode I and James and Sergey in Episode II) position themselves and each other as epistemic authorities in the situation, as they both exchanged ideas among each other, and took up the ideas offered.

The extent to which Mrs. Perry releases epistemic authority to the group, however, differs between episodes. In episode one, Mrs. Perry supports the students in working through the assigned task by directing Nick to mark his solution at a particular place on the graph and giving the status of the group's work. In doing so, Mrs. Perry shares some of the epistemic authority with the students yet still seems in command of the material as she guides the students toward the next step, implying she holds the solutions to the task. However, in the second episode, Mrs. Perry appears to release her epistemic authority almost entirely after James asks a question to which she does not immediately know the answer. The moment when Mrs. Perry says, "wait no" as she leans her head down, effectively bowing out of the conversation, appears to be when she fully releases her epistemic authority, as James and Sergey are not deterred by her refutation. Rather, they continue toward their solution in spite of her lack of direction or affirmation.

In both episodes, joint exploration ends when a student or students become the ones to determine whether knowledge was correct or incorrect, without confirmation from the teacher. This is particularly notable that students are able to resolve their own uncertainties in the presence of the teacher, yet without relying on the teacher as the source or confirmation of knowledge as valid. Additionally, as students worked to resolve their own uncertainty, students in both episodes utilized calculators as tools to verify their proposed solutions. While the student had to determine what to input into the calculator, and how to interpret the output, the calculator appeared to positioned by students as an authority for validating of knowledge.

Discussion and Conclusion

These episodes are encouraging as they suggest that joint exploration can happen in secondary mathematics classrooms with the teacher present. In fact, our findings suggest that the teacher's actions may be helpful to establish and maintain joint exploration. In both episodes, joint exploration was initiated by students while the teacher was present, with the teacher acting in ways that implicitly decenter herself as the epistemic authority and primary knower. It is not clear that this was the teacher's intended purpose in leaving the group briefly or expressing hesitation in response to James' question. However, it appears to have that result as Ellie redirects her question to Nick. Neither involves explicit or obvious positioning, such as the teacher directing the group investigate a question or asking how the group might figure it out. This suggests that the teacher's role in fostering joint exploration can be subtle. Future research could explore the possibility of whether teacher moves that more obviously and explicitly decenter herself as the authority can also be present when joint exploration is established.

The findings also show the teacher positioning students in more obvious and explicit ways while maintaining joint exploration. For example, the teacher takes stock of what solutions they know, frames the group's work as needing to figure out the other solutions, and restates a student's question with more clarity. These actions do not position students to provide information they already know about the problem in order to answer or solve a question, which would reflect the "knower" position in the literature (Gonzl�ez & DeJarnette, 2015; Lo & Ruef, 2020). Instead, these findings suggest that an additional position of "explorer" or "investigator" that better characterize the positions subtly implied by the teacher, which are taken up by students during the episodes of joint exploration. Future research could investigate whether coding schemes for positions of participants in groupwork (e.g., González & DeJarnette, 2015; Lo & Ruef, 2020) can be extended to distinguish between knowers, actors, and explorers.

We note that both of these episodes seem to be rich instances of productive disciplinary engagement (Engle & Conant, 2002), including mathematical sensemaking, reasoning, and a variety of mathematical practices. Future research could conduct an analysis of students' engagement with mathematical content and practices, and its relationship to positioning, in episodes of joint exploration. As previous research has done in relation to social positioning in collaborative groupwork in mathematics (e.g., Langer-Osuna, 2016), this line of future research could examine how different mathematical engagement may be afforded to different group members, potentially due to how they are positioned by other members in the group.

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