Investigating Growth of Representational Competencies by Knowledge-Component Model

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ABSTRACT

Instruction in many STEM domains heavily relies on visual representations, such as graphs, figures, and diagrams. However, students who lack representational competencies do not benefit from these visual representations. Therefore, students must learn not only content knowledge but also representational competencies. Further, as learning progresses, knowledge likely becomes more abstract, so that content knowledge may no longer be tied to a specific representation. This raises the question of whether students integrate representational competencies with content knowledge as learning progresses. The present study addresses this question by building knowledge-component models using log data collected from two studies in an introductory electrical engineering course. We compared knowledge-component models that separate representational competencies from content knowledge to knowledge-component models that integrate representational competencies with content knowledge. Our results show that as learning progressed, integrated knowledge-component models had better model fit. This finding indicates that over time, students' representational competencies become gradually integrated into content knowledge. Further, this suggests that different knowledge-component models might be needed at different times during a learning progression.

Keywords

Representational competencies, content knowledge, knowledge-component model, intelligent tutoring system

1. INTRODUCTION

The success of adaptive educational technologies depends on analyses of students' knowledge growth during their interaction with problem-solving activities. These analyses equip the educational technology with information about the students' current learning progress to mastery of the targeted knowledge [32] and enables it to provide adaptive feedback or to select appropriate interventions [16] based on the individual student's learning progress [7]. This capability has contributed to the success of adaptive educational technologies [30].

Therefore, much research has investigated how to analyze students' knowledge growth based on log data generated by students' problem-solving interactions in educational technologies. The first step in analyzing knowledge growth is to capture students' knowledge in a way that can then be used to trace their knowledge acquisition over time [4, 7]. Knowledge-component models are a common way of capturing students' knowledge [15]. The basic assumption of

knowledge-component models is that knowledge consists of fine-grained "atom-like" components [15]. Hence, knowledge-component modeling seeks to identify all knowledge components required for mastering the targeted knowledge [13].

Traditional knowledge-component models have focused on capturing content knowledge. However, focusing on only content knowledge may not adequately enable educational technologies to support students' learning, especially in STEM fields. Previous research showed that, students often have difficulties understanding visual representations, while most STEM instruction heavily relies on multiple visual representations [2, 17]. Such struggles can impede their content learning [24]. For example, electrical engineering courses on signal processing frequently use visual representations as shown in Fig. 1 to explain concepts related to sinusoids. While visual representations are often thought to support learning [1], they can impede learning for students who do not know how to interpret the visual representations. For instance, if the students are unfamiliar with time-domain graphs (Fig. 1a) or phasor graphs (Fig. 1b), they may struggle to understand the concept of the sinusoid. This example typifies that many instructional scenarios expect students to have *representational competencies*. Representational competencies are defined as the knowledge and skills that enable students to understand and use visual representations to reason and solve tasks [9].

While most research on knowledge-component models has focused on content knowledge, only a few studies show that capturing representational competencies in addition to content knowledge improves the fit of knowledge-component models [25]. A limitation of these studies is that they have assumed a static structure of knowledge-component models; that is, representational competencies and content knowledge were captured as separate knowledge components, and this did not change over time. However, as learning progresses, students' content knowledge likely becomes more abstract and their use of representational competencies becomes more automated. Thus, the goal of this paper is to address this limitation by comparing knowledge-component models that separate or integrate representational competencies and content knowledge in various ways.

2. LITERATURE REVIEW

In the following, we first review research on representational competencies. Then, we briefly review the few prior studies that have captured representational competencies in knowledge-component models.

2.1 Representational Competencies

The educational psychology has identified several types of representational competencies that enable students to learn content knowledge from visual representations [24].

First, students need *visual-understanding competencies:* the ability to map visual features to relevant to-be-learned content [28]. In the

previously mentioned example of a student learning about sinusoids, visual-understanding competencies allow the student to map a visual feature (e.g., the peak in the time-domain graph) to the concept it depicts (e.g., the amplitude of a sinusoid).

Second, students need *conceptual connection-making competencies:* the ability to conceptually understand similarities and differences between multiple visual representations [1]. This allows students to explain how domain-relevant concepts are depicted in different visual representations [29]. For example, the red arrow in Fig. 1 illustrates how a student should connect the amplitude in the time-domain graph (Fig. 1a) to the phasor's magnitude in the phasor graph (Fig. 1b). Conceptual connection-making competencies also involve identifying visual features that have surface similarities among visual representations but are conceptually irrelevant [11]. For example, a student may notice that both the time-domain graph (Fig. 1a) and the phasor graph (Fig. 1b) have two axes. Yet, the axes represent different concepts: time and amplitude in the time-domain graph, the imaginary and real parts of the phasor in the phasor graph.

Third, students need *perceptual connection-making competencies:* the ability to effortlessly and efficiently translate between multiple visual representations [12]. Students with perceptual connection-making competencies can intuitively translate between two visual representations and quickly judge whether they depict the same concept, without experiencing mental effort when executing this task [12]. For example, a perceptually proficient student would see "at a glance" the phasor graph in Fig. 1b represents the amplitude of the sinusoid in Fig. 1a.
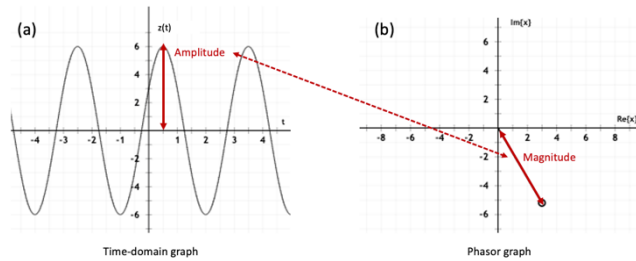

Figure 1. Mapping time-domain graph amplitude (a) to phasor graph magnitude (b).

2.2 Knowledge-Component Model

Adaptive educational technologies require information about the students' learning progress in order to provide individualized support. [30]. They must describe the knowledge students have already learned and what knowledge they still have to learn [6]. Furthermore, adaptive educational technologies rely on algorithms that predict which types of support (e.g., contextual feedback, choice of problem-solving activities) would help the student acquire the knowledge s/he has not yet learned [21].

Educational technologies rely on knowledge-component models to trace students' knowledge acquisition and to predict the growth of students' knowledge. Knowledge-component models represent "acquired units of cognitive function that can be inferred from performance on a set of related tasks" [15]. Here, knowledge components refer to a unit of distinct skills or concepts, which together compose the knowledge students learn in problem-solving activities. Therefore, the accuracy of a knowledge-component model depends on identifying all relevant knowledge components that describe the targeted knowledge [13, 16].

Cognitive Task Analysis (CTA) is one prominent method to describe the requisite knowledge components to perform a task [33]. However, since CTA relies on a thorough analysis of how experts solve tasks, it is time consuming. To increase the efficiency of knowledge-component modeling, educational data mining techniques can be used to automate the process of building models, including learning factors analysis (LFA) [4], Knowledge Spaces [31], and matrix factorization [8]. Typically, multiple potential models are compared using Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) scores [14]. However, since this research has focused mostly on modeling content knowledge, the interplay between representational competencies and content-focused knowledge components remains unexamined.

One study compared knowledge-component models that captured (1) only content knowledge and (2) content knowledge and representational competencies. Capturing both content knowledge and representational competencies resulted in better model fit [25]. A follow-up study tested whether adapting instructional support to students' representational competencies in addition to content knowledge resulted in higher learning outcomes than adapting only to content knowledge [23]. Results showed that adapting to students' representational competencies in addition to content knowledge enhanced students' learning of content knowledge.

However, this prior research is limited in two ways. First, while most prior research focuses on modeling content knowledge [18, 20], the few studies that have also modeled representational competencies [23, 25] have focused on a particular domain; namely chemistry. Therefore, we seek to replicate these findings in another domain. A second limitation is that the prior studies assumed that the structure of the knowledge-component model remains static over time. Yet, according to expert-novice research, students gradually acquire highly abstract schemas about the content knowledge relevant to a given domain [5, 10]. This yields the hypothesis that capturing representational competencies separately from content knowledge is most important early in a learning progression whereas later in a learning progression, representational competencies likely become integrated with content knowledge.

3. HYPOTHESES

To address the limitations of prior research just described, we test:

Hypothesis 1: A knowledge-component model that captures representational competencies and content knowledge is more accurate than a knowledge-component model that captures only content knowledge or a knowledge-component model that captures only representational competencies.

Hypothesis 2. As students' learning progresses, a knowledge-component model that integrates content knowledge and representational competencies is more accurate than a knowledge-component model that captures content knowledge separately from representational competencies.

4. DATASETS

To test these hypotheses, we use log data generated from students' problem-solving interactions in Signals Tutor, an intelligent tutoring system (ITS) for undergraduate electrical engineering. In the following, we first describe the problem-solving activities in Signals Tutor, and then the log data we used to test our hypotheses.
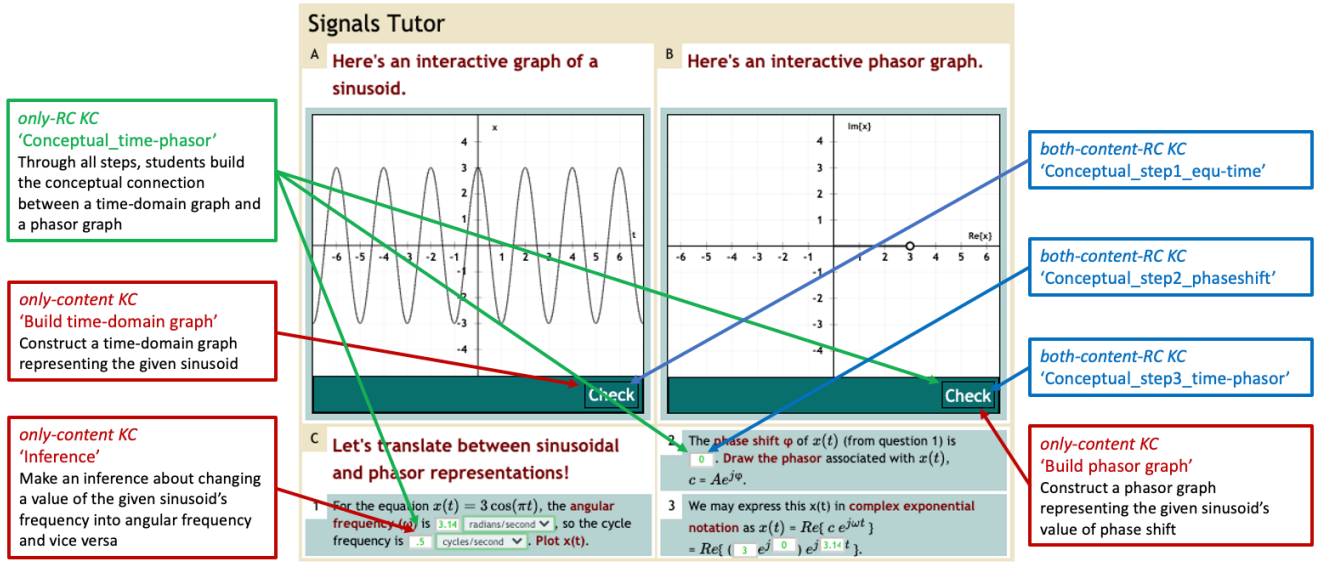
Signals Tutor

only-RC KC
'Conceptual_time-phasor'
Through all steps, students build the conceptual connection between a time-domain graph and a phasor graph

A **Here's an interactive graph of a sinusoid.**

B **Here's an interactive phasor graph.**

both-content-RC KC
'Conceptual_step1_equ-time'

only-content KC
'Build time-domain graph'
Construct a time-domain graph representing the given sinusoid

both-content-RC KC
'Conceptual_step2_phaseshift'

both-content-RC KC
'Conceptual_step3_time-phasor'

only-content KC
'Inference'
Make an inference about changing a value of the given sinusoid's frequency into angular frequency and vice versa

C **Let's translate between sinusoidal and phasor representations!**

1 For the equation $x(t) = 3\cos(\pi t)$, the angular frequency (ω) is 3.14 radians/second ▾, so the cycle frequency is .5 cycles/second ▾. Plot x(t).

2. The phase shift φ of $x(t)$ (from question 1) is 0. Draw the phasor associated with $x(t)$, $c = Ae^{j\varphi}$

3 We may express this x(t) in complex exponential notation as $x(t) = Re\{ c\, e^{j\omega t} \}$ = $Re\{ (3\ e^{j\ 0})e^{j\ 3.14}\ t \}$.

only-content KC
'Build phasor graph'
Construct a phasor graph representing the given sinusoid's value of phase shift

Figure 2 Example of a problem in Signals Tutor and steps that labelled with different knowledge components for hypothesis 1

4.1 Signals Tutor

Signals Tutor supports learning through problem solving [22, 27, 30]. As is typical for ITSs, Signals Tutor provides step-by-step guidance for complex problem-solving tasks [30], detects multiple possible solution paths, provides personalized feedback that addresses diagnosed misconceptions, and on-demand hints for each step. As illustrated in Fig. 2, students work with interactive visual representations to visually depict concepts related to sinusoids.

Because Signals Tutor incorporates multiple visual representations, it offers opportunities for students to practice representational competencies. Specifically, students practice *visual-understanding* competencies when they interact with one visual to make sense of sinusoid concepts. Students practice *conceptual connection-making* competencies when they have to integrate information from multiple visual representations to understand sinusoid concepts. They practice *perceptual connection-making* competencies when a task requires translating quickly among multiple visual representations to extract relevant information about sinusoids.

4.2 Log Data

We collected log data from two studies that were conducted as part of an introductory electrical engineering course on signal processing at a university in the Midwestern U.S. Study 1 was conducted in Fall 2020; Study 2 was conducted in Spring 2021. The course was taught online during both semesters. Students used Signals Tutor as part of the course for a study described elsewhere [26]. The present paper focuses on log data generated from students' interactions with two units of Signals Tutor that provided opportunities to practice the representational competencies described above. Specifically, we extracted 84,960 transactions generated by 136 students from Study 1 log data and 76,786 transactions generated by 145 students from Study 2. These transactions involved problem-solving steps where students constructed visual representations and equations and answered conceptual questions by selecting answers from drop-down menus or via text input.

5. ANALYSIS

To test hypothesis 1, we created knowledge-component models with and without representational competencies. To test hypothesis 2, we created several knowledge-component models that captures the integration of content knowledge and representational competencies. We compared the fit of each model in the earlier vs. the later unit of Signals Tutor.

5.1 Knowledge-Component Models with and without Representational Competencies

To test hypothesis 1, we created three knowledge-component models: (1) the *only-content-KC model* captures only content knowledge, (2) the *only-RC-KC model* captures only representational competencies, and (3) the *both-content-and-RC-KC model* captures both content knowledge and representational competencies. The knowledge components captured by each model were derived from manual cognitive task analysis relying on expert content knowledge provided by an engineering professor who taught the course for more than 30 years/decades.

First, the *only-content-KC model* contains 9 *only-content knowledge components* that describe concepts and skills irrespective of the representational competencies, listed in Table 1. For example, the knowledge component 'Inference' in Table 1 refers students' ability to make an inference about changing a value of the given sinusoid's frequency into the angular frequency.

Second, the *only-RC-KC model* captures only representational competencies but not content knowledge. It contains 11 knowledge components, listed in Table 1. As mentioned above, Signals Tutor offers opportunities to practice three types of representational competencies. The *only-RC-KC model* describes the competencies students need to understand the visual representations used in the problems; for example, conceptual connection-making competencies (e.g., 'Conceptual_time-phasor') and perceptual connection-making competencies (e.g., 'Perceptual_time-phasor') related to translating a time-domain graph to a phasor graph.

Finally, the *both-content-and-RC-KC model* captures both content knowledge and representational competencies. It contains 42 knowledge components, listed in Table 1. To develop content knowledge of translating a time-domain to a phasor graph, students practice competencies for making both conceptual and perceptual connections among visuals. For example, consider the following steps that provided practice opportunities for conceptual connection-making competencies. Students first built a time-domain graph

Table 1 Examples of knowledge components in Signals Tutor (For the value in [ ], students type in their answer).

Knowledge Components	Examples of knowledge components	Description	Example
only-content KC	Inference	Make inferences about changing the value of frequency to angular frequency or vice versa.	$x(t)$ is has an angular frequency ω of $\frac{\pi}{2}$ in radians/sec. The frequency in cycles/sec is [.25].
	Planning	Plan how to represent the sinusoid using different type of visual representation.	The complex amplitude can be represented visually by a [phasor].
	Build phasor graph	Construct a phasor graph of a given sinusoid	Plot the phasor corresponds to $x(t)$.
	Build time-domain graph	Construct a time-domain graph of a given sinusoid	Plot this sinusoid on the given time-domain graph.
only-RC KC	Conceptual_time-phasor	Make sense of how a time-domain visual correspond to a given phase-domain visual	The given graph shows a sinusoid $x(t)$. Plot the phasor corresponding to $x(t)$.
	Perceptual_phasor-time	Translate a phase-domain visual to a time-domain visual	Here's a phasor. Which cosine function represents that phasor?
both-content-and-RC KC	Conceptual_step1_equ_time	Given a cosine function, build a time-domain graph representing the given time-domain equation	For the equation $x(t) = 3\cos(\pi t)$, plot $x(t)$.
	Conceptual_step2_phaseshift	After building a time-domain graph in step 1, identify its value of phase shift	The phase shift φ of $x(t)$ is [0].
	Conceptual_step3_time-phasor	Based on identified information in step 2, translate a time-domain graph to a phasor graph	Draw the phasor associated with $x(t)$, $c = Ae^{j\varphi}$.
	Conceptual_step4_phasor-exp	After building a phasor graph in step 3, write corresponding complex exponential notation.	We may express $x(t)$ in complex exponential notation as $Re\{[3]\exp(j[0])\exp(j[3.14]t)\}$
	Perceptual_time-phasor (clockwise)	Translate a time-domain graph to a phasor graph (rotated in the clockwise direction)	Here's a sinusoid (cosine function). Which phasor represents that sinusoid?
	Perceptual_time-phasor (counter-clock)	Translate a time-domain graph to a phasor graph (rotated in the counter-clockwise direction)	Here's a sinusoid (cosine function). Which phasor represents that sinusoid?

representing a given sinusoid represented in the equation form (e.g., 'Conceptual_step1_equ-time' KC in Table 1). The next step is to find the value of phase shift (e.g., 'Conceptual_step2_phaseshift'), which is basis for building a corresponding phasor graph in the next step. In the third step, students translate the time-domain graph to a phasor graph (e.g., 'Conceptual_step3_time-phasor').

Hypothesis 1 predicts that the *both-content-and-RC-KC model* has better model fit than the other knowledge-component models.

5.2 Knowledge-Component Models with and without abstracted Knowledge Components

To test hypothesis 2, we created two knowledge-component models: (1) the *separate-RC-KC model* describes the knowledge structure of students starting at the novice-level (2) the *integrated-RC-KC model* describes the knowledge structure of students reaching to the expert-level through working on Signals Tutor, detailed in the following.

First, based on learning gains we observed between units [26], we assumed that structural changes in knowledge components would occur between units 1 and 2. Therefore, we chose to investigate knowledge components that were common to units 1 and 2 and examined how these knowledge components changed after finishing each unit. Thus, starting with the list of knowledge components in the *both-content-and-RC-KC model* described in 5.1, we identified 33 knowledge components that unit 1 and unit 2 had in common.

This yielded the *separate-RC-KC model*. Given that this knowledge-component model separately captures content knowledge and representational competencies, we anticipate that this might capture the knowledge structure of novice students in the first unit.

Second, based on previous studies' finding that sufficient training makes students' knowledge become abstracted [5, 10] or abstracted away from the type of representational competencies [25], we assumed that students' knowledge components started to be integrated into content knowledge after finishing unit 1. Thus, we identified lists of knowledge components from the *both-content-and-RC-KC model* that describe similar content knowledge. For example, in Signals Tutor, students learn about concepts related to the phase shift of sinusoids, which we classify as content knowledge. Depending on how students interact with the visual representations that depict these concepts, they practice different representational competencies. The *both-content-and-RC-KC model* contains six knowledge components related to phase shift, illustrated in Table 1: three knowledge components describe students' ability to conceptually connect a sinusoid's shifted amount and direction shown in a time-domain graph to a phasor's rotational direction and amount shown in a phasor graph (e.g., 'Conceptual_step1_equ_time', 'Conceptual_step2_phaseshift', 'Conceptual_step3_time-phasor'), and two knowledge components describe students' ability to make perceptual connections between a time-domain graph and a phasor graph by treating the representations holistically (e.g., 'Perceptual_time-phasor (0)',

'Perceptual_time-phasor (clockwise)', 'Perceptual_time-phasor (counter-clock)').

If these six representational competencies become more integrated with content knowledge about phase shift after practice (hypothesis 2), separating these representational competencies from content knowledge may no longer adequately describe students' abstracted knowledge structure. To capture this more abstracted understanding, we built new knowledge-component models that contained merged knowledge components. Starting with the knowledge components from the *separate-RC-KC model*, we first merged knowledge components that covered similar content knowledge. For example, 'Conceptual_step1_equ_time', 'Conceptual_step2_phaseshift', 'Conceptual_step3_time-phasor' and 'Perceptual_time-phasor (clockwise)' were merged into the 'Build phasor graph' knowledge component, which is one of the knowledge components from the *only-content KC model*. We then tested whether merging these knowledge components improved model fit using AIC and BIC scores. If it did, we kept the merged knowledge component; if it did not, we kept the separated knowledge components. We repeated these steps until there were no more opportunities to merge knowledge components. We carried out these steps separately for the data from Studies 1 and 2, yielding *integrated-RC-KC model-1* and *integrated-RC-KC model-2* as shown in shown in Table 2.

Hypothesis 2 predicts that the *integrated-RC-KC model-1* and the *integrated-RC-KC model-2* have a better model fit compared to the *separate-RC-KC model* in unit 2. Similarly, since students are novice at the beginning stage, hypothesis 2 predicts that the *separate-RC-KC model* shows better model fit compared to the *integrated-RC-KC model-1* and the *integrated-RC-KC model-2* in unit 1.

Table 2. Separate and integrated knowledge components.

Separate KCs in unit 1	Integrated KCs in unit 2
Conceptual_step3_time-phasor	Build phasor graph (*integrated-RC-KC model-1*)
Perceptual_time-phasor (0)	
Perceptual_time-phasor (clockwise)	
Perceptual_time-phasor (counter-clockwise)	
Perceptual_phasor-exp	Write complex exponential notation (*integrated-RC-KC model-1*)
Conceptual_step4_phasor-exp	
Perceptual_phasor-cartesian	Write cartesian form (*integrated-RC-KC model-2*)
Individual_phasor-cartesian (real part)	
Individual_phasor-cartesian (imaginary part)	

6.   RESULTS

To test hypothesis 1, we compared the model fit of the *only-content-KC model*, *only-RC-KC model*, and *both-content-and-RC-KC model* using data from Studies 1 and 2. Table 3 shows that the *both-content-and-RC-KC model* has a better model fit than the *only-content-KC model* and the *only-RC-KC model* in Study 1 and Study 2. For AIC scores, the lower AIC indicate a better-fit-model, and more than -2 is considered significantly better than model it is being compared [3]. Similarly, a decrement greater than 10 indicates very

strong evidence in terms of BIC [19]. Lower values of RMSE also indicate better fit. These results support hypothesis 1.

To address hypothesis 2, we compared the model fit of the *separate-RC-KC model,* the *integrated-RC-KC model-1*, and the *integrated-RC-KC model-2* as shown in Table 4. For Study 1, results show that in unit 1, the *separate-RC-KC model* shows the better fit than the *integrated-RC-KC model*s. By contrast, in unit 2, the *integrated-RC-KC model-1* shows better model fit than the *separate-RC-KC model* in terms of AIC and RMSE (but based on BIC the *integrated-RC-KC model-2* shows the best model fit). This supports hypothesis 2.

For Study 2, we found that in unit 1, the *separate-RC-KC model* shows a better model fit than the *integrated-RC-KC model*s in terms of AIC as shown in Table 5. However, the BIC and RMSE scores indicated that the *integrated-RC-KC model-1* had the best model fit. By contrast, in unit 2, the *integrated-RC-KC model-2* shows the best model fit. This result supports hypothesis 2 in terms of AIC scores. For BIC and RMSE scores, the results partially support hypothesis 2, because the *integrated-RC-KC model-1* shows the better model fit than the *integrated-RC-KC model-2*.

Table 3. Test of hypothesis 1: Model accuracy for study 1 and study 2. Bold stands for the best fit.

Study	KC Model	# of KC	AIC	BIC	RMSE
1	***both-content-and-RC***	**42**	**22,123**	**24,221**	**0.4422**
	only-RC	34	22,736	24,709	0.4456
	only-content	9	23,853	25,446	0.4537
2	***both-content-and-RC***	**42**	**18,559**	**20,358**	**0.4370**
	only-RC	34	19,120	20,796	0.4421
	only-content	9	20,133	21,418	0.4544

Table 4. Test of hypothesis 2: Model fit by unit (Study1). Bold stands for the best fit.

Unit	RC-KC Model	# of KC	AIC	BIC	RMSE
1	***separate-RC-KC model***	**33**	**9,733**	**11,488**	**0.4367**
	integrated-RC-KC model-1	30	9,779	11,491	0.4368
	integrated-RC-KC model-2	27	9,803	11,492	0.4374
2	*separate-RC-KC model*	33	10,387	12,133	0.4414
	integrated-RC-KC model-1	30	**10,372**	12,075	**0.4387**
	integrated-RC-KC model-2	27	10,396	**12,054**	0.4399

Table 5. Test of hypothesis 2: Model fit by unit (Study 2). Bold stands for the best fit.

Unit	RC-KC Model	# of KC	AIC	BIC	RMSE
1	separate-RC-KC model	33	**8,134**	9,587	0.4445
	integrated-RC-KC model-1	30	8,156	**9,567**	**0.4431**
	integrated-RC-KC model-2	27	8,201	9,570	0.4461
2	separate-RC-KC model	33	9,095	10,598	0.4278
	integrated-RC-KC model-1	30	9,085	10,546	0.4263
	integrated-RC-KC model-2	**27**	**9,080**	**10,498**	**0.4264**

7. DISCUSSION

Our results show that the knowledge-component model incorporating both representational competencies and content knowledge had the best model fit (hypothesis 1). This result aligns with findings from a previous study that investigated whether a knowledge-component model should incorporate representational competencies using chemistry students' learning data. Our results replicate this finding in the electrical engineering domain.

Further, we found that students' representational competencies become abstracted and integrated with content knowledge as students practice representational competencies (hypothesis 2). This shows that as students' learning progress, their representational competencies are gradually merged with content knowledge. We note that the time at which each representational competency is integrated into the content knowledge may not be uniform. For instance, while students' representational competency of translating a time-domain graph to a phasor graph were merged with content knowledge in unit 2, the representational competency of translating a phasor graph to a time-domain graph remained separate from content knowledge. It is possible that after more practice, the latter representational competency would also merge with content knowledge.

Additionally, our results indirectly suggest that students' timelines may differ depending on their learning rates. Specifically, we found differences between the Study 1 and Study 2 cohorts. The fact that BIC, AIC, RMSE disagreed as to whether the *integrated-RC-KC model-1* or the *separate-RC-KC model* had a better model fit for unit 1suggests that students in Study 2 started with somewhat more integrated knowledge and ended with more integrated knowledge compared to students in Study 1. We do not want to speculate what might have caused these cohort effects because there are numerous possible reasons, but it suggests that students may start and end at different points on a separate-to-integrated knowledge trajectory.

Our study makes novel contributions to the field of educational data mining because it is, to our knowledge, the first study capturing dynamic development of students' representational competencies using knowledge-component models. Knowledge-component modeling allowed us to identify dynamic, developmental patterns of representational competencies and to show that they are not static. Further, our finding that knowledge-component models should incorporate representational competencies in addition to content knowledge expands the search space for knowledge-component models in future work.

Our findings also have important implications for the design of adaptive educational technologies. First, technologies that use visual representations should trace students' acquisition of representational competencies in addition to content knowledge. Doing so is particularly important at the beginning of a learning sequence. When students first learn a new concept with visual materials, instructional supports should be designed with consideration of students' representational competencies and content knowledge. Second, as students' representational competencies change dynamically through practice, the educational technology may no longer need to monitor representational competencies separately from content knowledge. This, however, may need to be adapted to the rate at which students learn specific representational competencies and content knowledge.

8. LIMITATIONS & FUTURE WORK

Our results should be interpreted considering the following limitations. First, we collected log data from students working on Signals Tutor in the context of online learning. Online learning differs from in-person learning in multiple ways. Therefore, future studies should replicate our results in the context of in-person learning. Second, our experiment was constructed in a specific electrical engineering course. Even though visual representations are commonly used in many STEM instructions, representational competencies are domain-specific and highly dependent on the particular content knowledge covered. Thus, future research needs to test whether our results generalize to other STEM domains and topics. Finally, open questions remain about the suitable length of a learning intervention to ensure that all representational competencies become integrated with content knowledge. Although this study found that the students' representational competencies are gradually integrated with content knowledge, it did not examine when each representational competency becomes fully integrated into the content knowledge. To address this limitation, a longer intervention is needed. Such research could help establish the length of learning trajectories that relate to representational competencies.

9. CONCLUSION

The present paper shows that the structure of students' knowledge changes over time. Consequently, different knowledge-component models are best suited at different times during a learning trajectory. While modeling representational competencies is important, representational competencies become integrated with content knowledge with practice. Thus, educational technologies should employ dynamic knowledge-component models that capture representational competencies separately from content knowledge at the beginning of a learning trajectory while merging them with content knowledge later in a learning sequence. The way in which these knowledge components are merged may depend on the student's learning rate. Given that prior research shows that adapting instructional support to students' representational competencies can significantly enhance their learning of content knowledge and given the prevalence of visual representations in STEM instruction, our study may have considerable impact on the effectiveness of educational technologies.

10. ACKNOWLEDGMENTS

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