Conceptions of active learning held by undergraduate mathematics instructors participating in a statewide faculty development project

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We explore the perceptions about "active learning" among college and university mathematics faculty involved in early stages of the Mathematical Inquiry Project (MIP), which supports long-term collaboration across mathematics departments at the 27 public institutions of higher education in the state of Oklahoma. Our analysis indicates that faculty beliefs about active learning varied widely across individuals and significantly differed from the MIP characterization, even though participants believed their conceptions to be aligned. We document changes in participants' beliefs as a result of participation in the MIP that faculty attributed to engagement in rich mathematical tasks, conversations with other participants, small group discussions of research literature, and conversations with project team members. Participants also reported enacting their conceptions of active learning in their classrooms more often as a result of their involvement in the professional development.

Keywords: Community of Practice, Inquiry-Based Learning, Entry-Level Math, Active Learning, Professional Development

Introduction and Background

The Mathematical Inquiry Project (MIP) is a statewide collaboration among mathematics departments at the 27 public institutions of higher education in Oklahoma to foster sustainable, large-scale reforms to improve instruction in entry-level mathematics courses. To promote awareness of and attention to the mathematical, epistemological, and affective considerations in instructional design, the MIP is guided by definition of *mathematical learning through inquiry* that entails three interdependent components: (a) engaging students in *active* learning, (2) incorporating *meaningful applications*, and (3) supporting students' development of broader *academic success skills*. These components are defined as follows:

Students engage in active learning when they work to solve a problem whose resolution requires them to select, perform, and evaluate actions whose structures are equivalent to the structures of the concepts to be learned.

Applications are meaningfully incorporated in a mathematics class to the extent that they support students in identifying mathematical relationships, making and justifying claims, and generalizing across contexts to extract common mathematical structure.

Academic success skills foster students' construction of their identity as learners in ways that enable productive engagement in their education and the associated academic community.

The MIP aims to foster instructors' professional growth by fostering a *community of practice* (Wenger, 1998) in which participants engage in a joint enterprise to design, disseminate, and implement instructional resources, as well as develop as leaders within the community's emerging view of expertise. In this report, we focus on faculty perceptions of *active learning*.

In the summers of 2019-2021, the MIP led five multi-day initiation workshops during which Oklahoma mathematics faculty identified key priorities for courses in the four state pathways—

functions and modeling, college algebra/precalculus, quantitative reasoning, and Calculus I—as well as on academic success skills across all courses. Participants engaged in readings, presentations, and small- and whole-group discussions about the three components of inquiry, their dependence on conceptual analyses (Thompson, 2008), and their implications for constructing hypothetical learning trajectories (Simon & Tzur, 2004). The purpose of the initiation workshops was threefold: (a) to initiate a statewide community of practice, (b) to build the capacity for faculty to design and implement instructional materials that support learning through inquiry, and (c) to identify the key conceptual threads in these entry-level courses for future instructional design collaborations. Following the workshops, the MIP participants were encouraged to join Collaborative Research and Development Teams (CoRDs) comprised of groups of 2-5 faculty tasked with developing, testing, and refining an instructional module related to one of the conceptual threads identified in an initiation workshop. Later stages of the MIP will involve broadening the community of practice through disseminating resources via the MIP website, regional workshops, and peer mentoring.

Research on faculty professional development highlights that change strategies should seek to alter individual's beliefs as opposed to enacting top-down policy to impact teaching or disseminating "effective" curricular resources (Henderson et al., 2011). This demonstrates the importance of characterizing faculty's conceptions of mathematical learning through inquiry and describing how particular professional development experiences contributed to their evolution. Furthermore, by adopting communities of practice as a model of faculty change, the project forgoes attempting to directly impact participants' conceptions of inquiry-oriented mathematics instruction, leaving that to emerge as part of the community's shared priorities. As such, we sought to evaluate participants' conceptions of active learning after having participated in one or more of the MIP initiation workshops. Specifically, we were interested in the extent to which participants' conceptions of active learning aligned with the MIP's definition and what project activities facilitated any changes in these conceptions. We consider the following research questions:

1. What are participants' conceptions of active learning, and to what extent do they align with the MIP's definition of active learning?
2. What aspects of the MIP influenced participants' conceptions of active learning?

We expect our exploration of these research questions to inform how we might operationalize the general mechanisms of individual learning through social engagement articulated by Wenger (1998) to influence the conceptions of active learning held by mathematics faculty at different stages of participation in a statewide professional development project.

Theoretical Framing

The MIP seeks to effect changes in the cultural practices of mathematics instructors by cultivating a community of practice that enables professional growth through individual participation. A community of practice is a social entity in which individuals negotiate meaning through their mutual engagement in a joint enterprise around a shared repertoire of reified artifacts (Wenger, 1998). Each stage of the MIP seeks to engage mathematics faculty in experiences that require their negotiation of meaning around the MIP's three components of inquiry.

The purpose of this research study is to explore how individuals experience their involvement in the MIP community of practice as they negotiate meaning through their engagement with other members and through their interaction with the community's established set of reified artifacts. Through an individual's interactions with other members of the community and its reified artifacts, they become increasingly cognizant of the knowledge base and skillset required to participate competently within the community. A central goal of the MIP is to cultivate a community of mathematics faculty that, through their participation in the community's activities, negotiate a notion of competence reflecting the three components of the MIP's definition of mathematical learning through inquiry. Characterizing participants' initial and developing conceptions of the three components of inquiry is essential to this effort as these characterizations can inform the MIP Team's participation in the community and allow for the strategic introduction of reified artifacts into the community's activities.

Methods

We conducted semi-structured interviews over Zoom with 15 MIP participants in spring 2021. The interviews were audio recorded and transcribed for use in analysis. The interview questions included the following:

1. Please describe your image of active learning in entry-level college math courses.
   a. Why is this important for entry-level math courses?
   b. Can you describe a specific example of active learning in an entry-level math course, yours or someone else's?
   c. What made this example effective? What could have been better?
   d. Has your participation in the MIP activities changed your thinking about active learning? How?
2. Here is the MIP's definition of active learning. [Participants were presented with the definition].
   a. Are there parts of this that you think are important but haven't discussed yet?
   b. Do you particularly agree or disagree with emphasizing any aspect of the MIP definition for improving instruction in entry-level college mathematics?

We employed the constant comparative method (Strauss & Corbin, 1994) to identify themes in the data. One author read all the transcripts, highlighting words and phrases that characterized participants' images of active learning. When a new word or phrase was added to the list, the author reread all previous transcripts seeking instances of that word or phrase. This generated a list of 30 items. The author then grouped similar items into themes and described them using the words/phrases from the list (Table 1), resulting in a list of open codes. The author then re-coded the transcripts using the working descriptions of the codes, and then refined these descriptions until they captured all highlighted words from the first reading. We note that that these codes are not mutually exclusive; for instance, if a participant discussed motivating students with real-world examples, that was coded as both 'real world examples' and 'affective.' Similarly, a participant suggesting enacting active learning with a class discussion was coded both a 'format in which learning would take place' because it described the plan for the lesson, and as 'nature of student engagement' because students would be interacting with other people.

Results

Our analysis suggests that participants' conceptions of active learning focused on three major themes: the class setup, the mathematical content of a lesson, and the affective facets and benefits of active learning.

Table 1. Emergent themes

Theme	Sub-theme(s)	Description
Class setup	Nature of student engagement	Participant describes a way students might be involved, such as answering questions, interacting with other students, a class discussion, exploring, group work, students giving presentations
	Format in which the learning will take place	The participant describes something that could be thought of as lesson planning, like group work, class discussion, guided work, scaffolding information, using manipulatives, using Desmos or a graphing calculator, doing projects
Content	Problem solving	Participant mentions “problems” or “problem solving” without explicitly identifying that it is a novel task
	Problematic situations	Participant mentions engaging students in a productive struggle and/or in situations that are problems (not exercises), or describes students selecting, applying, and evaluating tools or actions
	Understanding	Participant describes that active learning should help students understand or know rather than memorize
	Real-world examples	Participant mentions that active learning should entail real world examples
Affective	n/a	Participant describes active learning in terms of its implications for, or dependence on, students’ interests, motivation, perseverance, mindset, anxiety, etc.

Regarding class setup, all participants mentioned that they associated active learning with particular instructional formats, such as collaborative group work or a class discussion. This demonstrated participants’ attention to ways students might interact with each other or with mathematical tools (e.g., graphing calculator or dynamic visualization software) and the class structure (e.g., scaffolding information instead of lecturing, making a class like a lab or workshop). Jack, who attended one workshop and was on one CoRD, described his class:

> I incorporate a lot of collaborative project learning.... I like to use a lot of manipulatives. I have a limit of how much I want to actually speak to them in a formal setting and having

them actually do things while I'm there as a mentor is a lot more helpful ... the more I can make my class like a lab, the better I am at really letting active learning [happen].

Most participants provided both examples of active learning that foregrounded the structure of students' mathematical activity and its relation to their conceptual development and an example that foregrounded students' participation without attention to how the activity might support students' construction of particular mathematical meanings. For example, Reagan, who had attended two workshops and was on a CoRD, said in response to interview Question 1b,

In the college algebra class when we talk about the function rate of change and like a main concept at the beginning.... I designed the pre class activity for students to work on [review] problems.... I also let a student to come up with examples. So I give problems, I give applications and let them come up with what kind of additional related example in the real life they can come up with.... so like in the rate of change when we talk about constant rate, normally we start with talking about the distance and the time.... some of students will come up with... go to the grocery store and you buy the grocery and it to sell by the pound, what is the unit price? That is also a constant rate.

Reagan did not connect her example of students working on review to a specific conception of rate of change that she intends students to construct. We consider this portion of Reagan's statement to be more aligned with a colloquial meaning of active learning because, although she referenced a mathematical topic (rate of change), she did not specifically attend to how the problems in question might elicit actions that reflect the multiplicative structure of a function's constant (if linear) or average rate of change. Instead, Reagan considered the example she proposed to be an instance of active learning because students were working on problems (instead of observing her work on these problems in lecture). On the other hand, she related students production of an example of average rate of change to the underlying structure of corresponding changes in quantities' measures such as changes in "distance and time" and the proportional relationship between a grocery item and its weight and cost. Her attention to aspects of the specific conceptual structure of rate of change in this activity indicates the potential for the students to select actions whose structures are equivalent to the concept to be learned.

All participants acknowledged the affective requirements and affordances of engaging in active learning. Actively engaging in meaningful applications of mathematics both requires and fosters academic success skills (e.g. mathematics self-efficacy, growth mindsets, persistence in problem solving). Adam and Eden's comments are representative of those coded under this theme. Adam had worked on a CoRD, and Eden had attended three workshops and was participating on a CoRD at the time of the interview.

Adam: I think [active learning] is important [because] students who engage with math tend to have better perception of it. ... I think it allows the students to gain a sense of autonomy and, um, confidence in math that they may not be accustomed to.

Eden: it's self-efficacy, the whole thing and that goes into the academic success skills, but I mean it's building, especially for students that are going to need to take math past that entry level, it's, you're, you're, you're creating if you like your own machine, you're, you're starting at the entry level and getting them used to this idea. So, as they progress through their math courses, they will be more successful.

Both Adam and Eden discuss the affective benefits of active learning: Adam's response focused on students' developing a sense of autonomy, potentially increasing their mathematical confidence, and Eden's remarks highlight the importance of mathematical persistence.

Affective affordances and requirements of active learning were not part of the MIP's definition of active learning, but do appear in the MIP definition of academic success skills. The MIP three components of inquiry are interdependent, and we agree that active learning can both require particular affective states (e.g., a willingness to engage in productive struggle) and afford particular affective states (e.g., foster increased willingness to engage in productive struggle). The MIP had explicitly stated affective affordances and requirements only in the definition of academic success skills. That participant talked about affective affordances and requirements in active learning indicates negotiating meaning in the community of practice. Specifically, participants seemed to favor a definition of active learning that incorporates the affective affordances and requirements of active learning explicitly.

The participants also reported that various aspects of their engagement in the MIP influenced their conceptions of active learning. Generally, participants cited specific examples of rich tasks, conversations with other participants, small group discussions of research literature, and conversations with project team members. For example, Ellison, who attended one workshop and was on a CoRD, felt the problem-solving literature she read for her CoRD had influenced her thinking that an important part of active learning was not to give the answer too soon. Gemma, who attended one workshop, said

> I really liked those types of problems that we did as samples... something that gets you to think outside of the box on math and not have to be like... sitting there doing x's and y's and whatever. Thinking about real life and how can I connect this and then you know problem solving

The sample problem Gemma referred to were generated by the MIP team. The goal of the problems was to model a conceptual analysis, hypothetical learning trajectory, and how those informed the design of tasks that operationalize the three MIP components of mathematical inquiry. We take Gemma's statement about the problems making students to "think outside of the box" and "problem solving" as possibly indicative that she attended to the tasks as problematic situations.

Discussion

We developed the MIP definition of active learning to foreground the implications of the nature of students' activity for their construction of particular mathematical meanings and to serve as a guide for the project design of engaging faculty in a community of practice. While we expect to see some change of participant's goals, values, and beliefs that might make aspects of the MIP definition more meaningful to them, we equally expect the community to develop its own priorities and standards. We present our characterization of participants' goals, values, and beliefs about active learning to inform subsequent project activities in ways that will better support participants to (a) understand the nature of their conceptual learning goals; (b) act in ways that foster those goals in their instruction; and (c) reinforce the development of the community of practice toward similar sensitivities.

While participants universally described general formats in which students might be participate in class (e.g., group work, class discussions, and projects), they often did not attend to the nature of students' engagement with carefully designed mathematical tasks that deliberately support abstraction of underlying mathematical structure. We note that many of these perceptions were internally consistent, based on individualized implicit learning theories and prior experience, and thus highly stable. However, it is important to the PD project to know that participants' definitions of active learning did not explicitly attend to students' selecting, performing, and evaluating specific mental actions, because we view the 'active' in 'active learning' as *mental activity*, and hence propose that focusing on the nature of the mathematical tasks (as opposed to whether they will be done in groups, or in back-and-forth question-and-answers between the class and instructor) is critical in effective instruction. In short, there is some misalignment with participants' definitions and the MIPs in regard to how each views what 'active' means in active learning.

The organic evolution of a normative conception of competence within a community of practice—which reflects the current and developing conceptions of those participating in it—restricts the range of possible interventions that seek to influence how participants conceptualize both the practice in which they engage and the nature of competence this engagement requires. Participation in the community is the mechanism of individual identity transformation (i.e., learning), and is directed towards the normative conception of competence implicitly negotiated by the community through its pursuit of a joint enterprise. A principal affordance of becoming aware of one's own conceptions of learning is that it positions an instructor to *purposefully* develop and implement instructional sequences that are consistent with it (Tallman, 2021). We view a central priority for the project to be fostering the community of practice to (a) make these goals, values, and beliefs explicit; (b) create the intellectual need for critical reflection on them; and (c) provide opportunities to develop, implement, evaluate, and refine new strategies based on the MIP characterization of inquiry. Based on our analysis, we recommend the following forms of interventions that might support the refinement of participants' conceptions of active learning through their engagement in the MIP community of practice:

- *opportunities to engage in instructional design with community-recognized experts to foster the MIP components of inquiry*
- *opportunities to critically evaluate curricular artifacts that reflect explicit operationalization of the MIP components of inquiry*
- *feedback from peers that suggest concrete ways to modify their proposed instructional materials to support more effective implementation of the MIP components of inquiry*
- *guided reflection on results of pilot lessons and refinement to improve implementation of the MIP components of inquiry*

Future research

One direction for our future research is to include additional data sources. For example, recordings of participants teaching or artifacts from their class materials could lend additional insight into participants' conceptions of active learning and the extent to which they align with the MIP's definition. An analysis of such data would provide more robust findings by allowing us to describe how participants enact their conceptions of active learning.

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References

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher*, *38*(3), 181-199. https://doi.org/10.3102/0013189X08331140

Harel, G. (2008). A DNR perspective on mathematics curriculum and instruction. Part II: with reference to teacher's knowledge base. *ZDM Mathematics Education, 40(5),* 893-907. https://doi.org/10.1007/s11858-008-0146-4

Henderson, C., Beach, A., & Finkelstein, N. (2011). Facilitating change in undergraduate STEM instructional practices: An analytic review of the literature. *Journal of Research in Science Teaching*, *48*(8), 952-984. https://doi.org/10.1002/tea.20439

Simon, M. A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning*, *6*(2), 91-104. https://doi.org/10.1207/s15327833mtl0602_

Strauss, A., & Corbin, J. (1994). Grounded theory methodology: An overview. In N.K. Denzin & Y.S. Lincoln (Eds.), *Handbook of qualitative research*, (pp. 273-285). Sage Publications.

Tallman, M. (2021). Investigating the transformation of a secondary teacher's knowledge of trigonometric functions. *Journal of Mathematical Behavior, 62*.

Thompson, P. W. (2008). Conceptual analysis of mathematical ideas: Some spadework at the foundations of mathematics education. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sepulveda (Eds.), *Proceedings of the Joint Meeting of PME 32* (pp. 31-49). PME.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* Cambridge University Press.