DESIGNING A VIDEO-BASED INTERVENTION TO ELICIT TEACHER CANDIDATES' MATHEMATICAL KNOWLEDGE: THE HEXAGON TASK

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Researchers increasingly take a design-based research approach to iteratively design, implement, and revise interventions. In this paper, we describe how our iterative design led to improvements in design principles aimed at supporting secondary mathematics teachers' attentiveness development. We describe issues we encountered and insights for developing video-based interventions to improve attentiveness.

Keywords: Design Experiments, Mathematical Knowledge for Teaching, Preservice Teacher Education, Teacher Noticing

Video-based interventions can lead to growth in teachers' and teacher candidates' (TCs') knowledge of mathematics (Jacob et al., 2009), professional noticing skills (van Es & Sherin, 2008), and knowledge of students' mathematical ideas (Powell et al., 2003). Lacking in the literature, however, are detailed accounts of how iterative curriculum design processes are used to improve video-based interventions (Cavey et al., 2020). We are in the fourth year of a design-based research (DBR) project (VCAST) focused on developing video-based modules to improve secondary mathematics TCs' ability to attend to student thinking (Carney et al., 2017). Data analysis revealed limitations in TC responses to prompts about student thinking. In this paper, we describe how the iterative design process led to improvements in our ability to elicit better evidence of TCs' mathematical knowledge associated with figural pattern tasks.

Background

For decades educators have leveraged online instructional materials to maximize in-person class time (Graham, 2006). We began this project interested in developing an intervention with online videos and supporting materials for mathematics courses, similar to earlier work (Goldman & Barron, 1990; Lampert & Ball, 1998). Instructional interventions are rarely iteratively designed, implemented in authentic environments, and improved over time (Amiel & Reeves, 2008). DBR focuses on solving educational problems by connecting research, theory, and practice through iterative theory-driven development focused on making an intervention effective in authentic settings (Anderson & Shattuck, 2012). We took a DBR approach when developing our intervention.

Attentiveness is the ability to analyze and respond to a student's mathematical ideas in ways that build upon student understanding towards formal mathematics and its conventions (Carney et al., 2017) and is grounded in professional noticing (Jacobs et al., 2010), mathematical knowledge for teaching (Ball et al., 2008), and progressive formalization (Freudenthal, 1973). It narrows the focus of professional noticing to an individual student and provides a lens through which to theorize how a teacher's mathematical knowledge and pedagogical stance are elicited in the analyses of student work. Two key aspects of this lens are the ability to recognize, articulate, and connect (1) a student's productive reasoning along with ideas that may be in need of refinement, and (2) the key ideas associated with a mathematical task.

The Intervention & Design Principles

Each VCAST module centers on a mathematical task and features video clips and written artifacts produced by secondary students during their engagement with the task. Each module has an online, in-class, and exit ticket component. In each module, TCs complete the online component, then take part in class discussions on the task and artifacts, and then complete the exit ticket. Each module is typically implemented over a week. Design principles guided our iterative development.

Design Principle 1 (DP1): Solve non-routine mathematical tasks. The use of non-routine tasks creates opportunities to exhibit mathematical reasoning and can be more accessible to students with a range of background knowledge (Schoen, 2003). Each module begins with TCs solving the same task as the secondary students. The Hexagon Task (Figure 1) elicits a range of approaches and is ideal for analysis of student reasoning (Cavey et al., 2020).

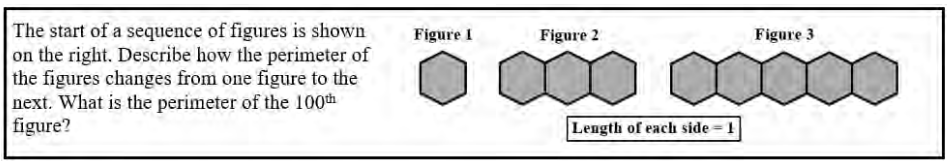

Figure 1: Adapted Hexagon Task; Hendrickson et al. (2012)

Design Principle 2 (DP2): Analyze a range of student evidence. Directed analysis of student evidence can help TCs improve their ability to notice students' mathematical reasoning (Sherin & van Es, 2009; Star & Strickland, 2008). To target DP2, we selected three students who used a range of different strategies and approaches to solving the Hexagon Task (Figure 2).

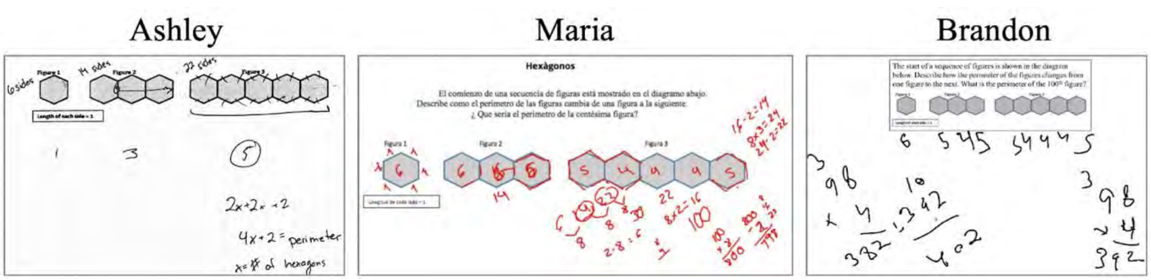

Figure 2: Student evidence featured in The Hexagon Task module

Design Principle (DP3): Engage in Cycles of Inference and Prediction. Effective teachers do more than analyze and interpret student strategies; the inferences they make about students' understanding and their predictions about students' next steps help inform their responses to students (Hill et al., 2005; Jacobs et al., 2010; Lesseig & Hine, 2019). We target DP3 by sequencing video clips from the three featured students throughout the module.

Design Principle (DP4): Describe the Mathematical Challenges for Students. Teachers who understand the specific mathematical challenges students may encounter with a non-routine task are better positioned to respond to students in ways that support students' productive struggle (Stein et al., 1996). We target DP4 by including multiple opportunities for TCs to recognize the challenges students face when focusing on how the configuration of hexagons in each figure contributes to the perimeter.

Methods

We report on this design-based research project following its third revision cycle and after implementation at six public US university Uteach replication sites (Uteach, n.d.). Participants (n=73) were undergraduate students enrolled in the study's partner instructors' mathematics courses. Six partner instructors either taught at the host university or were recruited through the Uteach listservs and annual conferences.

Data were collected via the project's digital platform. TCs submitted responses to open-ended, single- or multiple-selected response, ranking, and upload prompts. Prior to analyzing module responses, three researchers from the design team discussed the expected range of responses. Though we anticipated a range of quality across responses with respect to descriptions and inferences about student work (Jacobs et al., 2010; van Es, 2011), we assumed TCs would have sufficient mathematical knowledge to make sense of the task and the students' work. With each design cycle, the research team applied a variation of magnitude coding (Saldaña, 2016) to review the quality of alignment between expected response and the actual response data. Three researchers independently reviewed all TC responses for each module prompt and compared them to what the prompt had been intended to elicit. Then, researchers reviewed the sets of individual TCs' responses to each sequence of prompts focused on a featured secondary student in the module. Researchers met weekly to discuss emergent areas of concern and to reach consensus on module content warranting revision. The areas of concern connected to TCs' own mathematical knowledge which emerged from these cycles of analysis and discussion are the focus of this paper's results and inform our conclusion and its implications for future research.

Results

Due to limited space, we focus on DPs 3 and 4. An area of concern related to DP3 arose when we were unable to determine whether TCs' superficial predictions stemmed from weaknesses in mathematical knowledge. For example, in Maria's first clip, she narrates how the two outer hexagons in the third figure contribute five units each, while the interior three hexagons contribute four units each, to yield a perimeter of 22. When predicting Maria's next steps, some TCs predicted that Maria will try to solve the task. In the next video segment, Maria creates a sequence of the first three figures' perimeters and computes the common difference of 8 between them. When asked how Maria's next steps compared to TCs' predictions, some TCs made a judgment about her progress with the task, seemingly ignoring the shift in her problem-solving process. In the third video clip, Maria successfully solves the task. Interestingly, some TCs appeared to either not value or not notice some aspects of Maria's productive work. Sample TC evidence elicited from this cycle illustrates the challenge we encountered (Figure 6).

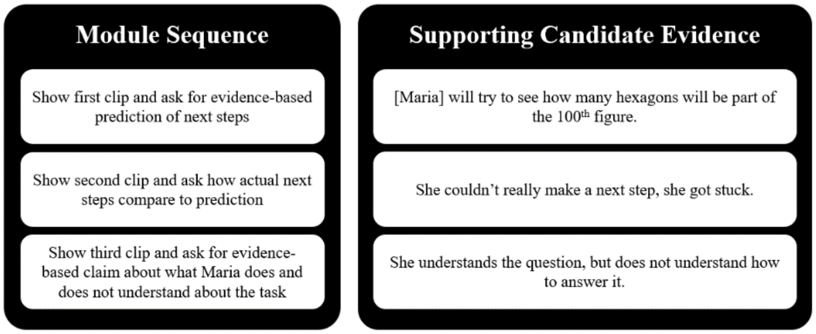

Figure 6: Sample TC Evidence Elicited During a Prediction Cycle

To address this concern, we followed the cycles of prediction in the online component with an adjusted version of the task, the Octagon Task, and provided selected-response prompts which asked TCs to apply each featured students' approach to the new task. This revision accomplishes two goals: (1) it provides scaffolded support to TCs who struggle with the task and (2) it allows us to differentiate between TCs who simply struggle to articulate the mathematical approach they might notice from those who struggle to make sense of the approach itself.

The DP4 area of concern is related to TCs' persistent inability to articulate the mathematical challenges for students. Many of these TCs also appeared to struggle with the same mathematical challenge themselves. To address this concern, we added an official Exit Ticket to the module where we explicitly direct TCs' attention to the challenge that arises when students focus first on the relationship between the number of hexagons and the perimeter and then try to connect their reasoning to the figure number. By Year 4, the Exit Ticket featured video of our third student, Brandon, who made an initial misstep by assuming there were 100 hexagons in the 100th figure. After he noticed his original perimeter of 402 was incorrect, he observed:

> It's going to be more. That's plus two, plus two. I guess it's going to be plus two again. Okay. Five, seven, nine. Okay, [the perimeter's] going to be more than 402. Because there's more than 100 hexagons in the 100th figure. [...] But I need to find how many hexagons the 100th figure has. Um, so it increases by two every time. One plus two equals three. Three plus two equals five. Seven, nine ..Ihat would take too long.

We follow this with two prompts: "Describe the challenge that Brandon encountered in this clip. Use evidence from the clip to support your answer" and "Describe how Brandon's approach to the Hexagon Task is contributing to the difficulty he is experiencing."

Conclusion

Given the importance of conceptual understanding in mathematics education (Ball, 1990), we feature non-traditional tasks that require conceptual understanding to solve in the VCAST modules. With each of our design principles, we work to position TCs in ways that support their ability to analyze, interpret, and make inferences about the range of student reasoning they might encounter in their future classrooms. Our intent is to support TCs' development of the knowledge and skills they will need to enact responsive teaching practices in the classroom.

Yet with each round of our iterative design process, we have discovered areas of ambiguity in the TC response data. We found that we cannot make assumptions about the reasons behind TCs' struggles to make sense of student reasoning evidence while engaged in those same tasks. Engaging in this revision process improves our ability to discern whether TCs' mathematical knowledge might be posing barriers to their attentiveness development and also improves our understanding of the attentiveness construct itself. By sharing some of the challenges encountered in refining the operationalization of design principles through iterative cycles of implementation data collection and analysis and illuminating implicit researcher assumptions which impact design decisions, we hope to support others in similar work.

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