Categorizing student interactions with manipulatives in statics

Abstract
This work in progress paper describes ongoing work to understand the ways in which students make use of manipulatives to develop their representational competence and deepen their conceptual understanding of course content. Representational competence refers to the fluency with which a subject expert can move between different representations of a concept (e.g. mathematical, symbolic, graphical, 2D vs. 3D, pictorial) as appropriate for communication, reasoning, and problem solving.

Several hands-on activities for engineering statics have been designed and implemented in face-to-face courses since fall 2016. In the transition to online learning in response to the COVID-19 pandemic, modeling kits were sent home to students so they could work on the activities at their own pace and complete the associated activity sheets. An assignment following the vector activities required students to create videotaped or written reflections with annotated pictures using the models to explain their thinking around key concepts. Students made connections between abstract symbolic representations and their physical models to explain concepts such as a general 3D unit vector, the difference between spherical coordinate angles and coordinate direction angles, and the meaning of decomposing a vector into components perpendicular and parallel to a line.

The video and written data analyzed to inform the design of think-aloud exercises in one-on-one semi-structured interviews between researchers and students that are currently in progress. This paper presents initial work analyzing and discussing themes that emerged from the initial video and written analysis and plans for the subsequent think-aloud interviews, all focused on the specific attributes of the models that students use to make sense of course concepts. The ultimate goal of this work is to develop some general guidelines for the design of manipulatives to support student learning in a variety of STEM topics.

Introduction
In engineering statics courses, students learn foundational concepts such as vectors, forces, moments, and free-body diagrams. These interrelated concepts can be applied in a broad range of real-world contexts. Drawing free-body diagrams in particular is a fundamental statics skill that students often approach in a procedural way, leading to strategies that focus on memorizing support reaction conventions and reproducing previously worked examples [1], [2], [3]. A strictly procedural approach to statics can lead to superficial understanding of fundamental concepts such as the force interaction between contacting bodies [4].

Engineers communicate and apply concepts such as force interactions using a language of multiple representations that include pictorials, diagrams, graphs, symbols, numbers, and narrative language [5]. Representation translations can be an effective strategy for building conceptual understanding because they provide opportunities for students to resolve misconceptions (or naïve conceptions) and build mental models of the underlying meaning the representations communicate [6]. Studies in chemistry education have shown the potential of concrete models as effective scaffolds for building understanding through representation translations [7], [8]. Manipulatives have also been widely used in K-12 mathematics classrooms to provide concrete representations that allow students to explore mathematical concepts with a hands-on approach. When teaching important concepts, moving in a systematic way from concrete models to representational models to abstract representations of concepts can support students in developing conceptual understanding. This idea of concreteness fading suggests that students should have learning experiences with all three types of representations that includes explicit links between them [9]. Moving in order from concrete to representational to abstract supports transfer.

We have been reporting on an analogous approach in Statics instruction that leverages models as concrete representations to help students make connections between a structure and common abstract representations such as symbolic vector notation, free-body diagrams, and associated equilibrium equations [10], [11]. We hypothesize these hands-on activities can support students in developing their representational competence. Representational competence refers to the ability to use and fluently move between multiple representations of a concept for the purpose of problem solving and communicating thinking [12]. While using manipulatives and hands-on models are widely suggested as an effective instructional strategy, the specific ways in which students interact with the tools connected to their representational competence has not been studied [13]. This study seeks to understand the ways in which students use 3D models of statics concepts to solve problems, communicate their thinking, and make connections to representational (e.g. free-body diagrams) and abstract (e.g. symbolic representations) concepts.

Methodology

The fall 2020 statics course was taught online with a single required 90-minute Zoom session per week and a second optional 60-minute session added midway through the term. The remainder of learning activities were administered asynchronously through the Canvas learning management system. Asynchronous learning included modeling activities and associated worksheets that were adapted from those originally developed for group learning in face-to-face instruction [14], [15]. The scaled down model kits were checked out to every student for the duration of the term. Worksheet submissions were awarded full credit based on completion regardless of accuracy. Example worked solutions were provided after each due date.

Students completed three modeling activities on 3D vector representations and operations during the first two weeks. After completion of these activities, students were given an assignment to produce a discussion post that used their model to explain a vector concept chosen from the following three options:

- Explain the concept of a general unit vector in 3D and why it is useful.
- Explain the difference between spherical coordinate angles and coordinate direction angles.
- Explain what it means to decompose a vector into components perpendicular and parallel to a line.

They had the option to produce their explanation as a 3-4 minute video or as a written narrative with annotated photos. Students' responses were subsequently analyzed using thematic analysis as described below in the next section.

Data Analysis
The written and video-recorded student responses provided a great deal of information about students' thinking and understanding. Students interacted with the 3D models in a wide variety of ways. Students used the 3D model as a tool for making sense of definitions and basic formulae. They also referenced the models to discuss issues of precision and directionality. Two common ways that students used the 3D model were to connect components of the model to values in a formula or calculation and to use the model to describe what effect changing a parameter would have on the problem situation.

Samples of Student Responses
While the analysis is ongoing and incomplete at this time, samples of student thinking are provided to illustrate ways in which students connected between abstract course concepts and the 3D modeling kit. Translating between representations is an important component of representational competence. These excerpts of student thinking from written and recorded work demonstrate the flexibility with which students worked with multiple representations.

Several students described concepts in ways that made explicit connections to previous coursework as well as used directional language that described the 3D models. By annotating in writing or describing their models orally in a video format, students were able to show their understanding in sophisticated ways.

Student 1 shared:

> *Spherical coordinate angles are probably more familiar to many beginning statics students. They are learned in Calculus 3 and are a relatively natural extension of 2 dimensional polar coordinates as learned in precalculus. To find polar coordinates in an xy coordinate system you start at the x-axis, sweep a line fixed at the origin an angle θ in a counterclockwise direction, then travel out that line distance r...*

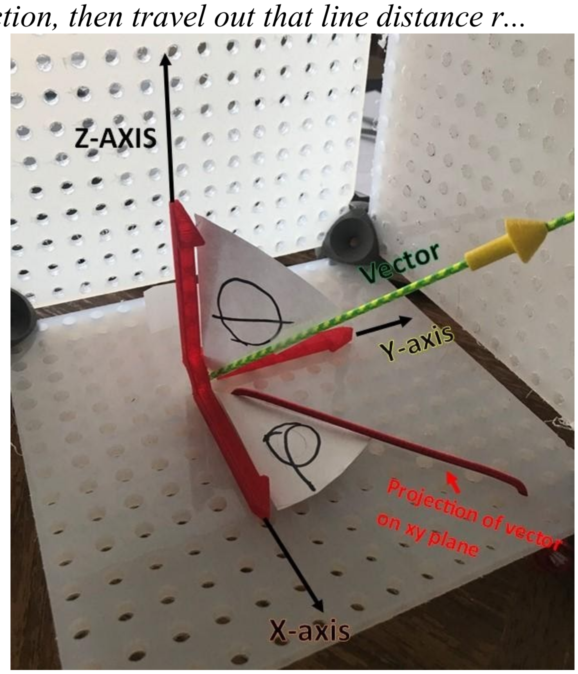


Figure 1. Image from Student 1's response

...In the image above the red cord represents the sweep in the xy plane that allows us to form the plane with the z axis. It's important to make sure that the vector is found by moving "down" from the z axis rather than "up" from our projection in the xy plane.

Student 2 made direct connections between the physical model and an abstract symbolic representation of the relationships.

Going back to the example vector, we can use the following equations above to get the unit vector.

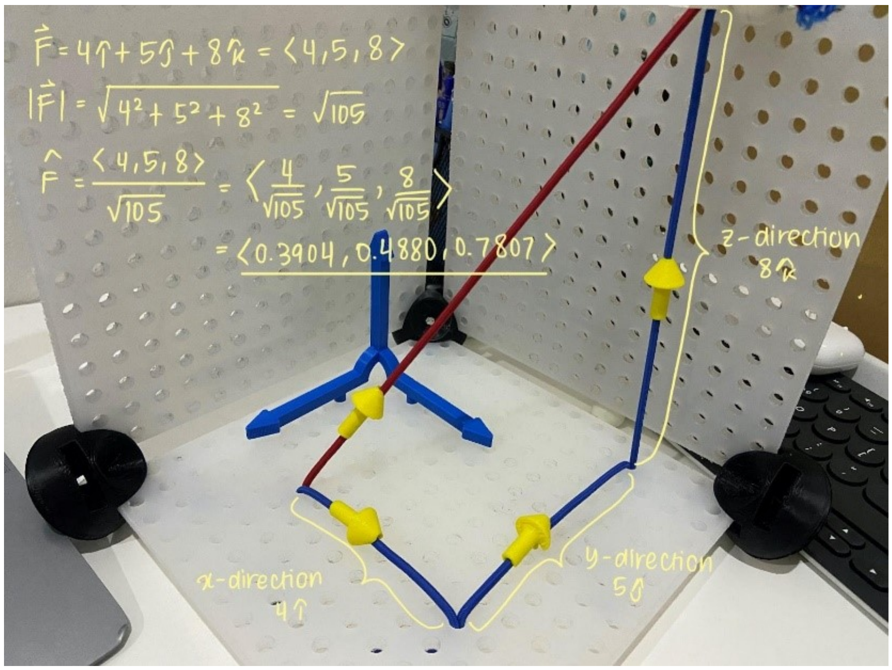


Figure 2. Image from Student 2's response

*So why exactly is the unit vector so useful? One of the characteristics of a unit vector is that when you multiply it with a scalar value, the unit vector will be **scaled** according to the value. This is very useful when calculating the force vector. The force magnitude (say, 200N) acts as the scalar value and the unit vector is the direction of vector. Multiplying the two will get you the force vector. One of the most common types of force vector we usually look for is tension.*

Ongoing Work-in-Progress

This initial analysis of classwork data informed the design of a protocol for a more detailed follow-up study. Students from the fall 2021 statics course are being recruited for semi-structured interviews that will feature think-aloud problem solving with the models and follow-up questions. The fall 2021 statics section was taught in a hybrid modality, but students completed the same take-home individual version of the activities as used in the fall 2020 online section.

The interview protocol has three phases. In the first phase, participants are asked questions to understand their past experiences in STEM courses. The questions asked during this phase include: (1) How did you come to study engineering? (2) What have your past experiences in STEM courses been like? (3) How did you use 3D models in your statics course last quarter? (4) Did you find them to be helpful? If so, how were they helpful for you?

The second phase of the interview requires participants to use the 3D model and solve related statics tasks (Figure 1). During this phase, the interviewer asks probing questions based on their work such as *How did you get this answer? Can you describe what you are thinking about as you use this model? Tell me more about your thinking.* Details of the tasks are provided in Figure 2. Note that the task chosen for this study is abstracted from an activity on two- and three-force members that is included in the full face-to-face version of the modeling curriculum but was dropped from the scaled-down online version.

The activity requires students to identify two and three force members, specifically with attention to developing an understanding of force interactions in the structure (Figures 3 and 4).

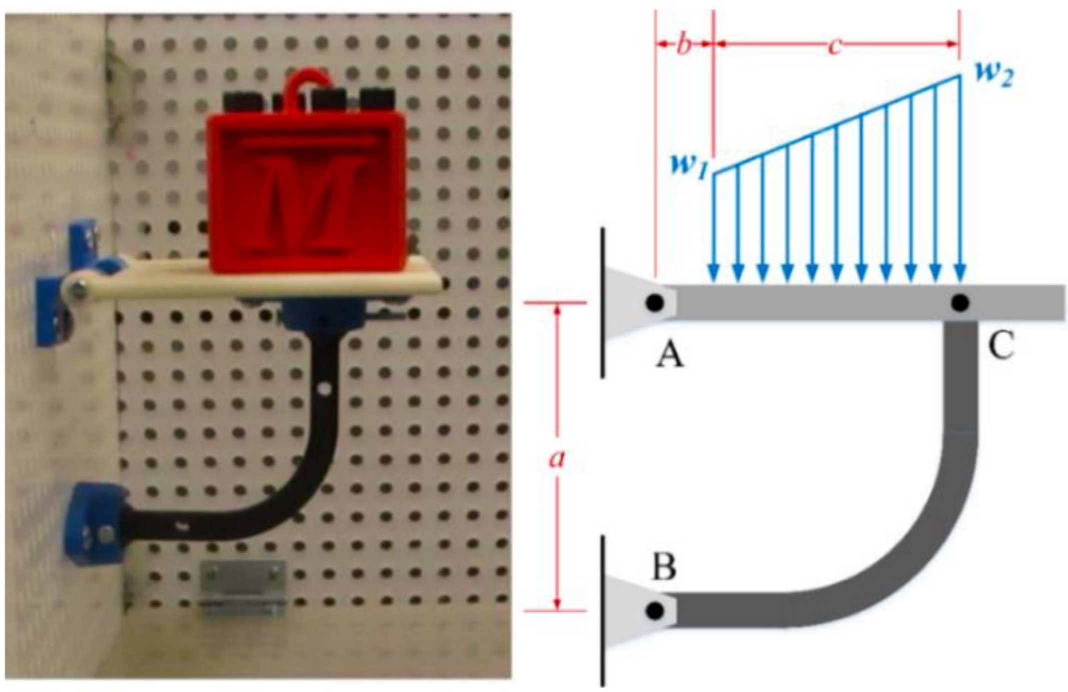
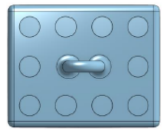


Figure 3. 3D model and diagram of two and three force members

Finally, after completing the tasks, the participants are asked to reflect on their use of the model and ways in which they found it helpful. (1) In what ways do you see a connection between the 3D model and your solutions? (2) Did you find that using the model supported your understanding of these ideas? If yes, how? (3) What else would you like me to know about your experiences using 3D models to solve these problems or in your Statics course?

1. Place six bolts in the box of statics knowledge in an arrangement that, when placed on the platform, will model the distributed load in the figure. Then place the box on the platform. Indicate the bolt arrangement you used on the figure below.



2. Compute the **equivalent concentrated load** for the distributed loading and the location of its **line of action.**

3. Add the force vectors necessary to complete the free-body diagram of the curved member BC. You should be able to feel the direction that the forces need to be by holding the 3D printed model in equilibrium between two fingers. If the direction is known, then you should draw each vector with known direction (indicated by an angle) but unknown magnitude.

4. Draw a complete free-body diagram of the *platform*. Be sure to account for what you know about the forces on **two-force member** BC. Include relevant dimensions and coordinate axes.

5. Suppose the dimension *a* increases but all other dimensions are held constant. Complete the table below to explain how the forces acting on the platform would change. Use **I** for increase, **D** for decrease, and **U** for unchanged.

Force	Magnitude	Direction (angle measured CCW from horizontal)	Notes
Reaction at A			
Reaction at C			

Figure 4. Statics tasks from semi-structured interview

Conclusion

While manipulatives are used widely in K-12 mathematics, less is known about the ways students interact with manipulatives in college-level courses. Students enrolled in an online statics course completed several activities that were explicitly designed to help them make connections between the concrete 3D models and important abstract concepts. This study analyzed data from one assignment in which students used their model kits to explain a 3D vector concept. The analysis informed the development of an interview protocol for a more in-depth analysis of how students interact with the models as learning aids. The semi-structured interviews will provide information about how students use models to make sense of problems and concepts with tasks that allow for concrete, representational, and abstract thinking. The ultimate goal is to use this information to develop guidelines for model design features that may be supportive of specific instructional goals related to coordinating information across multiple representations.

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