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Width Confinement in 3D Dielectric Waveguides and Comparison to 2D Analytical Models

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Abstract—Two-dimensional (2D) analytical models are only approximations for 3D structures where one cross-sectional dimension is much larger than the other. This paper uses the finite-difference time-domain method (FDTD) to perform numerical experiments on fully 3D dielectric waveguide structures to compute the wave-impedance and the propagation-constant for finite-width dielectric waveguides. These data are used to determine the width required to achieve good correlation against 2D analytical models. Results show that width $\geq 10 \times$ height is the limit for good approximation.

I. INTRODUCTION

Nano-scale dielectric waveguides are a critical component in modern integrated circuit design for both signal/power integrity and ultra high-speed data transfer. Analytical formulation of dielectric waveguides has historically been limited to the spatially two-dimensional (2D) canonical form of the *slab waveguide* [1]–[7]. However, realistic dielectric waveguides are fully three-dimensional (3D) structures. Despite the dimensional discrepancy, 2D analytical models are often used as approximations, provided that there is only tight confinement in one of the two cross-sectional dimensions, where length is functionally infinite along the longitudinal-section in both directions. 3D dielectric waveguides with width much larger than height are spatially inefficient. Therefore, in this paper we conduct numerical experiments in FDTD to determine the limit for width where 2D analytical models may be considered a *good* approximation of 3D dielectric waveguides. The 2D analytical models for wave-impedance Z_w (Ohm) and the propagation-constant β (rad/m) are used for comparison.

II. FORMULATION

The waveguides of interest are fully 3D and have step index contrast between core and cladding regions with refractive indices $n_1 = 3.5$ and $n_2 = 1.5$ (corresponding to Si/SiO₂), respectively, where the core is surrounded uniformly on all sides by a cladding which is assumed to be infinite in extent. The core region geometry is shown in Fig. 1, where we note the top and bottom walls of the waveguide are separated by the height (δ), the left and right walls are separated by the width (w), and all walls are smooth.

The 2D analytical Z_w model is defined in (1) [4] may be used as an approximation for comparison with 3D simulations data.

$$Z_w^{\text{TE}} = \frac{E_y}{H_z} = -j \frac{\omega \mu_0}{\gamma}, \quad (\Omega) \quad (1)$$

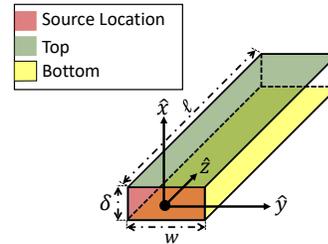


Fig. 1: 3D waveguide geometry.

where $j = \sqrt{-1}$, $\gamma = \sqrt{\beta^2 - n_2^2 k_0^2}$ (rad/m), μ_0 is the free-space magnetic permeability (H/m), angular frequency $\omega = 2\pi f$ (rad/s), cyclic frequency f (Hz), and field components E_y (V/m) and H_z (A/m) are time-harmonic frequency-domain phasors, and Z_w is purely imaginary for a smooth dielectric waveguide with no added loss mechanism (such as surface roughness or lossy material).

The propagation constant can be evaluated from the relation of E-field components using the imaginary component of the complex logarithm in (2) [8]

$$\beta = \frac{1}{\ell} (\arg(E_1) - \arg(E_2)) = n_{\text{eff}} k_0, \quad (\text{rad/m}) \quad (2)$$

where k_0 is the free-space wave number at the source wavelength λ_0 , n_{eff} is the effective refractive index as calculated from the *Effective Index Method* [2, §D.C], $\log(E_1/E_2) = \alpha + j\beta$, $\alpha = 0$ in the absence of any additional loss mechanism, and ℓ is the distance between E_2 and E_1 . The arg function returns the complex angle including any and all additional $2\pi m$ turns with $m \in \mathbb{Z}$.

III. RESULTS AND DISCUSSION

For computing Z_w using (1), each numerical experiment uses $\lambda_0 = 1.54 \mu\text{m}$ and $\delta = 200 \text{ nm}$, and reported values are evaluated at $f = 194.8 \text{ THz}$. FDTD field data are collected at the midpoints along ℓ and w , with a three-cell offset from the bottom of the waveguide core region.

These data are collected over widths varying from 200 nm to $5 \mu\text{m}$. Those results are shown in Fig. 2, where $\Im\{Z\}$ is the imaginary component of the complex number Z . We see therein that Z_w calculated with FDTD approaches the 2D analytical model and saturates at $w \approx 800 \text{ nm}$. There is a noticeable offset between the 3D FDTD data and the 2D analytical approximation below that point but little variation

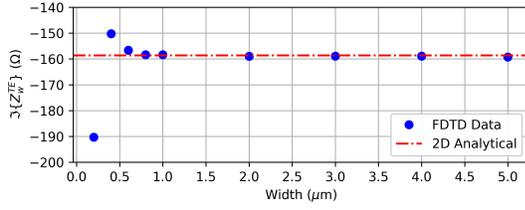


Fig. 2: Z_w across widths.

between FDTD and analytical calculations above that point. In this case, the 2D analytical model is a good approximation for $w \geq 4\delta$.

Sample FDTD Z_w data are shown with all discrete cells in the $w \times \ell$ cross-section in Fig. 3. The boundary of the region directly below the waveguide core is shown as dotted lines. Within the *below core* region there is minimal variation along length. Z_w settles to a stable value within $2 \mu\text{m}$ from the source location at $z = 0$, and the variations along w appear only near the region boundary. Outside the boundary there are several null points appearing periodically along length. The length interval between nulls seems to be inversely proportional to waveguide width. The null points are likely the result of 3D multi-modal behavior as more propagating modes exist in the waveguide with increasing w .

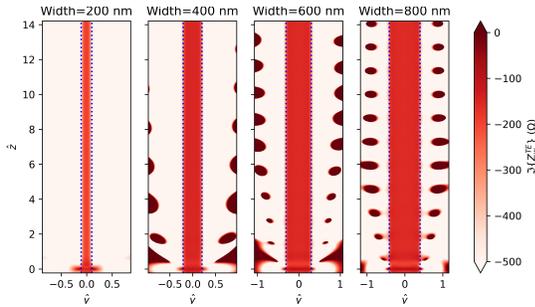


Fig. 3: Z_w vs. w and length.

FDTD fields data are also collected at two $w \times \delta$ planes along length. The average of all E_y values in those planes is then related to β using (2), where $\ell = 5 \mu\text{m}$. We see saturation-like behavior in Fig. 4 similar to Fig. 2. However, the saturation point for β appears to be at $w \approx 2 \mu\text{m}$. This implies that

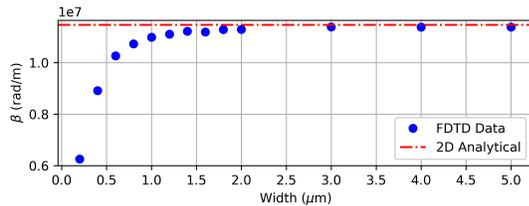


Fig. 4: β across widths.

the 2D analytical model is a good approximation for the 3D dielectric waveguide where $w \geq 10\delta$. Since Z_w is an implicit function of β through γ , this w limit should be used when utilizing the 2D analytical approximations.

Fig. 5 shows the point-to-point β calculations (without field-averaging), where the boundary between core and cladding is shown as a red dotted line. The mode configuration changes as w increases, hence the use of field-averaging in Fig. 4.

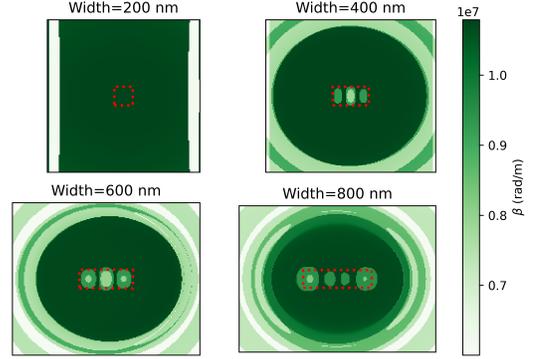


Fig. 5: Point-to-point β across w and δ .

IV. CONCLUSION

The experiments conducted in this paper showed that fundamental parameters Z_w and β have a strong dependence on the w/δ ratio, where in this case, good correlation to 2D analytical model is achieved for $w \geq 10\delta$; the data suggests an order-of-magnitude difference would be sufficient. Further experiments should be conducted for additional parameters, e.g., α , in both *smooth* and *rough* waveguides.

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