

A ROBUST OPTIMAL GUIDANCE STRATEGY FOR MARS ENTRY

Emily M. Palmer* and Anil V. Rao†

A robust optimal guidance strategy is proposed. The guidance strategy is designed to reduce the possibility of violations in inequality path constraints in the presence of modeling errors and perturbations. The guidance strategy solves a constrained nonlinear optimal control problem at the start of every guidance cycle. In order to reduce the possibility of path constraint violations, the objective functional for the optimal control problem is modified at the start of a guidance cycle if it is found that the solution lies within a user-specified threshold of a path constraint limit. The modified objective functional is designed such that it maximizes the margin in the solution relative to the path constraint limit that could potentially be violated in the future. The method is validated on a path-constrained Mars entry problem where the reference model and the perturbed model differ in their atmospheric density. It is found for the example studied that the approach significantly improves the path constraint margin and maintains feasibility relative to a guidance approach that maintains the original objective functional for each guidance update.

INTRODUCTION

As space exploration continues to progress, Mars has become a subject of great interest. When posed as an optimal control problem, Mars entry, descent, and landing (EDL) is a challenging problem due to the path constraints imposed on the vehicle during entry. Beyond satisfying these constraints, it is also necessary to maintain a gap between the values of the constraint functions relative to the constraint boundaries in order to improve the constraint margins. During entry, it is likely that the actual motion will deviate from the reference (planned) motion due to modeling errors and environmental perturbations. In order for the vehicle to move in such a way that it does not come in close proximity of the constraint boundaries, it is necessary for the guidance command to be robust to these modeling errors and environmental disturbances. This paper provides a demonstration of a method for optimal guidance that reduces sensitivity in the solution to modeling errors and disturbances in order to maintain feasibility and increase path constraint margin during atmospheric entry.

Methods for guidance of aerospace vehicles include linear-quadratic (LQ) methods,¹⁻³ proportional navigation (PN),^{4,5} neighboring optimal control (NOC),⁶ and acceleration guidance.⁷ These methods, are limited because the guidance law does not take into account the full capability of the vehicle. In addition, these methods are not designed to be robust to modeling errors or perturbations. An approach called desensitized optimal control⁸⁻¹¹ (DOC) has been conceived to improve robustness to modeling errors and environmental disturbances. The goal of DOC is to generate

*Ph.D. Student, Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, Florida 32611-6250. Email: emilypalmer@ufl.edu.

†Professor, Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL 32611-6250. E-mail: anilvrao@ufl.edu. Fellow AAS and Associate Fellow AIAA. Corresponding Author

a trajectory that is less sensitive to variations due to modeling errors or other uncertainties in the system. The early work of Ref. 8 provided a foundation for the DOC method. In particular, in Ref. 8 the parameters of interest were elevated to the level of states and the standard optimal control problem is augmented by a sensitivity matrix that captures the sensitivity of the state at an arbitrary time with respect to changes in the initial state. The sensitivity of the state at the terminal time t_f can then be found with respect to the state at any time t using the chain rule. These sensitivities then can be incorporated into the objective functional. The DOC approach was first formulated for unconstrained optimal control problems, then later developed for problems with control and state constraints.¹¹ This methodology was used to solve a Mars Entry problem with respect to uncertainty in atmospheric density.⁹ One issue with using the sensitivity matrix is that the optimal control problem increases in dimension, with the number of states increasing quadratically as a function of the number of original states and parameters of interest. The increase in dimensionality was later reduced using traditional sensitivity functions.¹⁰

One fundamental characteristic of DOC is that the entire trajectory is designed with the goal of reducing sensitivity. It is noted, however that increasing the robustness of the entire trajectory decreases overall performance. As a result, it is preferable to focus the increase in robustness to only those aspects of the trajectory where such robustness is advantageous. This research is aimed at reducing the effect of uncertainty and disturbances when the solution lies in close proximity to the constraint limits and maintains the original formulation of the problem when the solution lies far from the constraint limits. While desensitized optimal control often requires manipulation of the state matrix and objective functional, in this paper a method for guidance is developed that reduces computational complexity relative to previous methods for desensitized optimal control. In the approach of this paper, two separate objective functionals are employed: (1) the original objective functional and (2) a modified objective functional that is intended to increase the margin between the value of a path constraint function and the constraint limit. The guidance strategy is then to solve the original optimal control problem in real time when the solution is distant from the constraint limit and switches to solving the modified optimal control problem when the solution lies near a path constraint limit. This approach ensures feasibility throughout the solution, while reducing performance only when necessary in order to maintain feasibility. The approach developed in this paper is demonstrated on a Mars entry guidance problem where the objective is to maximize the terminal altitude subject to a terminal constraint on speed and path constraints on acceleration load, dynamic, pressure, and heating rate. The steps in the method of this paper are given as follows: (1) the reference optimal trajectory is computed using adaptive Gaussian quadrature collocation; (2) the flight is simulated with the optimal control produced from the reference solution over a guidance cycle using a perturbed (off-nominal) model. During each guidance cycle the values of the path constraint functions are evaluated. If it is found that the value of the path constraint function is approaching the path constraint limit and lies within a certain range of this limit, the modified objective functional is used when re-solving the optimal control problem for use on the next guidance cycle. It is noted that the modified objective functional is designed to minimize the maximum path constraint value experienced by the vehicle over the remaining horizon. On the other hand, if the path constraint values do not lie within the user-chosen range of the path constraint limit or are moving away from this limit, then the original objective functional is employed.

BOLZA OPTIMAL CONTROL PROBLEM

Without loss of generality, consider the following optimal control problem in Bolza form. Determine the state, $\mathbf{x}(\tau) \in \mathbb{R}^{n_x}$, and the control, $\mathbf{u}(\tau) \in \mathbb{R}^{n_u}$, on $\tau \in [-1, +1]$ that minimize the objective functional

$$J = \mathcal{M}(\mathbf{x}(-1), t_0, \mathbf{x}(+1), t_f) + \frac{t_f - t_0}{2} \int_{-1}^{+1} \mathcal{L}(\mathbf{x}(\tau), \mathbf{u}(\tau), t(\tau; t_0, t_f)) d\tau \quad (1)$$

subject to the dynamic constraints

$$\frac{d\mathbf{x}}{d\tau} - \frac{t_f - t_0}{2} \mathbf{f}(\mathbf{x}(\tau), \mathbf{u}(\tau), t(\tau; t_0, t_f)) = \mathbf{0}, \quad (2)$$

the inequality path constraints

$$\mathbf{c}_{\min} \leq \mathbf{c}(\mathbf{x}(\tau), \mathbf{u}(\tau), t(\tau; t_0, t_f)) \leq \mathbf{c}_{\max}, \quad (3)$$

and the boundary conditions

$$\mathbf{b}_{\min} \leq \mathbf{b}(\mathbf{x}(-1), t_0, \mathbf{x}(+1), t_f) \leq \mathbf{b}_{\max}. \quad (4)$$

It is noted that domain $t \in [t_0, t_f]$ and the domain $\tau \in [-1, +1]$ are related via the affine transformation

$$t(\tau; t_0, t_f) = \frac{t_f - t_0}{2} \tau + \frac{t_f + t_0}{2} \quad (5)$$

LEGENDRE-GAUSS-RADAU COLLOCATION

Consider now the following partition of the Bolza optimal control problem described in Eqs. (1)–(4) into K mesh intervals. First, let $(T_0, \dots, T_K) \in [-1, +1]$ be such that $T_0 < T_1 < \dots < T_K$. Furthermore, let $T_0 = -1$ and $T_K = +1$. Finally, let $\mathcal{S}_k = [T_{k-1}, T_k]$. Next, let $\mathbf{x}^{(k)}(\tau)$ and $\mathbf{u}^{(k)}(\tau)$ be the state and control, respectively, in mesh interval $\mathcal{S}_k$. The Bolza optimal control problem given in Eqs. (1)–(4) is then formulated as a K mesh interval problem as follows. Minimize the objective functional:

$$J = \mathcal{M}(\mathbf{x}^{(1)}(-1), t_0, \mathbf{x}^{(K)}(+1), t_f) + \frac{t_f - t_0}{2} \sum_{k=1}^K \int_{T_{k-1}}^{T_k} \mathcal{L}(\mathbf{x}^{(k)}(\tau), \mathbf{u}^{(k)}(\tau), t(\tau; t_0, t_f)) d\tau, \quad (6)$$

subject to the dynamic constraints:

$$\frac{d\mathbf{x}^{(k)}}{d\tau} - \frac{t_f - t_0}{2} \mathbf{f}(\mathbf{x}^{(k)}(\tau), \mathbf{u}^{(k)}(\tau), t(\tau; t_0, t_f)) = \mathbf{0} \quad (k = 1, \dots, K) \quad (7)$$

the inequality path constraints:

$$\mathbf{c}_{\min} \leq \mathbf{c}(\mathbf{x}^{(k)}(\tau), \mathbf{u}^{(k)}(\tau), t(\tau; t_0, t_f)) \leq \mathbf{c}_{\max} \quad (k = 1, \dots, K) \quad (8)$$

and the boundary conditions

$$\mathbf{b}_{\min} \leq \mathbf{b}(\mathbf{x}^{(1)}(-1), t_0, \mathbf{x}^{(K)}(+1), t_f) \leq \mathbf{b}_{\max}. \quad (9)$$

It is noted that, in order to maintain continuity at each interior mesh point, the condition $\mathbf{x}(T_k^-) = \mathbf{x}(T_k^+)$ ($k = 1, \dots, K-1$) is enforced.

The partitioned optimal control problem given in Eqs. (6)–(9) is discretized using Legendre-Gauss-Radau (LGR) collocation as described in Refs. 12–15. In the LGR method, the state in every mesh interval is approximated as

$$\mathbf{x}^{(k)}(\tau) \approx \mathbf{X}^{(k)}(\tau) = \sum_{j=1}^{N_k+1} \mathbf{X}_j^{(k)} \ell_j^{(k)}(\tau) \quad (10)$$

where $\ell_i^{(k)}(\tau)$ is the basis of Lagrange polynomials

$$\ell_j^{(k)}(\tau) = \prod_{i=1, i \neq j}^{N_k+1} \frac{\tau - \tau_i^{(k)}}{\tau_j^{(k)} - \tau_i^{(k)}} \quad (11)$$

and $(\tau_1^{(k)}, \dots, \tau_{N_k}^{(k)})$ are the LGR points in mesh interval S_k and $\tau_{N_k+1}^{(k)} = T_k$ is a noncollocated point. The derivative of the state approximation $\mathbf{X}^{(k)}(\tau)$ is given as

$$\frac{d\mathbf{X}^{(k)}}{d\tau} = \sum_{j=1}^{N_k+1} \mathbf{X}_j^{(k)} \frac{d\ell_j^{(k)}}{d\tau} \quad (12)$$

The dynamic constraints given by Eq. (7) are collocated at the N_k LGR points in mesh interval $k \in [1, \dots, K]$ as

$$\sum_{j=1}^{N_k+1} D_{ij}^{(k)} \mathbf{X}_j^{(k)} - \frac{t_f - t_0}{2} \mathbf{f}(\mathbf{X}_i^{(k)}, \mathbf{U}_i^{(k)}, t_i^{(k)}) = \mathbf{0} \quad (i = 1, \dots, N_k) \quad (13)$$

where $t_i^{(k)} = t(\tau_i^{(k)}; t_0, t_f)$ and

$$D_{ij}^{(k)} = \left[\frac{d\ell_j^{(k)}}{d\tau} \right]_{\tau=\tau_i} \quad , \quad (i = 1, \dots, N_k; j = 1, \dots, N_k+1) \quad (14)$$

are the elements of the $N_k \times (N_k+1)$ Legendre-Gauss-Radau differentiation matrix¹² in mesh interval S_k , $k \in [1, \dots, K]$. The LGR approximation then leads to the following nonlinear programming problem (NLP). Minimize

$$\mathcal{J} = \mathcal{M}(\mathbf{X}_1^{(1)}, t_0, \mathbf{X}_{N_K+1}^{(K)}, t_f) + \sum_{k=1}^K \sum_{j=1}^{N_k} \frac{t_f - t_0}{2} w_j^{(k)} \mathcal{L}(\mathbf{X}_i^{(k)}, \mathbf{U}_i^{(k)}, t_i^{(k)}) \quad (15)$$

subject to the collocation constraints of Eq. (13), the discretized path constraints

$$\mathbf{c}_{\min} \leq \mathbf{c}(\mathbf{X}_i^{(k)}, \mathbf{U}_i^{(k)}, t_i^{(k)}) \leq \mathbf{c}_{\max} \quad (i = 1, \dots, N_k), \quad (16)$$

the discretized boundary conditions

$$\mathbf{b}_{\min} \leq \mathbf{b}(\mathbf{X}_1^{(1)}, t_0, \mathbf{X}_{N_K+1}^{(K)}, t_f) \leq \mathbf{b}_{\max}, \quad (17)$$

and the continuity constraints

$$\mathbf{X}_{N_k+1}^{(k)} = \mathbf{X}_1^{(k+1)}, \quad (k = 1, \dots, K-1). \quad (18)$$

METHOD ROBUST OPTIMAL GUIDANCE USING LGR COLLOCATION

In this section a method for robust optimal guidance is described. The method developed in this section employs LGR collocation as a guidance algorithm on a *shrinking horizon* using a modification of the method for guidance and control developed in Ref. 16-17. In the method of Ref. 16, following optimal control problem was re-solved at the start of every guidance cycle $t_0 + k\Delta T$, where k is the guidance cycle and ΔT is the guidance cycle duration. It is noted that the method given in Ref. 16 did not explicitly consider path constraints. In particular, on any guidance it is possible that the optimal control problem may have no solution because disturbances or perturbations may make it such that the path constraints cannot be satisfied given the initial state for that guidance cycle. In order to overcome the issues with path constraints, an approach to guidance is developed that makes it possible to re-solve the optimal control while maintaining feasibility within the path constraints.

The following approach is developed to improve robustness when solving the optimal control problem with path constraints. First, two models for the entry vehicle are used. First, a reference model is used to solve the optimal control problem on the remaining horizon. Second, a perturbed model is used to simulate the dynamics over each guidance cycle. The following strategy is then implemented for optimal guidance. First, the reference optimal solution is generated using LGR collocation using the reference model. Then, using the perturbed (actual) model, the control obtained from LGR collocation is used to simulate the motion of the system over a guidance cycle of duration ΔT time units. The perturbed (actual) trajectory obtained using the reference optimal control is referred to as the *actual trajectory*. The simulated state at the time ΔT time units from the start of the guidance cycle produces a state at the end of the guidance cycle, and this state is used as the initial condition when re-solving the optimal control problem over the remaining horizon, that is, from the end of the guidance cycle to t_f . This process of simulating the motion of the vehicle using the control from GPOPS – III and solving the optimal control problem over the remaining horizon (which in this case is a shrinking horizon as the guidance cycles evolve) is repeated until the remaining horizon is smaller than ΔT . Once the remaining horizon is less than ΔT time units the optimal control problem is not solved again and the control obtained from the last solution obtained from GPOPS – III is used for the remainder of the flight.

The method of this paper differs from that of Ref. 16 in the following key manner. Specifically, in Ref.¹⁶ did not take into account path constraints, whereas the method of this paper includes the path constraints in the solution of the guidance problem. In order to incorporate the path constraints, the optimal control problem is formulated using one of two possible objective functionals. If the solution does not lie in proximity of the path constraint limits then the original optimal control problem is solved. If, however, it is found that the solution on a given guidance cycle is encroaching on a path constraint limit, then a modified objective functional is used that imposes a penalty on the path constraint. Denoting c as the path constraint of interest, four possible cases exist for the solution to lie in proximity of a path constraint limit: (1) $c_{\max} > 0$ and $\dot{c} > 0$; (2) $c_{\max} < 0$ and $\dot{c} > 0$; (3) $c_{\min} > 0$ and $\dot{c} < 0$; and (4) $c_{\min} < 0$ and $\dot{c} < 0$. It is beyond the scope of this paper to develop the modified objective functional for all four cases and, thus, only the case where $c_{\max} > 0$ and $\dot{c} > 0$ will be considered here. For this particular case, the following function is used in order to determine if the solution lies within a specified proximity of a path constraint limit:

$$P(t) = \frac{c(\mathbf{x}(t), \mathbf{u}(t), t)}{c_{\max}} \quad t \in [t_0 + k\Delta T, t_0 + (k+1)\Delta T], \quad (19)$$

where k is the current guidance cycle and c is the scalar path constraint of interest. It is noted in Eq. 19 that $P(t)$ is the fraction of the path constraint limit reached on the current guidance cycle. The maximum value of $P(t)$ is computed and compared to a user-specified threshold, α , where α is a fraction of the total allowable path constraint limit. The quantity α represents the proximity of the path constraint function relative to the path constraint limit, and the value α is a user-specified parameter. It is only necessary to take action if the path constraint is approaching the path constraint limit. While the method works if the path constraint function lies within the threshold α for either the lower or upper limit, the discussion here focuses on the case where the path constraint function is approaching its upper limit (as the approach is simply reversed if the path constraint function is approaching its lower limit). Suppose that the percentage of the total allowable value reached is greater than the user chosen percentage and the path is increasing. Then the following two conditions will be satisfied:

$$\max P(t) \geq \alpha \quad (20)$$

and

$$\dot{c} > 0 \quad (21)$$

If the conditions given in Eqs. (20) and 21 are satisfied, then the original objective functional is replaced by a modified objective functional when re-solving the optimal control problem over the remaining horizon. The modified objective functional, denoted J_2 , is designed to increase the margin between the value of the path constraint function and the path constraint limit. This improvement in margin is achieved by minimizing the maximum value of the path constraint function over the remaining horizon which is equivalent to minimizing

$$\lim_{p \rightarrow \infty} \left(\int_a^b f^p(t) dt \right)^{1/p}. \quad (22)$$

While using Eq. (22) as the objective functional is not computationally tractable, it is possible to approximate Eq. (22) using a finite and relatively small value of p . Then the modified objective functional is given as

$$\left(\int_{t_0}^{t_f} f^p(t) dt \right)^{1/p} \quad (23)$$

where p is an even number and is kept relatively small (for example p might take on the value of four or six). In order to reduce the size of the integral in Eq. (22), the path constraints of interest are scaled by the maximum allowable value which leads to the following modified objective functional

$$J_2 = \min \left(\int_{t_0}^{t_f} \left(\frac{c(t)}{c_{\max}} \right)^p dt \right)^{1/p} \quad (24)$$

If the conditions listed in Eq. (20)-(21) are not met, then the original objective functional, which is defined as J_1 is used when re-solving the optimal control problem.

MARS ENTRY OPTIMAL CONTROL PROBLEM

In this section the Mars entry optimal control problem of interest is formulated. First, the equations of motion for a point mass over a spherical nonrotating planet are given as¹⁸

$$\begin{aligned}
 \dot{r} &= v \sin \gamma, \\
 \dot{v} &= -D - g \sin \gamma, \\
 \dot{\theta} &= \frac{v \cos \gamma \sin \psi}{r \cos \phi}, \\
 \dot{\phi} &= \frac{v \cos \gamma \cos \psi}{r}, \\
 \dot{\gamma} &= \frac{1}{v} \left[L \cos \sigma + \left(\frac{v}{r} - \frac{g}{v} \right) \cos \gamma \right], \\
 \dot{\psi} &= \frac{L \sin \sigma}{v \cos \gamma} + \frac{v \cos \gamma \sin \psi \tan \theta}{r},
 \end{aligned} \tag{25}$$

where r is the radial distance above the center of Mars, v is the speed, θ is the latitude, ϕ is the longitude, γ is the flight path angle, ψ is the heading angle, and $g = \mu/r^2$. Next, the lift and drag specific forces are given, respectively, as

$$\begin{aligned}
 L &= \frac{q C_L S}{m}, \\
 D &= \frac{q C_D S}{m},
 \end{aligned} \tag{26}$$

where C_L is the coefficient of lift, C_D is the coefficient of drag, S is the characteristic length, m is the vehicle mass, $q = \rho v^2/2$ is the dynamic pressure, $\rho = \rho_0 \exp(-h/H)$ is the atmospheric density, where, $h = r - R$ is the altitude. It is noted that ρ_0 , H , and R are the Martian sea level density, the density scale height, and the radius of the planet, respectively. The physical constants used in the study are given in 1. During entry, the vehicle is subject to path constraints on the

Table 1. Physical constants.

Quantity	Value	Quantity	Value
R	$3.386 \times 10^6 \text{ m}$	r_n	0.6 m
μ	$4.284 \times 10^{13} \text{ m}^3/\text{s}^2$	M	3
g_0	9.80665 m/s^2	N	0.5
S	15.9 m^2	K_q	$1.9027 \times 10^{-8} \text{ W/cm}^2$
C_D	1.45	H	9354 m
C_L	0.348	ρ_0	0.0158 kg/m^3
m	3300 kg		

dynamic pressure, the sensed acceleration, and the heating rate. Denoting the acceleration load as $A = \sqrt{L^2 + D^2}$ and the heating rate as $\dot{Q} = K_q \left(\frac{\rho}{r_n} \right)^N v^M \leq \dot{Q}_{\max}$, these constraints are given,

respectively, as

$$q \leq q_{\max}, \quad (27)$$

$$A \leq A_{\max}, \quad (28)$$

$$\dot{Q} \leq \dot{Q}_{\max}, \quad (29)$$

where N , M , and K_q are constants, and g_0 is the Earth sea level acceleration due to gravity. Next, the initial state and time are fixed and these initial values are provided in Table 2. The final state is free with the exception of the final speed which is fixed. Additionally, bounds placed on the control, σ . Finally, limits on the state are provided in Table 3. The objective is to maximize the altitude at the terminus of the entry, that is, minimize

$$\min J_1 = -r(t_f). \quad (30)$$

Table 2. Boundary conditions.

Quantity	Value	Quantity	Value
r_0	3.5112×10^6 m	θ_0	0 rad
v_0	6000 m/s	ϕ_0	-0.0873 rad
γ_0	-0.2007 rad	ψ_0	1.6581 rad
v_f	540 m/s		

Table 3. Bounds on variables and constraints.

Quantity	Value	Quantity	Value
$r_{\min}$	3.3862×10^6 m	$r_{\max}$	3.5112×10^6 m
$v_{\min}$	0.1 m/s	$v_{\max}$	6000 m/s
$\theta_{\min}$	-2π rad	$\theta_{\max}$	2π rad
$\phi_{\min}$	-1.2217 rad	$\phi_{\max}$	1.2217 rad
$\gamma_{\min}$	-1.0472 rad	$\gamma_{\max}$	0 rad
$\psi_{\min}$	-2π rad	$\psi_{\max}$	2π rad
$\sigma_{\min}$	0.05236 rad	$\sigma_{\max}$	2.0944 rad
$q_{\min}$	0 Pa	$q_{\max}$	10000 Pa
$A_{\min}$	0 m/s ²	$A_{\max}$	$5g_0$ m/s ²
$Q_{\min}$	0 W/cm ²	$Q_{\max}$	70 W/cm ²

OPTIMAL CONTROL PROBLEM SOLVER

All optimal solutions in this paper were generated using the optimal control software GPOPS – III an *hp* adaptive Gaussian quadrature collocation solver.^{12–14, 19–23} The *hp* method allows for smaller meshes to be utilized than typical *h* methods (fixed-order methods), yet still achieves the desired accuracy tolerances. This is achieved by concentrating mesh points where the solution is nonsmooth and rapidly changing and placing fewer mesh points where the solution is smooth. The decay of the Legendre polynomial coefficients as a function of the coefficient index was used to determine the

smoothness of the solution.²³ GPOPS – III implements the nonlinear problem solver, IPOPT, an interior point method. IPOPT was employed in full Newton mode and supplied with the objective function gradient, Lagrangian Hessian, and the constraint Jacobian. The first and second derivatives were calculated from sparse central finite differencing. The boundary conditions, model parameters, an initial guess, an initial mesh, and functions defining the dynamic constraints as well as the objective functional were used as inputs to GPOPS – III. GPOPS – III then outputs the optimal solution, containing the control matrix and corresponding state matrix.

RESULTS

In this section the aforementioned robust optimal guidance strategy is demonstrated on the Mars entry optimal control problem defined earlier. The objective functional for the reference optimal control problem is given in Eq. (30), while the modified objective functional is given in Eq. (24) using the value $p = 4$. The trajectories obtained using the robust optimal guidance strategy are compared against the reference solution. In order to demonstrate the robust optimal guidance strategy, the reference and the perturbed models differ in the density model used. The reference and perturbed density models are given, respectively, as

$$\rho = \rho_0 \exp(-h/H), \quad (31)$$

$$\rho = \tilde{\rho}_0 \exp(-h/H), \quad (32)$$

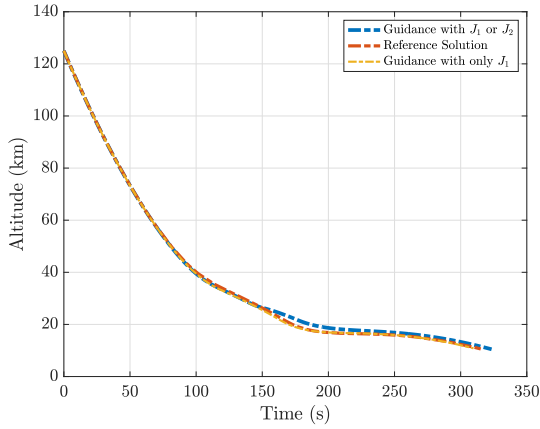
where ρ_0 and $\tilde{\rho}_0$ are the reference and perturbed surface level densities, h is the altitude, and H is the density scale height. When simulating a modeling error, two different cases will be presented. The first modeling error consists of a constant five percent difference between ρ_0 and $\tilde{\rho}_0$, while the second modeling error is drawn from a uniform distribution on each guidance cycle on the interval $[\rho_0, 1.05\rho_0]$. All results were obtained for $\alpha = 0.9$ (that is, the threshold where the objective functional is changed is when the path constraint function lies within ten percent of the upper limit on the path constraint). Additionally, a guidance cycle duration $\Delta T = 10$ s is chosen.

Constant 5% Error in Surface Level Density

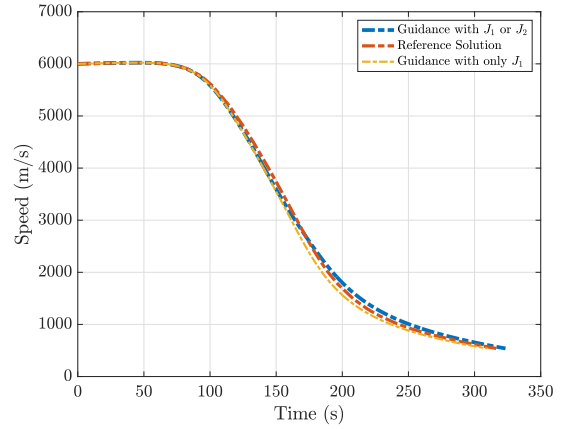
Table 4 shows the components of the final state along with the maximum path constraint function values for the case where the perturbed surface level density is five percent higher than the nominal surface level density (that is, the case where $\tilde{\rho}_0 = 1.05\rho_0$). It can be seen from Table 4 that the robust guidance method developed in this paper not only prevents the flight from violating the constraints, but it also produces a final altitude that is closer in value to the final altitude obtained from the reference optimal solution when compared with the guidance strategy that does not use switch objective functionals. Additionally, it is noted that for the robust guidance method developed in this paper, the modified objective functional was used on only two guidance cycles but still keeps the path constraints from being violated. Furthermore, Fig. 3 shows that the robust guidance strategy developed in this paper does not drastically increase computational complexity and follows a similar trajectory as a guidance strategy that employs only the original objective functional, where computational time generally decreases as the problem is being resolved each cycle. Finally, again from Table 4, it is seen that the terminal time obtained using the robust guidance strategy is quite similar to the terminal time obtained on the reference optimal solution.

Table 4. Final state and maximum path values

Quantity	Reference Solution	Guidance with only J_1	Guidance with J_1 or J_2
$h(t_f)$	10.43 km	10.84 km	10.46 km
$v(t_f)$	540 m/s	538 m/s	539 m/s
$\theta(t_f)$	0.306 rad	0.299 rad	0.308 rad
$\phi(t_f)$	-0.137 rad	-0.135 rad	-0.137 rad
$\gamma(t_f)$	-0.246 rad	-0.243 rad	-0.254 rad
$\psi(t_f)$	2.07 rad	2.07 rad	2.06 rad
t_f	316.4 s	309.7 s	369. s
$q_{\max}$	6.825 KPa	6.918 KPa	6.606 KPa
$A_{\max}$	$5.00g_0$ m/s ²	$5.07g_0$ m/s ²	$4.84g_0$ m/s ²
$Q_{\max}$	66.2 W/cm ²	69.4 W/cm ²	68.9 W/cm ²

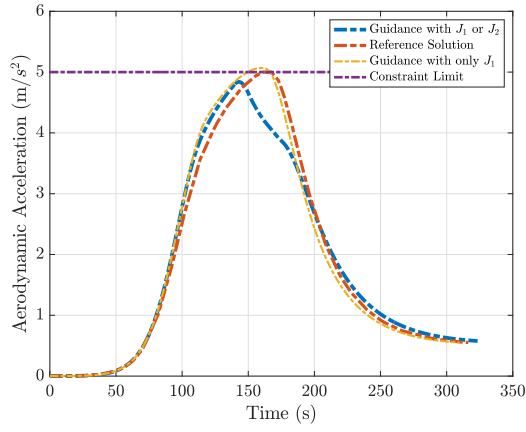


(a) $h(t)$ vs. t .

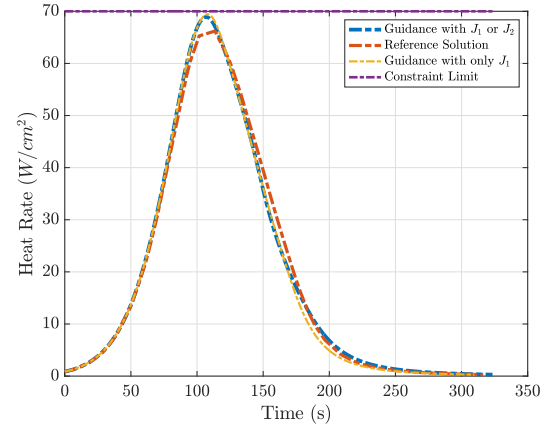


(b) $v(t)$ vs. t .

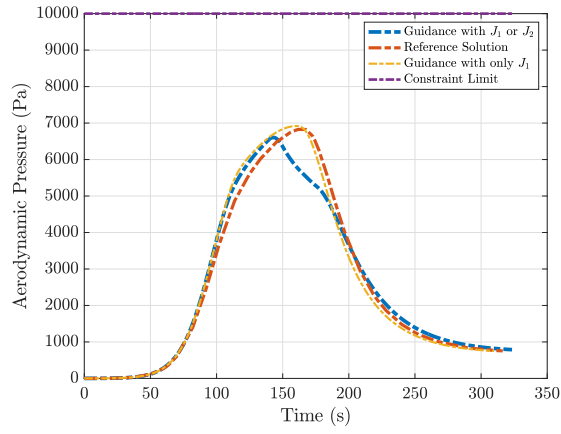
Figure 1. Altitude and speed using a +5 % error in the atmospheric density



(a) $A(t)$ vs. t .



(b) $\dot{Q}(t)$ vs. t .



(c) $q(t)$ vs. t .

Figure 2. Path constraint values using a +5 % error in the atmospheric density.

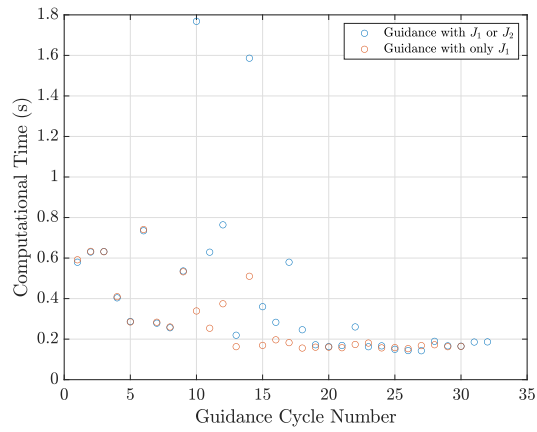


Figure 3. Computation time when re-solving the optimal control problem at each guidance cycle.

Random Perturbation in Surface Level Density

As stated earlier, for the case of a random perturbation, the value of the surface level density at the start of each guidance cycle was drawn from a uniform distribution on $[\rho_0, 1.05\rho_0]$. It should be noted that the actual realizations of the random sequence of values of $\tilde{\rho}_0$ differs every time the guidance simulation is performed (due to the use of a random number generator) which in turn produces different results for each realization. While the random number generator can be seeded to produce the same sequence each time, such an approach is beyond the scope of this paper. Therefore, in this section the results obtained for one realization of a sequence of values of $\tilde{\rho}_0$ is shown in order to see the effect of using a random perturbation in surface level density. In particular, it is seen from Fig. 5(a) that the robust guidance method described in this paper is able to avoid path constraint violations. Furthermore, as with the constant offset in surface level density, in this case the guidance method switched the objective functional twice throughout the guidance updates. Additionally, from Fig. it is seen that the computational times are of the same order of magnitude as seen in the previous section and generally decrease as the horizon shrinks. While the results obtained in this paper are for a single realization of random values of surface level density, a thorough analysis using Monte Carlo simulation is required in order to obtain a more accurate assessment of the approach described in this paper. Such a Monte Carlo analysis is, however, beyond the scope of this paper.

Table 5. Final state and maximum path values

Quantity	Reference Solution	Guidance with J_1 or J_2
$h(t_f)$	10.43 km	10.26 km
$v(t_f)$	540 m/s	539.2 m/s
$\theta(t_f)$	0.306 rad	0.310 rad
$\phi(t_f)$	-0.137 rad	-0.138 rad
$\gamma(t_f)$	-0.246 rad	-0.256 rad
$\psi(t_f)$	2.07 rad	2.07 rad
t_f	316.4 s	325.2 s
$q_{\max}$	6.825 KPa	6.54 KPa
$A_{\max}$	$5g_0$ m/s ²	$4.79g_0$ m/s ²
$Q_{\max}$	66.2 W/cm ²	69.04 W/cm ²

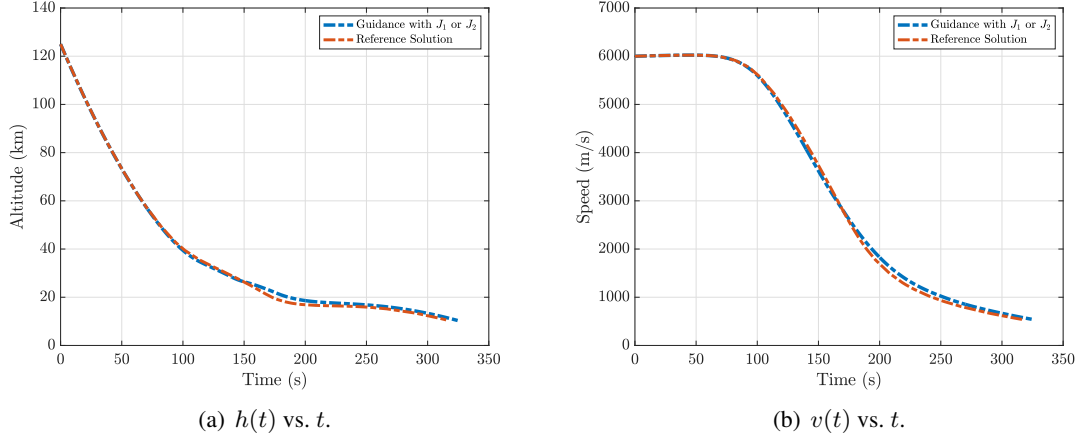


Figure 4. Altitude and speed using a time varying error in the atmospheric density

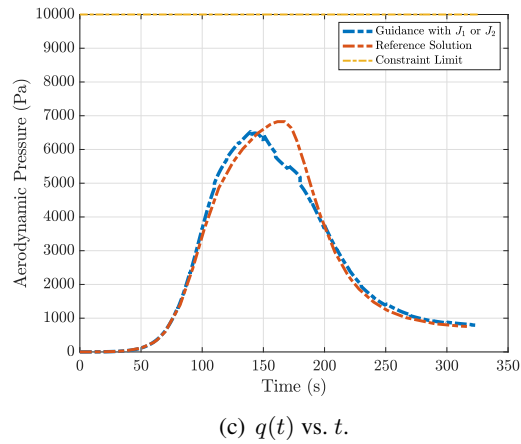
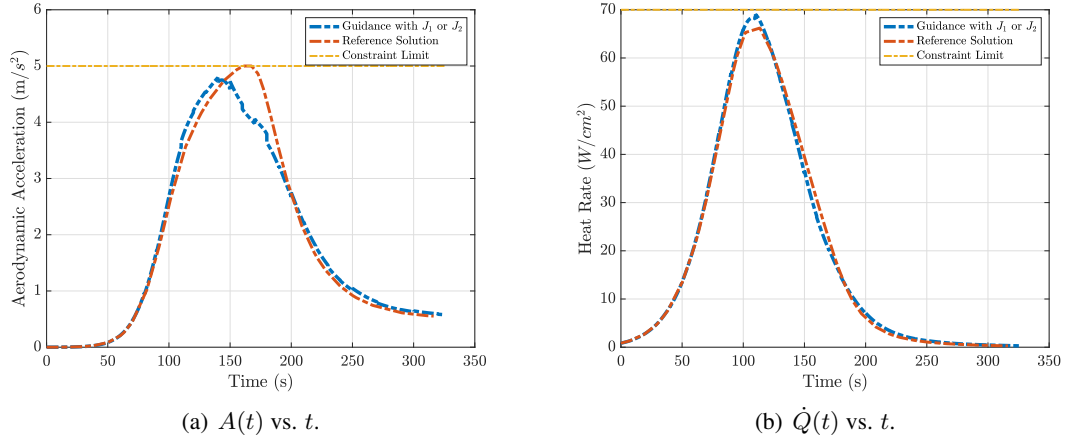


Figure 5. Path constraint values using a time varying error in the atmospheric density.

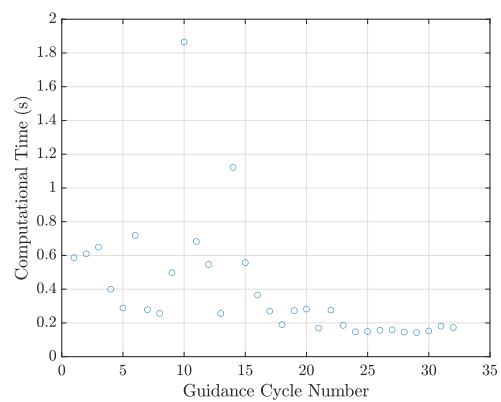


Figure 6. Computational time when re-solving the optimal control problem at each guidance cycle

CONCLUSIONS

An robust optimal guidance method has been proposed and applied to path-constrained a Mars entry problem. The goal was to implement real-time optimal control using mid-course corrections to guide the vehicle in the presence of modeling errors and perturbations. Feasibility was ensured by preventing path constraint violations with the newly developed optimal guidance strategy. In particular, the possibility of maintaining feasibility has been improved by switching the objective functional in the optimal control problem when the solution lies within a user-specified threshold of a path constraint limit. This alternative objective functional is designed to maximize the margin between the path constraint function value and the path constraint limit. In order to demonstrate the approach, two cases of an altitude maximization Mars entry problem have been considered. The first case is one where the reference surface level density is is offset by a constant five percent in the perturbed model, while the second case is one where the surface level density is drawn from a uniform distribution. For both cases considered the objective functional needed to be switched on only two guidance cycles, and for both cases the guidance strategy maintained feasibility with respect to the path constraints. On the other hand, when the objective functional was not switched, the acceleration load limit was violated. The approach developed in this paper appears to be a viable method for improving robustness in optimal guidance.

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