Exponential Stabilization of Periodic LIP Walking on a Horizontally Moving Surface

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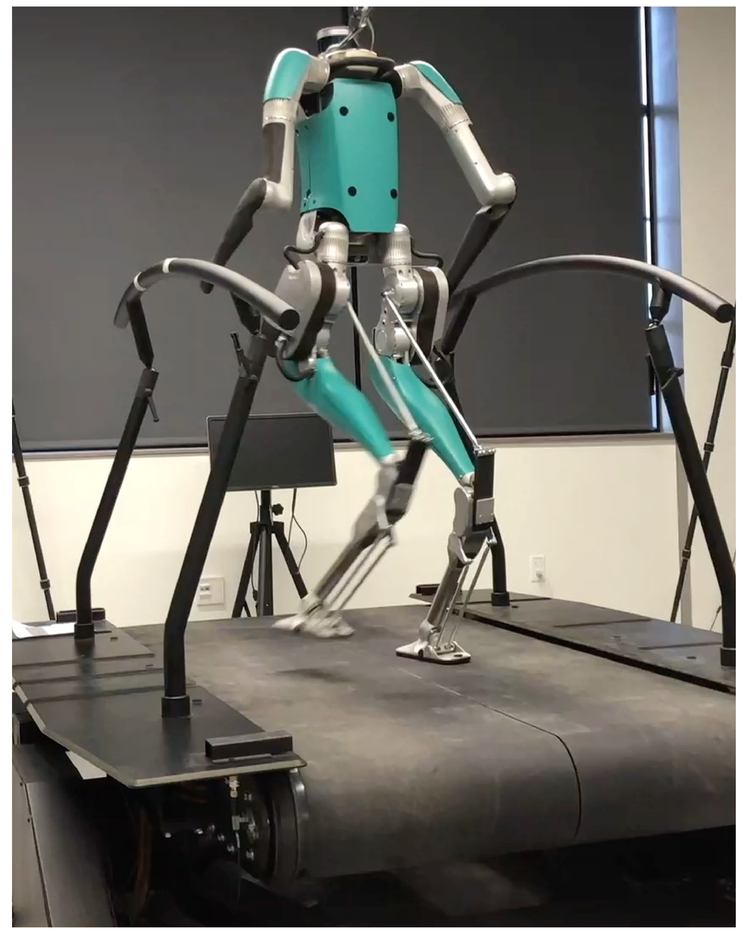
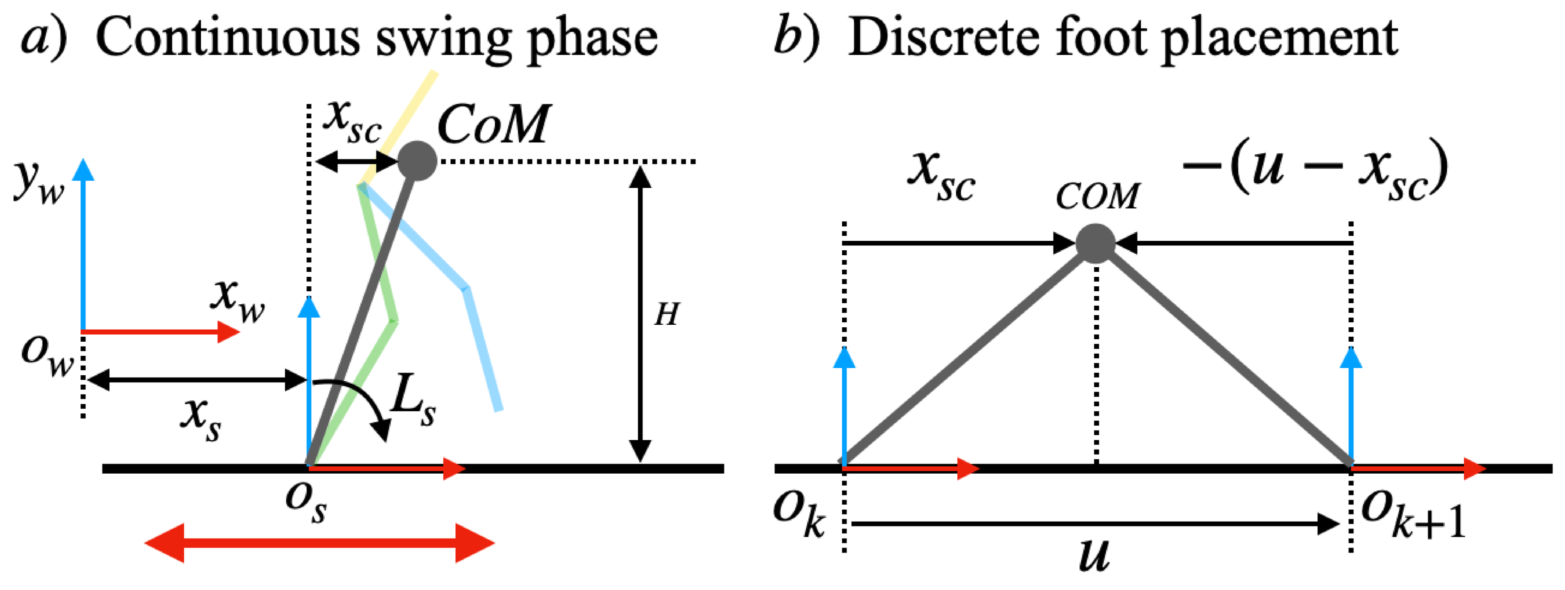
ABSRTACT

- Controlling a bipedal robot that walks on a dynamic rigid surface (DRS) is a challenging task due to the complexity of the associated robot dynamics [1-3].
- We introduce a hybrid linear inverted pendulum (LIP) model for underactuated bipedal walking on a DRS.
- We also propose a discrete-time stepping controller to provably stabilize the periodic gait of the hybrid LIP.



MOTIVATION: *Digit's proprietary controller seems to fail to sustain walking on a swaying treadmill.*

METHODOLOGY



Illustrate of the hybrid walking cycle of an LIP

- ***Continuous-time dynamics during a swing phase*** [4]:

$$\dot{x}_{sc} = \frac{L_s}{mH} - \dot{x}_s(t),\ \dot{L}_s = mgx_{sc}$$

Discrete-time dynamics at a foot landing:

$$\Delta x_{sc} = -(u - x_{sc}) - x_{sc} = -u,\ \Delta L_s = 0$$

- ***Discrete-time feedback control via foot stepping*** [5]:

$$u = u^* + \mathbf{K}(\mathbf{x} - \mathbf{x}^*)$$

where $\mathbf{x} = [x_{sc}, L_s]$ is the state, u^* is the desired step length, $\mathbf{x}^*$ is the desired pre-impact state, and $\mathbf{K}$ is the state feedback gain.

- ***Hybrid, nonhomogeneous LIP dynamics under stepping control***:

$$\begin{cases} \begin{bmatrix} \dot{x}_{sc} \\ \dot{L}_s \end{bmatrix} = \begin{bmatrix} 0 & 1/mH \\ mg & 0 \end{bmatrix} \begin{bmatrix} x_{sc} \\ L_s \end{bmatrix} - \begin{bmatrix} \dot{x}_s(t) \\ 0 \end{bmatrix}, & t \neq \tau_k\ (k = 1, \dots, N) \\ \begin{bmatrix} \Delta x_{sc} \\ \Delta L_s \end{bmatrix} = \begin{bmatrix} -\mathbf{K} \\ \mathbf{0}_{1\times 2} \end{bmatrix} \begin{bmatrix} x_{sc} \\ L_s \end{bmatrix} + \begin{bmatrix} \mathbf{K}\mathbf{x}^* - u^* \\ 0 \end{bmatrix}, & t = \tau_k\ (k = 1, \dots, N) \end{cases}$$

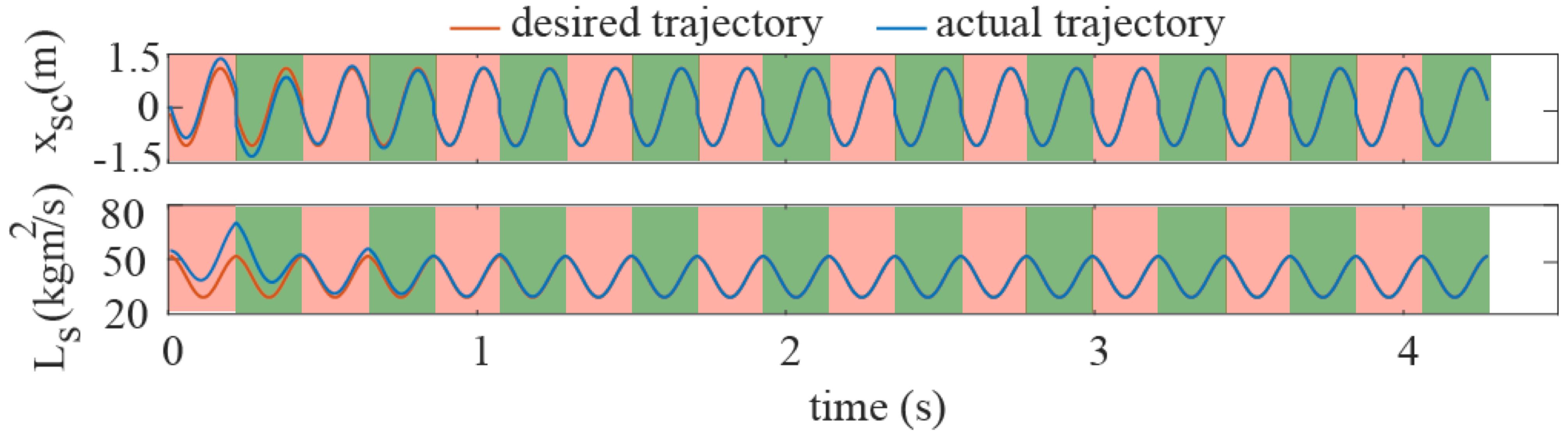
where $x_s(t)$ is the horizontal position of the support point with $x_s(t) = x_s(t+T)$ for some $T > 0$.

- ***Stability condition*** [6]:
  - If all eigenvalues of the monodromy matrix associated with the homogeneous equation of the hybrid LIP system are less than one in modulus, then the origin of the homogeneous eqution is exponentially stable.
  - Accordingly, the T-periodic solution $\varphi(t)$ (with $\varphi(t) = \varphi(t+T)$) of the hybrid nonhomogeneous system is exponentially stable.

- ***Trajectory Optimization:***
  We obtain the physically feasible solution $\varphi(t)$ and the stabilizing feedback gain $\mathbf{K}$ through trajectory optimization.

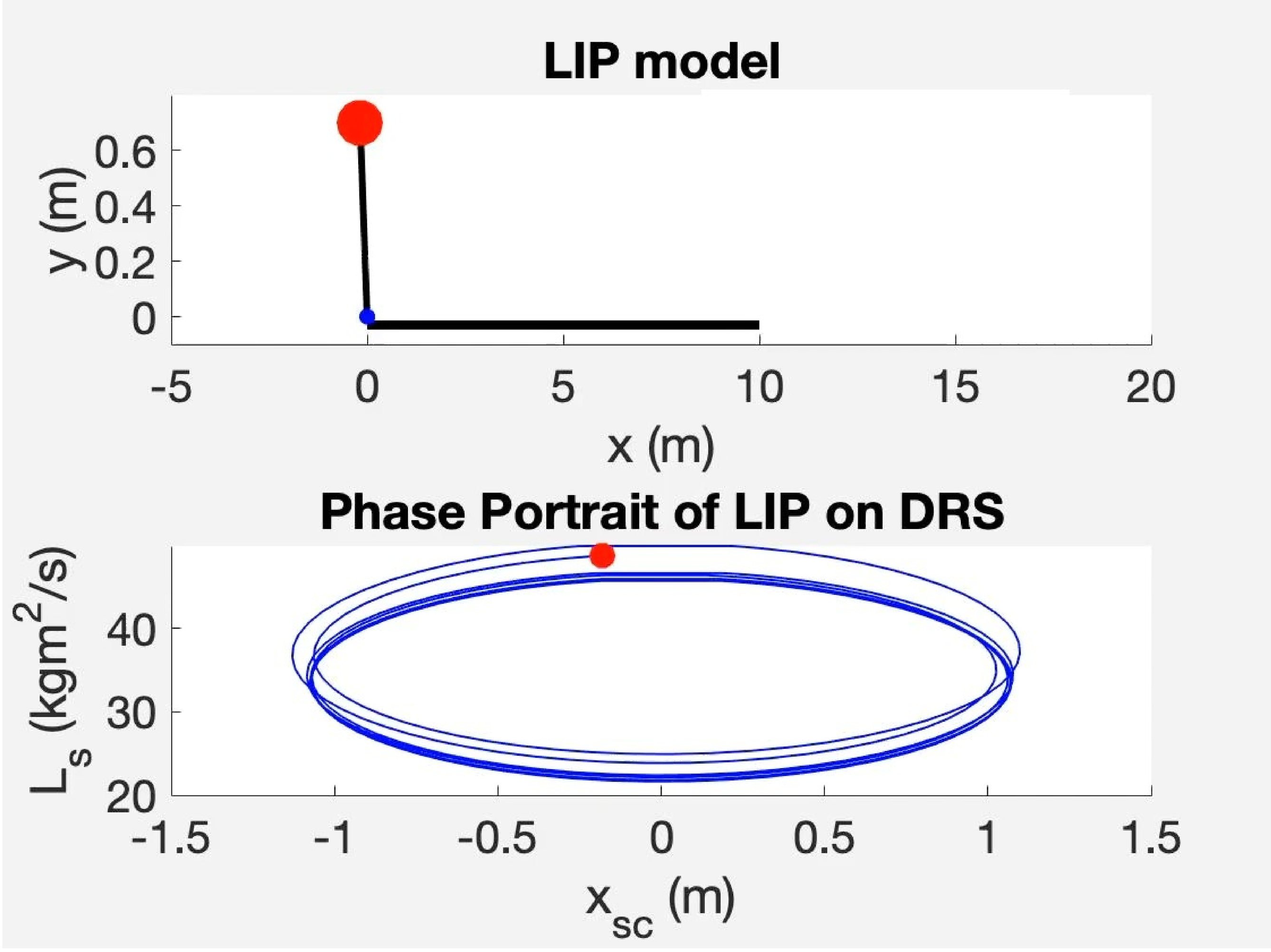
RESULTS

- The proposed stability condition and controller design sustain stable walking on a horizontally moving surface.
- Surface displacement: $x_s = \sin(27.4t)$ m.



Exponential trajectory tracking results of the hybrid LIP under the proposed stability conditions and stepping controller.

(The edges between different background colors indicate the foot landing timings.)



Animation of exponential stabilization of the T-period solution

FUTURE WORK

Our group is currently implementing the proposed LIP stepping controller as a high-level planner of the Digit bipedal humanoid robot to enable stable walking on a periodically swaying treadmill.

REFERENCE

[1] A. Iqbal, Y. Gao, and Y. Gu, "Provably stabilizing controllers for quadrupedal robot locomotion on dynamic rigid platforms," IEEE/ASME Trans. Mechatron., 2021

[2] Y. Gao, C. Yuan, and Y. Gu, "Invariant extended Kalman filtering for hybrid models of bipedal robot walking," IFAC-PapersOnLine 54, 2021.

[3] Y. Gao, C. Yuan, and Y. Gu, "Invariant filtering for legged humanoid locomotion on dynamic rigid surfaces," IEEE/ASME Trans. Mechatron., 2022, accepted

[4] Y. Gong, and J. Grizzle. "Angular momentum about the contact point for control of bipedal locomotion: Validation in a lip-based controller," arXiv preprint arXiv:2008.10763, 2020.

[5] X. Xiong, and A. Ames. "3-D underactuated bipedal walking via H-LIP based gait synthesis and stepping stabilization," IEEE Trans. Robot., 2022.

[6] D. Bainov, and P. Simeonov. "Impulsive differential equations: periodic solutions and applications," Vol. 66. CRC Press, 1993.