Examination of the Volumetric Strain Potential of Liquefied Soil with a Database of Laboratory Tests

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ABSTRACT

The primary basis of the empirical methods for estimating post-liquefaction ground settlement is the laboratory data presented in Ishihara and Yoshimine (1992). These data are from one series of cyclic simple shear tests performed on one uniform clean sand reconstituted to three relative densities and tested at one effective confining stress. It is not clear if these data are applicable to other clean sands with other gradations, nonplastic silty sands, and nonplastic silts. A comprehensive database of post-liquefaction volumetric strain test results on nine additional clean sands, two gravels, three silty sands, and five silts is compiled to examine trends over a wider range of soils. The general trends of the larger database of clean sand test data provide the basis for a new relationship between post-liquefaction volumetric strain and the cyclic-induced maximum shear strain for clean sand. Relationships are then extended to nonplastic silty sand and nonplastic silt using relative density to characterize the volumetric strain potential of clean sand and nonplastic silty sand and silt.

INTRODUCTION

Saturated soil under cyclic loading undergoes an accumulation of shear strain that can increase pore-water pressure. Depending on the magnitude and duration of the cyclic loading, the generated excess pore-water pressures can trigger liquefaction. At a free-field, level ground site, the dissipation of the excess pore-water pressure is followed by an accumulation of volumetric strain in the soil due to sedimentation and reconsolidation processes that cause ground settlement. The quantification of the likely amount of ground deformation resulting from this liquefaction effect is important because excessive ground deformation can result in significant losses. Hence, researchers have developed empirical methods based on soil and seismic demand proxies that capture these complex phenomena for estimating liquefaction-induced ground settlement due to the volumetric strain potential of liquefiable soil. Empirical models are typically used in earthquake engineering practice because they provide reliable estimates of soil response, whereas most soil constitutive models do not capture well all key aspects of post-liquefaction soil deformation in post-liquefaction reconsolidation settlement.

The primary basis of the empirical methods for estimating post-liquefaction ground settlement (e.g., Zhang et al. 2002, Idriss and Boulanger 2008) is the laboratory data developed by Nagase and Ishihara (1988) as interpreted by Ishihara and Yoshimine (1992). These laboratory data which are shown in Figure 1 are informative. However, the data are from one series of cyclic simple shear tests performed on one uniform clean sand, the Fuji River sand, reconstituted to three relative densities of 47%, 73%, and 93%, and tested at one vertical

effective stress (196 kPa). Engineers and researchers have questioned if the volumetric strain vs. shear strain relationships developed by Ishihara and Yoshimine (1992) based on just one uniform clean sand are applicable to other clean sands with other gradations, nonplastic silty sands, and nonplastic silts (e.g., Bray et al. 2017). To address this question, a comprehensive database of post-liquefaction volumetric strain test results on nine additional clean sands, two gravels, three silty sands, and five silts is compiled and interpreted. The general trends of the larger database of clean sand test data provide the basis for a new relationship between post-liquefaction volumetric strain and the cyclic-induced maximum shear strain for clean sand. Relative density is employed as a proxy for the state of the soil in the development of relationships for nonplastic silty sand and nonplastic silt in addition to clean sand. The proposed model of post-liquefaction volumetric strain provides a basis for re-interpreting current post-liquefaction ground settlement procedures.

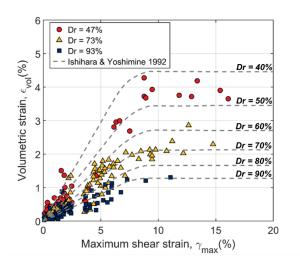


Figure 1. Uniform clean sand volumetric strain data of Ishihara and Yoshimine (1992).

VOLUMETRIC STRAIN POTENTIAL LABORATORY DATABASE

With the objective of expanding the original work of Ishihara and Yoshimine (1992) on uniform clean Fuji River sand, a comprehensive database of laboratory testing of soil ranging from clean gravel to silty soils was compiled. The laboratory data included in the database contain information of grain size distribution, initial void ratio or relative density, test type and conditions, shear strain and volumetric strain measurements. Information that summarizes the volumetric strain potential laboratory data collected through this study are presented in Table 1. In this table, the source of the laboratory test data, the soil type tested, the type of cyclic test, number of tests, and test specimen conditions (i.e., specimen preparation method, initial relative density (D_r) , and effective confinement stress (σ'_{vc}) are shared. Figure 2 displays the range of particle gradations of the soils in the assembled laboratory cyclic testing database using the Test Series Identification (ID) number provided in the left column of Table 1.

Seed (1979) used the term cyclic mobility to describe the overall mechanical response of sand to cyclic loading leading to liquefaction with different strain potential. Cyclic mobility with limited strain potential is typically observed in medium to dense angular sands and loose to dense silts as a result of cyclic stress applications in which the pore-water pressure ratio (r_u) momentarily achieves high values. The limited nature of this response is because the rate of

strain development is restrained by a tendency for soil dilation that temporarily increases soil stiffness during cyclic loading which prevents large, uncontrollable deformation. This type of response is referred to as cyclic mobility. Conversely, cyclic mobility with unlimited strain potential is associated to clean rounded loose sand undergoing uncontrolled deformation due to a sustained drop in the effective confining pressure accompanied by a low residual strength. This condition is typically triggered by high r_u values that are sustained during the cyclic loading. This type of soil response is termed flow liquefaction because the soil has the potential to flow if the static driving shear stress exceeds the soil's residual strength. The generation of high r_u values in level soil sites undergoing cyclic mobility or flow liquefaction eventually leads to the dissipation of excess pore-water pressure which in turn leads to post-liquefaction reconsolidation volumetric strain. Large volumetric strains can be produced if flow liquefaction and resedimentation occur.

Table 1. Liquefied Soil Strain Potential Laboratory Test Data

ID	Study	Soil Type	Type of test	Number of tests	USCS	Test Specimen Conditions		
				ϵ_{vol} - γ_{max}		Preparation method	Initial Dr (%)	Confinement (kPa)
1	Tatsuoka et al. (1984)	Sengenyama sand	CTS	12	SP	Dry pluviation	55-86	196
2	Chin (1987)	Li-kong sand	CTX	16	SP	Moist tamping	60	73.6
3	Ishihara and Yoshimine (1992) (1)	Fuji River sand	CSS	200	SP	Water pluviation	47, 73, 93	196
4	Shamoto et al. (1996)	Toyoura sand	CTX	12	SP	NA	50	100
5	Toriihara et al. (2000)	Silty sand	CTX	25	SM	Dry pluviation	72-112	NA
6	Wu (2002)	Monterrey No. 0/30	CSS	35	SP	Moist pluviation	38-87	34-182
7	Sancio (2003)	Adapazari silty soils	CTX	32	CL, ML, MH	Undisturbed	NA	25-300
8	Tsukamoto et al. (2004)	Toyoura sand	Large CTX	43	SP	Dry pluviation	60-80	98
		Kobe gravelly soils	Large CTX	29	GW	Moist tamping	65-90	98
9	Donahue (2007)	Adapazari silty soils	CTX	2	CL	Slurry deposition / wet pluviation	NA	50
10	Porcino and Caridi (2007)	Ticino silica sand	CSS	2	SP	Water sedimentation	40-75	100
11	Cetin et al. (2009)	Kizilirmak River sand	CTX	35	SP	Dry pluviation / moist tamping	35-85	100
12	Bilge (2010)	Adapazari and Ordu soils	CTX	41	ML - CH	Undisturbed	NA	NA
13	Thevanayagam and Shenthan (2010)	Otawa F-55 sand	CTX	6	SP	Moist tamping	32-81	100
14	Wang and Luna (2014)	Collinsville Silt	CTX	12	ML	Slurry consolidation	NA	90
15	Markham (2015)	Christchurch CBD soils	CTX	21	SP-SM / ML	Undisturbed	58.3-86.3	36.7-210
16	Parra (2016)	Ottawa F-65 sand	CSS	14	SP	Dry deposition / air pluviation	24-85	50-404
17	Beyzaei (2017)	Christchurch soils	CTX	38	ML, CL, SP-SM	Undisturbed	NA	35-113
18	Hubler (2017)	Ottawa C109 sand	Large CSS	2	SP	Dry tamping	50, 90	100
		Pea Gravel	Large CSS	2	GP	Dry tamping	44, 81	100

^{1.} Ishihara and Yoshimine (1992) reinterpreted the test results first published by Nagase and Ishihara (1988)

^{2.} CTS: Cyclic torsional shear; CSS: Cyclic simple shear test; CTX: Cyclic triaxial test

^{3.} NA: Not available

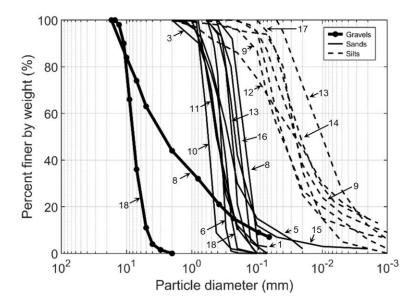


Figure 2. Grain size distribution of the soils in the compiled database. Numbers indicate soil ID listed in Table 1.

In the studies presented in Table 1, liquefaction was typically defined by the test specimen achieving a transient $r_u \approx 100\%$ condition or reaching a single-amplitude shear strain, γ_{cyc} , s_A of 3-5%. Only in a few cases was the testing continued to large strain or data provided to accurately classify the liquefaction phenomenon as either cyclic mobility or flow liquefaction. Because the subsequent development of post-liquefaction volumetric strain (ε_{vol}) is the focus of this study, for practical purposes in the following sections, the term "liquefaction" is used to characterize the condition of a soil that was sheared enough to reach either cyclic mobility or flow liquefaction.

The uniform clean sand data presented in Table 1 was obtained from Tatsuoka et al. (1984), Chin (1987), Shamoto et al. (1996), Wu (2002), Porcino and Caridi (2007), Cetin et al. (2009), Thevanayagam and Shentan (2010), Parra (2016), and Hubler (2017) in addition to Ishihara and Yoshimine (1992). These test results form an enlarged database, provide salient insights on the ε_{Vol} - γ_{max} relationship for clean sand and importantly provides the opportunity to evaluate the applicability of the commonly used empirical methods based on the Ishihara and Yoshimine (1992) to a wide range of clean sands.

The uniform gravel (GP) γ_{max} - ε_{vol} data from Hubler (2017) are comparable to most of the uniform sand data. Similar trends, but with slightly higher ε_{vol} values, were observed in the Tsukamoto et al. (2004) gravel (GW-GM), FC = 8%. Test results from Markham (2015) and Beyzaei (2017) provide results on silty sand which are worth comparing to the larger amount of data on uniform clean sand. Non-plastic silt response is explored through examination of the test data by Beyzaei (2017). Sancio (2003), Donahue (2007), Bilge (2010), Wang and Luna (2014) and Beyzaei (2017) provide data to examine low-plasticity silt and plastic clayey soil using state indexes other than D_r , which is outside the scope of this paper. Overall, the laboratory test database collected through this study contains a 579 γ_{max} - ε_{vol} data points. The enlarged test database provides the opportunity to better characterize the volumetric strain and shear strain potential of liquefied soil. In this paper, the datasets on γ_{max} - ε_{vol} for clean sands and gravel materials are first examined. Silty sand test data and non-plastic silt are then explored.

VOLUMETRIC STRAIN POTENTIAL OF CLEAN SAND

Data from nine different uniform clean sands were collected and processed to produce 177 datapoints in addition to those presented in Ishihara and Yoshimine (1992). The cyclic testing is on sands from different origins, formation process, gradation, and D_r values ranging from 24% to 92%. This enlarged database supports the development of a new generalized γ_{max} - ε_{vol} model for clean uniform sands over a wider range of D_r . Accordingly, the data were binned in 10% D_r bins to explore the influence of the sand's initial state on its post-liquefaction response and to develop representative central values for ε_{vol} in addition to plus and minus one standard deviation ($\pm 1\sigma$) ranges for each bin.

Examination of the expanded clean uniform sand test data provides useful insights (as noted also by others, e.g., Tsukamoto et al. 2004, and Ishihara and Yoshimine 1992), including: (1) ε_{vol} measurements have significant scatter for a given set of initial density and loading conditions; (2) isotropic reconsolidation (triaxial conditions) and K_o reconsolidation (zero lateral strain conditions) produce similar amounts of ε_{vol} ; (3) ε_{vol} depends primarily on the induced γ_{max} and not the type of loading; (4) ε_{vol} correlates better to γ_{max} than to the ratio of pore-water pressure ratio ($r_u = u_e / \sigma'_{vo}$) because while r_u reaches a maximum of unity at initial liquefaction ($r_{u,max}$ = 100%), ε_{vol} keeps increasing for strain levels beyond initial liquefaction; (5) a direct relationship between ε_{vol} and γ_{max} exists, (6) an inverse relationship between ε_{vol} and D_r exists; (7) ε_{vol} increases rapidly as the soil approaches initial liquefaction; and (8) ε_{vol} increases linearly with increasing γ_{max} up to a limiting shear stain of about γ_{max} = 6% to 10%, and then ε_{vol} remains relatively constant (within the limits of its inherent variability) at larger shear strain. Lastly, the ε_{vol} results can be smaller or higher than those estimated with the Ishihara and Yoshimine (1992) relationships developed from tests on solely Fuji River Sand.

Data of four of the eight D_r bins generated are shown in Figure 3, where additional uniform clean sand γ_{max} - ε_{vol} data are shown along with the Ishihara and Yoshimine (1992) original data points in lighter color for comparison. Data from Wu (2002), Markham (2015), and Parra (2016) illustrate that the amount of spread in the measured ε_{vol} could be as large as ±1%. Despite this inherent variability, the data across the different D_r bins support a linear relationship between γ_{max} - ε_{vol} up to $\gamma_{max} \approx 7\%$ to 10%, and past that range a ε_{vol} plateau is observed. The additional data shown in Figure 3a and 3b indicate that for low D_r values (< 50%) the Ishihara and Yoshimine (1992) relationship shown in gray broken lines could overestimate slightly ε_{vol} . Conversely, for the high D_r (>70%) data shown in Figure 3c and 3d, the Ishihara and Yoshimine (1992) relationship slightly underestimates ε_{vol} . The additional sand data also enable the investigation of the effects of combining different datasets. For the Ishihara and Yoshimine (1992) data, the variation of ε_{vol} about its mean trend is less than 0.5%; however, if two different datasets are analyzed together, the variation can exceed 1%.

VOLUMETRIC STRAIN RESPONSE OF NONPLASTIC SILTY SAND

Cubrinovski and Ishihara (2000) and Jefferies and Been (2016) showed that, with all other conditions maintained, nonplastic fines increase the sand's compressibility which reduces its penetration resistance while also reducing its cyclic resistance. Empirical liquefaction triggering methods deal with this difference in resistance in more compressible nonplastic silty sand by introducing a corrected equivalent-clean-sand penetration resistance which is obtained by adding a correction for the effect of fines to the measured resistance (known as a fines content (FC)

correction). This correction maps the penetration resistance of a silty sand to that of an equivalent-clean-sand so that the liquefaction evaluation can be performed in the clean sand domain where most of the adjustment factors to the cyclic resistance and magnitude scaling factor to the cyclic demand are better constrained. However, researchers have questioned whether this corrected equivalent-clean-sand penetration resistance should be used directly with empirically based clean sand γ_{max} - ε_{vol} models (e.g., Bray et al. 2017). This issue warrants the examination of silty soil test data to better understand the post-liquefaction response of silty soil. An empirical evaluation that could help bridge both approaches is to explore if clean sands and nonplastic silty sands prepared at similar D_r and sheared to similar γ_{max} develop also similar ε_{vol} .

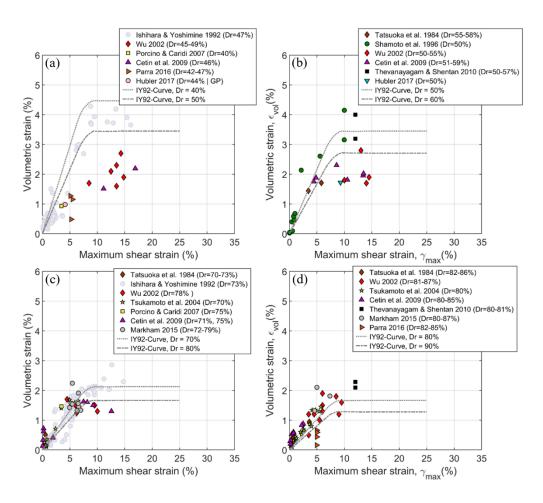


Figure 3. Clean uniform sand $\varepsilon_{vol} - \gamma_{max}$ data for (a) $D_r = 40\%$ - 50%, (b) $D_r = 50\%$ - 60%, (c) $D_r = 70\%$ - 80%, and (d) $D_r = 80\%$ - 90%. Ishihara and Yoshimine (1992) data are shown in light blue for reference.

In estimating D_r , it should be remembered that e_{max} and e_{min} are not intrinsic soil properties. Instead, they represent the loosest and densest states possible in a standard test (e.g., ASTM D4254 and JIS A 1224:2000). Cubrinovski and Ishihara (2002) found e_{max} and e_{min} could be reliably estimated for sands with FC up to 30% using the Japanese standard JIS A 1224:2000. Ishihara et al. (2016) later obtained e_{max} reliably using the Japanese standard for silty sands with FC up to 50%. Mijic et al. (2021) recently determined reasonable e_{max} and e_{min} values using the

Japanese standard test method on nonplastic silt from Christchurch with FC up to 70%. Also, their e_{min} and e_{max} values were not unreasonable for nonplastic silt with 100% fines. Thus, D_r is used in this study to enable sand, silty sand, and nonplastic silt data to be compared and interpreted. The silty sand data available to this study were taken from Beyzaei (2017) with FC = 34%, and from Markham (2015) with FC = 44% (for a soil with PI = 4). In these studies, e_{max} and e_{min} were measured using the Japanese standard.

Figure 4 shows data corresponding to the 60% - 70% D_r bin. The silty sand test results are plotted in solid black whereas clean sand data are plotted in lighter colors for comparison. In this case, both the nonplastic and low-plasticity silty sands were sheared to about $\gamma_{max} \approx 6\%$ and after liquefaction both developed $\varepsilon_{vol} \approx 2\%$, which is consistent with the observed response of clean sands. Although reasonable trends are observed, the amount of data available on silty sands is limited. Additional empirical data on silty sands is warranted for the advancement of our understanding of liquefaction effects on silty sand deposits.

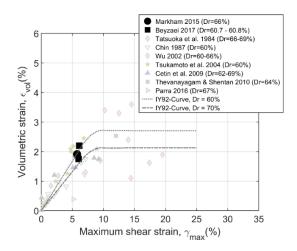


Figure 4. Nonplastic to low plasticity sands $\varepsilon_{vol} - \gamma_{max}$ data for Dr = 60% - 70%. Clean sand data shown in light colors.

VOLUMETRIC STRAIN RESPONSE OF NONPLASTIC SILT

Liquefaction of fine-grained soils has been reported after major earthquakes such as the 1999 Kocaeli and 1999 Chi-Chi earthquakes, and more recently identified after the 2010-2011 Christchurch earthquake sequence. After a comprehensive field and laboratory investigation, Bray and Sancio (2006) found that liquefaction of shallow low-plasticity silts can induce significant post-liquefaction deformation. Bray and Sancio (2006) found that fine-grained soils of low plasticity ($PI \le 12$) and high water contents (w/LL > 0.85) could undergo cyclic mobility if intensely shaken. Cubrinovski and Ishihara (2000) showed that the flow potential of sands with FC > 30% is higher than clean sand and mainly controlled by the fine matrix. Later, Cubrinovski and Ishihara (2002) investigated the e_{max} and e_{min} characteristics of clean sands, sands with fines, and silty soils. They identified that for soils with FC greater than 20% to 30%, the finer fraction controls particle fabric. These two observations indicate that sands with a FC greater than about 30% respond more like a silt than a clean sand. Hence, silty sand with FC greater than about 30% are combined with the data on silts classified as per USCS to examine the volumetric response of nonplastic silt.

Bray and Sancio (2006), Markham (2015), and Beyzaei et al. (2018) showed that nonplastic silt liquefies in a similar manner as medium angular clean sand in what is termed cyclic mobility. A total of eleven γ_{max} - ε_{vol} data points were collected and following the previously discussed findings of nonplastic silts showing a similar cyclic response as medium angular clean sands, the volumetric reconsolidation of nonplastic silts is compared to the larger clean sand data, shown in lighter colors, in the two of the D_r bins in Figure 5. The non-plastic silts do reconsolidate similar amounts compared to clean sand across a wide range of densities. For example, the nonplastic silts with $D_r = 61\%$ to 67% in Figure 5a reconsolidate similar amounts compared to Cetin et al. (2009) Kizilirmak River sand ($D_r = 62\%$ to 69%) and Tsukamoto et al. (2004) Toyoura sand with $D_r = 60\%$. Similar agreement between nonplastic silts and clean sands post-liquefaction reconsolidation is observed for dense sands as shown in Figure 5b. The test data provided in Figure 5 combined with that in Figure 4 when compared to the data in Figure 3 suggest nonplastic silty sands and nonplastic silts reconsolidate similar amounts to that of clean sand when at similar D_r . Additional studies are warranted to confirm these findings, but at this time, there is a reasonable basis to treat sand and nonplastic silty sand and silt similarly in terms of their relationship of γ_{max} - ε_{vol} lab data.

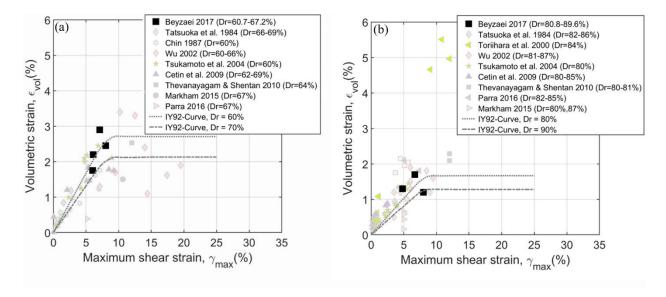


Figure 5. Nonplastic silts ε_{vol} — γ_{max} data for (a) Dr = 60% - 70% and (b) Dr = 80% - 90%. Clean sand data shown in light colors.

VOLUMETRIC STRAIN POTENTIAL OF NONPLASTIC SOIL

All data on nonplastic soil (i.e., clean sand, silty sand, and nonplastic silt) are combined to develop a model to characterize their volumetric strain potential. The regression equation was set to be a function of D_r up to a limiting plateau. Linear, quadratic, and exponential forms were evaluated considering not only how well the data are fit but also considering that the model should produce mechanistically sound responses over a wide range of densities. For example, data from Ishihara and Yoshimine (1992), Wu (2002), and Cetin et al. (2009) show that ε_{vol} increases at a higher rate as D_r decreases towards low D_r values. The resulting functional model to estimate ε_{vol} (in %) as function of γ_{max} (in %) for a specified value of D_r (in decimal) is:

$$\varepsilon_{vol} = 1.14 \cdot \exp(-2.0 \cdot D_r) \cdot \min(\gamma_{max}, \bar{\gamma}) \pm \varepsilon$$
 (1)

where $\bar{\gamma}$ is the limiting shear strain value of 8%. Using $\bar{\gamma} = 8\%$ from the possible range of 7% to 10% described previously, rendered slightly higher R² and slightly smaller standard deviations. ε represents the model residual with standard deviation $\sigma = 0.62$ in natural log units. The model's contours corresponding to D_r from 30% to 90% are shown in Figure 6 along with the Ishihara and Yoshimine (1992) clean sand curves. The results of the regression analyses using the enlarged database indicate ε_{vol} should vary within a narrower range than envisioned previously. For example, the proposed model estimates a maximum $\varepsilon_{\text{vol}} \approx 4.1\%$ at large shear strain for $D_r = 40\%$ soil, which is lower than the Ishihara and Yoshimine (1992) estimate of 4.5%. At a high $D_r = 90\%$, the proposed model calculates a maximum $\varepsilon_{\text{vol}} \approx 1.5\%$, which is higher than the Ishihara and Yoshimine (1992) estimate of 1.3%.

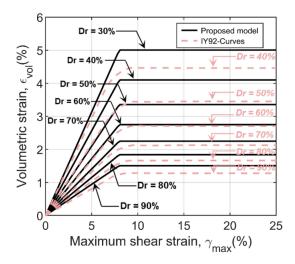


Figure 6. Nonplastic soils $\varepsilon_{vol} - \gamma_{max}$ relationships in terms relative density.

CONCLUSIONS

The primary basis of current empirical methods for estimating post-liquefaction ground settlement is one series of cyclic tests performed at one effective confining stress on one uniform clean sand reconstituted to three relative densities. A comprehensive database of post-liquefaction volumetric strain test results on ten clean sands, two gravels, three silty sands, and five silts was compiled to examine volumetric strain results over a wider range of soils. The general trends of the larger database of clean sand test data provide the basis for a new relationship between post-liquefaction volumetric strain and the cyclic-induced maximum shear strain for clean sand. The available cyclic tests on silty sand and nonplastic silt indicate that relative density can be used to characterize these materials so they can be combined with the clean sand data. The proposed volumetric strain model is fairly consistent with that of Ishihara and Yoshimine (1992), although slightly lower ε_{vol} values are calculated at low relative densities and slightly higher ε_{vol} values are calculated at high relative densities. The proposed model is supported by a database containing clean sands, silty sands, and nonplastic silts of different gradations. Thus, it provides greater confidence in its use for estimating post-liquefaction ground settlement for a wide range of soils, including soils with fines.

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