Control of the laser-induced vacuum decay by electronic phases

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We examine the laser-assisted electron-positron pair-creation process from the quantum vacuum in the presence of a binding potential with one optically active bound electron. If this core electron is initially prepared in a coherent superposition state of two resonant bound states, the electronic phase properties between both state excitations can be transferred to the positron during the pair-creation process. For example, the periodic Rabi population exchange between both electronic states modulates the temporal growth of the pair-creation probability and also leads to an Autler-Townes split positron energy spectra. Even more astonishing, for the case of different phases, for which the internal electronic dynamics (in the absence of pair creation) is identical, the positron's creation probability is different, suggesting that the vacuum decay process can "sense" the phase and not just the occupation number of the core electron. The field theoretical model of the laser assisted pair-creation process with subsequent electron capture can be mapped exactly onto two mutually independent (single-electron) ionization-like processes. This mathematical equivalency permits us to derive analytical solutions for the time evolution of the vacuum decay process under the rotating-wave approximation.

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There are two main and independent mechanisms in quantum electrodynamics by which two fermions can affect each other. The first mode of interaction is based on a direct exchange of photons, leading to the Coulomb force in the classical limit. The second mode is based on the Pauli exclusion principle, which prohibits any multiple occupation of a fermionic quantum state. Here the resulting Pauli blocking provides an alternative avenue by which the presence of one fermion can affect the state of another fermion without relying on any photons. To include the photonic exchange into quantum field theoretical descriptions is often difficult. For example, in nearly all models of the strong-field induced vacuum decay processes [1], where an external electromagnetic (or static electric) field is predicted to break down the quantum vacuum and create electron-positron pairs [2], the complicated photon-fermion couplings are approximated by the classical field approximation. Here only the Pauli-blocking mechanism induces multiparticle correlations. A well-known example of such a blocking effect is the Klein paradox [3,4], where an initial electron that has been injected into the spatially localized supercritical electric field region can suppress the positron creation probability during the scattering process, as the required final states of the associated created electron are already initially occupied (blocked) by the scattering electron.

In this paper, we suggest that, in addition to photon exchanges or the occupation number-based Pauli blocking, there is a third and nontrivial mechanism by which an already existing electron can modify the dynamics of the field-induced vacuum decay process. It turns out that the temporal growth pattern of the created electron-positron pairs can even be controlled by the *phase* information of this electron [5–7].

In order to illustrate this phase transfer mechanism in its simplest possible form, we examine the vacuum's decay in the presence of a highly charged nucleus, which carries an initial core electron (see Fig. 1) and can capture the created electrons. If this core electron is prepared in a coherent superposition of the nucleus' two resonant bound states, the electronic phase properties between both state excitations can be transferred to the positron during the pair-creation process beyond the usual (occupation number-based) Pauli-blocking mechanism [8]. In fact, the vacuum can even distinguish between those electronic phases that lead to identical occupation numbers, and (in the absence of pair creation) would preserve these occupation numbers at all times.

Let us begin our discussion by specifying the dynamics of the core electron in the *absence* of any pair creation. Here we assume that this optically active electron is initially prepared in the linear superposition quantum state $[\exp(i\phi)|1\rangle + |2\rangle]2^{-1/2}$ of the ground state $|1\rangle$ of energy E_1 and the first excited state $|2\rangle$ of energy E_2. The corresponding time dependence of the amplitudes under the action of the resonant laser field for the state $C_1(t)|1\rangle + C_2(t)|2\rangle$ is described by the well-known [9,10] two-level equations $i\hbar\, dC_1/dt = E_1 C_1 + \hbar\Omega_0 \sin(\omega t)\, C_2$ and $i\hbar\, dC_2/dt = E_2 C_2 + \hbar\, \Omega_0 \sin(\omega t) C_1$, where Ω_0 denotes the product of the electric field amplitude and the coupling strength between the two states. If we assume full resonance, E_2–$E_1 = \hbar\omega$, the evolution of the occupation number $|C_1(t)|^2$ depends crucially on the choice of the electron's initial phase ϕ relative to the initial

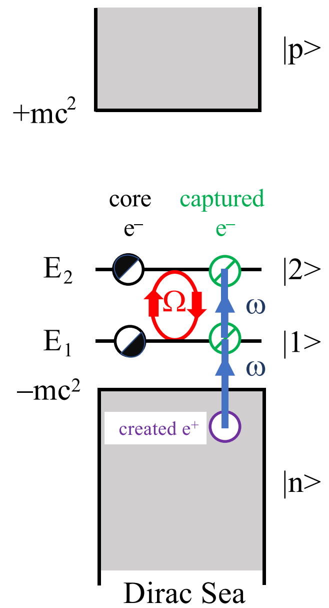

FIG. 1. The relevant continuum and discrete energy levels describing the laser field-induced electron-positron creation process from the quantum vacuum in the presence of a nuclear potential, which supports two bound states. While the created positron (hole in the Dirac sea) can escape to infinity, the associated created electron (green circle) is captured by the nucleus, which binds already one resonantly driven core electron (black circle). The parameters in our numerical simulations are $E_1 = -0.9\,mc^2$, $E_2 = -0.4\,mc^2$ and the laser's time dependence is given by $\Omega(t) = \Omega_0 \sin(\omega t)$ with (scaled) amplitude $\Omega_0 = 0.005\,mc^2/\hbar$ and frequency $\omega = 0.5\,mc^2/\hbar$.

phase of the laser field, as we illustrate in Fig. 2 for the four choices: $\phi = 0, \pi/2, \pi$, and $3\pi/2$.

The direct comparison with the probabilities obtained from the rotating-wave approximation (RWA) shows that this

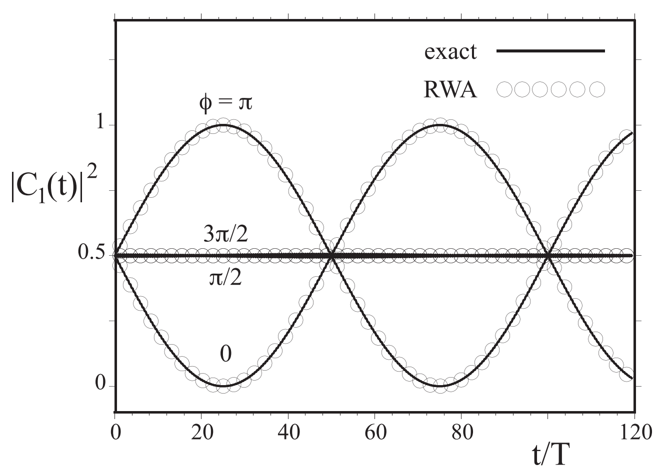

FIG. 2. The time evolution of the probability $|C_1(t)|^2$ to find the core electron in state $|1\rangle$. For the initial excitation amplitudes we chose $C_1(t=0) = \exp(i\phi)/2^{1/2}$ and $C_2(t=0) = 1/2^{1/2}$, with the four phases $\phi = 0, \pi/2, \pi$, and $3\pi/2$. The predictions based on the rotating-wave approximation are indicated by the open circles. The time is in units of the laser period $T = 2\pi/\omega$ with all parameters as in Fig. 1.

assumption is valid for our parameters. In fact, here we would derive $|c_1(t)|^2 = [1 - \sin(\Omega_0 t)]/2$, ½, $[1 + \sin(\Omega_0 t)]/2$ and ½, for the four choices: $\phi = 0, \pi/2, \pi$, and $3\pi/2$, respectively. We have used the lowercase letter c_1 for the rotating frame. We note that the constant population $|c_1(t)|^2 = 1/2$ for $\phi = \pi/2$ (and similarly for $3\pi/2$) can be explained by the fact that this initial state matches exactly one of the two dressed states, and not a superposition of both.

Next, we include the vacuum decay process, where we assume that the created electron can be captured by the bound state $|1\rangle$, while the associated created positron can escape to infinity. If the vacuum's decay rate is less then Ω_0, then at early times the pair-creation process affects the dynamics of the core electron only minimally. This means that the created electrons find their final state partially occupied with a time-dependent occupation number $|c_1(t)|^2$. If the Pauli blocking was the only interaction mode between the initially bound core electron and the vacuum decay process, then this mechanism would suggest that for times close to a quarter of the Rabi period $\pi/(2\Omega_0)$ the initial electronic phase $\phi = 0$ would *increase* the positron creation probability as the blocking population (proportional to $|c_1(t)|^2$) decreases. Similarly, the initial increase of $|c_1(t)|^2$ for the other phase $\phi = \pi$ should *decrease* the positron creation. The most important question in this paper, however, is whether the positron's creation probability can even detect any difference between those two phase choices ($\phi = \pi/2$ and $3\pi/2$), for which the "Pauli-blocking" occupation number $|c_1(t)|^2 = 1/2$ is identical and even remains so at all times.

To address this intriguing question, we have to change from the simple quantum-mechanical approach, which was sufficient to describe the single-particle dynamics of the core electron, to a fully quantum field theoretical formalism in order to predict the vacuum decay process. In the framework of computational quantum field theory [11], all dynamical features of the pair-creation process are modeled by the electron-positron field operator Ψ, whose space-time evolution is obtained by the Dirac equation $i\hbar\,\partial\Psi/\partial t = H\Psi$, with the usual Hamiltonian [1] given by

$$H = c\boldsymbol{\alpha}\cdot[\mathbf{p} - e\,\mathbf{A}(\mathbf{r},t)/c] + mc^2\boldsymbol{\beta} + eV(\mathbf{r}). \qquad (1)$$

The energy eigenstates of the Hamiltonian $H_0 \equiv c\boldsymbol{\alpha}\cdot\mathbf{p} + mc^2\boldsymbol{\beta} + eV(\mathbf{r})$ in the absence of the time-dependent field $\mathbf{A}$, defined by $H_0|\alpha\rangle = E_a|\alpha\rangle$, can be categorized according to their energy into three groups. If $E_\alpha \geqslant mc^2$, we denote these positive continuum energy states as $|p\rangle$, if their energy is inside the mass gap $-mc^2 < E_\alpha < mc^2$, we denote these discrete electronic bound states as $|i\rangle$, and if their energy $E_\alpha \leqslant -mc^2$ is part of the negative energy continuum, we denote these states as $|n\rangle$. If we introduce the sets of (anticommuting) creation operators $(B_p{}^\dagger, B_i{}^\dagger, D_n{}^\dagger)$ and annihilation operators (B_p, B_i, D_n) asscociated with these states, the mode expansion of the quantum field operator is given by

$$\begin{aligned}\Psi(t) &= \Sigma_p B_p(t)|p\rangle + \Sigma_i B_i(t)|i\rangle + \Sigma_n D_n(t)^\dagger|n\rangle \\ &= \Sigma_p B_p|p(t)\rangle + \Sigma_i B_i|i(t)\rangle + \Sigma_n D_n{}^\dagger|n(t)\rangle,\end{aligned} \qquad (2)$$

where $|\alpha(t)\rangle$ is the single-particle solution to $i\hbar\,\partial|\alpha\rangle/\partial t = H|\alpha\rangle$ with the initial state $|\alpha(t=0)\rangle = \alpha\rangle$. We note that this particular mode expansion is different from the traditional approach [11], where one usually uses field-free states of H_0 with $\mathbf{A} = V = 0$, labeled by their (conserved) momentum. If we use the orthogonality among the dressed eigenstates, we can find for the time evolution of the operators

$$B_p(t) = \Sigma_{p'} B_{p'} \langle p|p'(t)\rangle + \Sigma_i B_i \langle p|i(t)\rangle + \Sigma_n D_n{}^\dagger \langle p|n(t)\rangle, \tag{3a}$$

$$B_i(t) = \Sigma_{p'} B_{p'} \langle i|p'(t)\rangle + \Sigma_{i'} B_{i'} \langle i|i'(t)\rangle + \Sigma_n D_n{}^\dagger \langle i|n(t)\rangle, \tag{3b}$$

$$D_n(t)^\dagger = \Sigma_{p'} B_{p'} \langle n|p'(t)\rangle + \Sigma_i B_i \langle n|i(t)\rangle + \Sigma_{n'} D_{n'}{}^\dagger \langle n|n'(t)\rangle. \tag{3c}$$

We see that the matrix elements $U_{\alpha',\alpha}(t) \equiv \langle\alpha'|\alpha(t)\rangle$ of the unitary time evolution operator are the basic building blocks of computational quantum field theory.

The initial quantum field theoretical state is given here by the superposition $|\Phi(t=0)\rangle = [B_1{}^\dagger \exp(i\phi) + B_2{}^\dagger]2^{-1/2}|\text{vac}\rangle$. Here $|\text{vac}\rangle$ denotes the vacuum state, defined as $B_p|\text{vac}\rangle = B_\text{i}|\text{vac}\rangle = D_n|\text{vac}\rangle = 0$. In its quantum-mechanical (single-particle) analog, the field theoretical state $|\Phi(t=0)\rangle$ would correspond to the one-electron quantum state given by the superposition $[\exp(i\phi)|1\rangle + |2\rangle]/2^{1/2}$ as discussed above.

The total number of electrons $N(e^-, t)$ and positrons $N(e^+, t)$ follow from the quantum field theoretical expectation values:

$$N(e^-, t) = \langle\Phi(t=0)|\Sigma_p B_p(t)^\dagger B_p(t) + \Sigma_i B_i(t)^\dagger B_i(t)|\Phi(t=0)\rangle, \tag{4a}$$

$$N(e^+, t) = \langle\Phi(t=0)|\Sigma_n D_n(t)^\dagger D_n(t)|\Phi(t=0)\rangle, \tag{4b}$$

where we consistently have $N(e^-, t) = N(e^+, t) + 1$ as the result of the total charge conservation. If we insert the specific initial state $|\Phi(t=0)\rangle = [B_1{}^\dagger \exp(i\phi) + B_2{}^\dagger]2^{-1/2}|\text{vac}\rangle$ into these expressions and use the solutions Eqs. (3), we obtain $N(e^-, t) = N(e^-, 1, t) + N(e^-, 2, t) + \Sigma p \Sigma n|U_{pn}(t)|2$, where the occupation numbers of the two electronic bound states $|1\rangle$ and $|2\rangle$ can be derived as

$$\begin{aligned} N(e^-, 1, t) &\equiv \langle\Phi(0)|B_1(t)^\dagger B_1(t)|\Phi(0)\rangle \\ &= |\exp(i\phi)U_{1,1}(t) + U_{1,2}(t)|^2/2 \\ &\quad + \Sigma_n|U_{1,n}(t)|^2, \end{aligned} \tag{5a}$$

$$\begin{aligned} N(e^-, 2, t) &\equiv \langle\Phi(0)|B_2(t)^\dagger B_2(t)|\Phi(0)\rangle \\ &= |\exp(i\phi)U_{2,1}(t) + U_{2,2}(t)|^2/2 \\ &\quad + \Sigma_n|U_{2,n}(t)|^2. \end{aligned} \tag{5b}$$

These expressions rely on all transition matrix elements $U_{\alpha,n}(t)$ and $U_{\alpha,i}(t)$ and therefore illustrate the complex many-body character of the vacuum. For example, in this description, the vacuum state is formally described by all states $|n\rangle$ with energy $E < -mc^2$ to be initially fully occupied (see the Dirac sea in Fig. 1).

If we assume that the nucleus is highly charged such that the two electronic states are deeply bound, we can neglect the creation of any uncaptured electrons, i.e., $\Sigma p \Sigma n|\text{U}p, n(t)|^2 \approx 0$ and the total number of created positrons can be obtained as $N(e^+, t) = N(e^-, 1, t) + N(e^-, 2, t)$–1. Furthermore, due to the resulting completeness of the basis states, we have $\Sigma_n|U_{1,n}(t)|^2 = 1$–$|U_{1,1}(t)|$–$|U_{1,2}(t)|^2$, such that we derive for the two occupation numbers the final remarkably simple expressions:

$$N(e^-,\ 1,\ t; \phi) = 1 - |\exp(i\,\phi)\,U_{1,1}(t) - U_{1,2}(t)|^2/2, \tag{6a}$$

$$N(e^-,\ 2,\ t; \phi) = 1 - |\exp(i\,\phi)\,U_{2,1}(t) - U_{2,2}(t)|^2/2. \tag{6b}$$

As the computation of the time evolution of each of the continuum energy states $|n(t)\rangle$ is no longer required, the vacuum decay can be obtained solely from the time evolution of the two initial states $|1\rangle$ and $|2\rangle$. This means we have successfully mapped the vacuum decay process to the (mathematically fully equivalent) description in terms of two mutually independent (single-electron) "ionization-like" processes with two different sets of initial conditions. We use the initial conditions $\{C_1(0) = 1,\ C_2(0) = 0\}$ to determine $\{\text{U}_{1,1}(t), U_{2,1}(t)\} = \{C_1(t), C_2(t)\}$ and the second set $\{C_1(0) = 0, C_2(0) = 1\}$ to determine $\{U_{1,2}(t), U_{2,2}(t)\} = \{C_1(t), C_2(t)\}$. Note that the knowledge of the important phase ϕ (characteristic of the quantum field theoretical initial state) is *not required* at this particular first calculational stage.

The required set of amplitudes can be obtained as solutions to the following set of Dirac equations:

$$i\,\hbar\, dC_2(t)/dt = E_2 C_2(t) + \hbar\,\Omega_0\, \sin(\omega t) C_1(t), \tag{7a}$$

$$i\,\hbar\, dC_1(t)/dt = E_1 C_1(t) + \hbar\,\Omega_0\, \sin(\omega t) C_2(t) + \int_{-\infty}^{-mc^2} dE\, \sin(\omega t)\,\kappa(E)\, C_E(t), \tag{7b}$$

$$i\,\hbar\, dC_E(t)/dt = E\, C_E(t) + \sin(\omega t)\,\kappa(E) C_1(t), \tag{7c}$$

where the energy-dependent factor $\kappa(E) = \kappa_0[1 + (E + mc^2)^2/(m^2c^4)]^{-1}$ models the density of the negative continuum states and their coupling strength to the ground state. We also neglected any multiphoton transitions.

In Fig. 3 we present our main results. We show the resulting number of created positrons $N(e^+, t; \phi) = N(e^-, 1, t; \phi) + N(e^-, 2, t; \phi)$–1 as a function of time for the four differently prepared superposition states of the initial core electron.

The observed largest growth of the positron number $N(e^+, t)$ occurs for the phase $\phi = 0$. This is fully consistent with our expectation as here the Rabi oscillation depletes the level $|1\rangle$; therefore, the amount of the Pauli blocking decreases, which increases the capture probability for the created electron. The opposite pattern is observed for $\phi = \pi$, where the growth of the positron's creation probability $N(e^+, t; \phi = \pi)$ comes even momentarily to a halt after a time of about $\pi/(2\Omega_0)$, when the occupation number of the ground state $|c_1(t)|^2$ approaches unity and we have perfect Pauli blocking.

For this paper, the most important observation is that $N(e^+, t; \phi)$ is *different* for those two phases ($\phi = \pi/2$ and $\phi = 3\pi/2$), which originally led to an *identical occupation* $|c_1(t)|^2 = 1/2$ for $\kappa_0 = 0$ (see Fig. 2). This unexpected response suggests that—even if the underlying Pauli-blocking strength is identical—the decay of the quantum vacuum state can "sense" the electronic phase ϕ. Quite interestingly, while for $k_0 = 0$ the solution $|c_1(t)|^2 = 1/2$ is valid for any

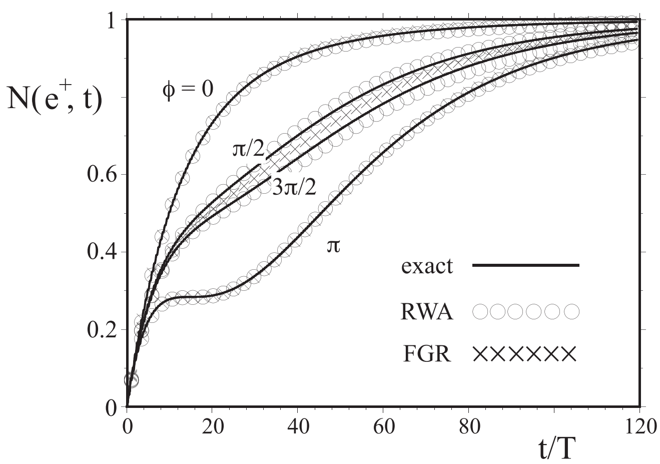

FIG. 3. The time dependence of the created positrons as a function of time (in units of the laser period $T = 2\pi/\omega$) $N(e^+, t) = N(e^-, 1, t) + N(e^-, 2, t)–1$. Here the initially bound electron was in the superposition state $[\exp(i\phi)|1\rangle + |2\rangle]/2^{1/2}$ with four different initial phases ϕ. All other parameters as in Fig. 1, except that $\kappa_0 = 0.1\, m^{1/2}c$. The open circles represent the predictions based on the rotating-wave approximation and the crosses are the analytical predictions based on Fermi golden rule given by Eqs. (6) and (9), where the vacuum decay constant is $\Gamma \equiv 2\pi[\kappa(E_r)/2]^2/\hbar$, with $E_r = –1.4\, mc^2$.

Ω_0, the detected difference between $N(e^+, t; \phi = \pi/2)$ and $N(e^+, t; \phi = 3\pi/2)$ does depend on Ω_0. This reflects the crucial importance of the time dependence of the actual *phase* of the complex amplitude $c_1(t) = \exp[i\phi(t)]2^{-1/2}$ in contrast to the mere occupation number $|c_1(t)|^2$.

In order to shine some more light on this observed phase dependence, we examine its robustness with regard to two standard theoretical approximation schemes. In Fig. 3 the exact predictions for $N(e^+, t; \phi)$ were compared with those obtained based on the rotating-wave approximation to Eqs. (7). The good agreement of the data in Fig. 3 (especially for $\phi = 0$ and $\phi = \pi$) suggests that the RWA can describe the positron number $N(e^+, t; \phi)$ very well.

The third set of comparative data (crosses) was obtained under the additional single-pole (Fermi golden rule) approximation, which permits even a fully analytical solution for $N(e^+, t; \phi)$. If we solve Eq. (7c) under the RWA for $c_E(t)$ as a function of $c_1(t)$, and insert this solution into the RWA version of Eq. (7b), we obtain the set of integrodifferential equations:

$$i\hbar\, dc_2/dt = -\hbar\, \Omega_0/(2i)c_1(t), \tag{8a}$$

$$i\hbar\, dc_1/dt = -\hbar\, \Omega_0/(2i)c_2(t) + {\kappa_0}^2/(4i\hbar) \times \int_{-\infty}^{-mc^2} dE\, \rho(E) \int_0^t d\tau \times \exp[-i(E - E_r)(t - \tau)/\hbar]\, c_1(\tau), \tag{8b}$$

where the resonant continuum energy is $E_r \equiv E_1–\hbar\omega$. Under the usual single-pole approximation, we can assume that the integration kernel in Eq. (8b) is real and proportional to $\hbar\pi\,\delta(t–\tau)$. This simplifies Eq. (8b) to $i\hbar\ dc_1/dt = \hbar\Omega_0/(2i)c_2(t)–i\hbar\Gamma/2\ \ c_1(t)$, where the Fermi golden rule (FGR) inverse timescale $\Gamma \equiv 2\pi(\kappa(E_r)/2)^2/\hbar$ is the vacuum's decay rate. The resulting set of two coupled equations can be solved analytically, leading to

$$u_{1,1}(t) = \exp(-\Gamma t/4)[\cos(\Omega t/2) - \Gamma \sin(\Omega t/2)/(2\Omega)], \tag{9a}$$

$$u_{1,2}(t) = u_{2,1}(t) = \exp(-\Gamma t/4)\, \Omega_0 \sin(\Omega t/2)/\Omega, \tag{9b}$$

$$u_{2,2}(t) = \exp(-\Gamma t/4)[\cos(\Omega t/2) + \Gamma \sin(\Omega t/2)/(2\Omega)], \tag{9c}$$

where the vacuum decay process modifies the Rabi frequency to $\Omega \equiv [{\Omega_0}^2–(\Gamma/2)^2]^{1/2}$. While these analytical solutions (crosses in Fig. 3) approximate $N(e^+, t; \phi = 0)$ and $N(e^+, t; \phi = \pi)$ remarkably well, they incorrectly predict $N(e^+, t; \phi = \pi/2) = N(e^+, t; \phi = 3\pi/2)$. This means that the important observed sensitity of the vacuum, to be able to distinguish between the two phases $\phi = \pi/2$ and $3\pi/2$, has disappeared under this standard (FGR) approximation, which is usually rather accurate in ionization applications. This sheds also some light on the dynamical significance of the imaginary part of the integration kernel in Eq. (8b).

In summary, as this study has introduced a phase-based mechanism by which a coherently prepared electron can affect the vacuum decay process, it provides naturally many challenges. For example, as the phase ϕ has a clear temporal impact on $N(e^+, t)$, we would also expect energetic implications with regard to the positronic spectrum beyond the Autler-Townes splitting [12–14], by which the core electron's coherence manifests itself in the positron's momenta and angular distributions as well as other electron-positron and likely spin-related correlation properties.

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