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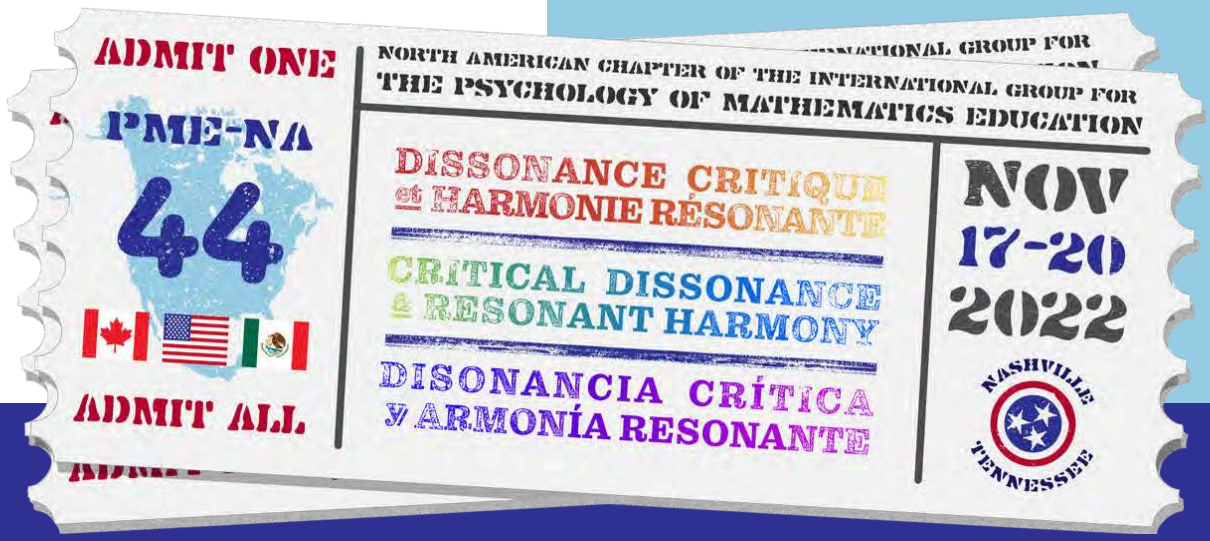
CRITICAL DISSONANCE AND RESONANT HARMONY



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Critical Dissonance and Resonant Harmony

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A CLASSROOM OBSERVATION TOOL FOR EQUITY-ORIENTED TEACHING OF MATHEMATICAL MODELING IN THE ELEMENTARY GRADES

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Mathematical modeling can be a lever for equity in the elementary math classroom, as it empowers teachers to build on the knowledge and cultural resources that children bring to the classroom and empowers students to draw on their experiences and identities to inform their mathematical work. To better support this transformative synergy between mathematical modeling and equity-oriented practices, we need a tool to deepen our understanding of variations and potential trajectories of teacher practice. In this report, we briefly describe our process for developing an equity-oriented mathematical modeling classroom observation protocol. We then discuss two sample dimensions from our tool to illustrate our integrated attention to equity-focused and mathematical modeling-specific teaching practices.

Keywords: Modeling; Elementary School Education; Instructional Activities and Practices; Equity, Inclusion and Diversity

In elementary mathematics education, classroom observation tools can support teacher learning by making visible key features of instructional practice and outlining potential trajectories for developing new practices (Bostic et al., 2021; Boston et al., 2015). Observation tools also serve research by generating evidence of teachers' enactment of specific practices which can inform the design of professional learning experiences, or evaluate the effectiveness of interventions. While there are numerous observation tools focused on high-quality math teaching (e.g., MQI; Learning Mathematics for Teaching Project, 2011; M-SCAN, Walkowiak et al., 2014), tools focused specifically on teaching mathematical modeling are limited.

Mathematical modeling is an iterative process involving problem posing, and testing, validation, and revision of mathematical models to inform decision-making (Lesh & Zawojewski, 2007; Pollak, 2012). Observation tools that attend to modeling often include modeling as a single dimension (e.g., Bostic, et al., 2019; Gleason et al., 2017). Although researchers have begun to conceptualize modeling-specific protocols (e.g., Hwang, 2020), validating tools for use in elementary grades is still needed. Additionally, while there is growing recognition that mathematical modeling can be a lever for equity (Aguirre et al., 2019; Anhalt et al., 2018; Carlson et al., 2016; Cirillo et al., 2016; Suh et al., 2018; Turner et al., 2022), tools that integrate an explicit focus on modeling and equity-oriented practices are lacking. Equity-oriented tools often lack a content focus (e.g., CLASS, Pianta & Hamre, 2009; Salazar, 2018), or focus generally on standards-based mathematics instruction, but not modeling (e.g. Nava, et al., 2019).

The lack of a validated classroom observation protocol that attends, in substantive ways, to both mathematical modeling and equity-oriented, culturally responsive instruction is problematic because of the synergy between mathematical modeling and advancing equity. To address this need, our project is building on prior exploratory work that involved adapting dimensions of an existing classroom observation protocol for elementary mathematics (the M-Scan, Walkowiak et

al., 2014) to focus on culturally responsive mathematical modeling in grades 3-5. In this report, we describe our development process and discuss tool dimensions that illustrate our integrated attention to equity-focused and modeling-specific teaching practices. We end with a discussion of potential uses of the classroom observation tool, and next steps in our validation process.

Equity-Focused Teaching Practices for Mathematical Modeling

Teaching mathematical modeling is challenging because modeling includes processes like posing problems, making assumptions, and testing and revising models, that are not typical in mathematics classrooms (Niss, Blum, & Galbraith, 2007). Modeling tasks are more open and less predictable than those in most lessons (Cai et al., 2018) and require teachers to know about the real-world contexts that motivate modeling problems, potential mathematical solutions, and ways to maintain rigor and support for students as they develop, refine, and communicate their models (Suh et al., 2021; Turner et al., 2021). However, there is synergy between equity-focused mathematics teaching and mathematical modeling. Modeling empowers teachers to elicit and build on the knowledge and cultural resources that students bring to the classroom and empowers students to draw on their identities and experiences to inform mathematical work and take action (Aguirre et al., 2019; Turner & Bustillos, 2017). Classroom modeling also encourages diverse student contributions and gives teachers opportunities to "recognize and reward a broader range of mathematical abilities than those traditionally emphasized" (Lesh & Doerr, 2003, p. 23). To better support teachers' learning and practices related to the transformative integration of equity-focused teaching and mathematical modeling, we need a tool to sharpen our vision and deepen our understanding of variations and potential trajectories of teacher practice.

Overview of Classroom Observation Tool Development

To develop our classroom observation tool, we followed a multi-stage process for protocol validation outlined by Bostic and colleagues (2019). *Stage 1* involved a review of observation protocols for mathematical modeling and equitable teaching practices, and research on effective teaching practices for mathematical modeling, especially in grades K-2. *Stage 2* focused on synthesizing key outcomes from the literature, and using these ideas to draft an initial version of the tool. This version included 6 dimensions, each of which focused on equity-oriented teaching practices for a specific phase of the modeling process (e.g., making sense of the context and posing problems; identifying important quantities and making assumptions). Several of the dimensions were adapted from similar dimensions in existing, exploratory tools (Foote, Aguirre, Turner & Roth McDuffie, 2020. In *Stage 3*, we assembled an expert review panel consisting of 15 scholars with expertise in mathematical modeling, equitable teaching practices, and classroom observation tools. Scholars reviewed the draft protocol and provided feedback on its alignment with the construct (i.e., to what extent do the dimensions capture key features of mathematical modeling across the elementary grades?); the range of practice captured (i.e., do the indicators capture teacher moves and supports that are appropriate for K-5 students from diverse cultural, racial, linguistic, and geographic backgrounds?); and the clarity and usability of each dimension to describe and inform practice. In *Stage 4,* we revised the tool based on our review of the literature and feedback from our expert panel. *Stage 5,* our current stage, is focused on testing the revised observation tool with video of modeling lessons from different grade levels, and from teachers with various levels of experience teaching modeling.

Dimension 1: Connections to Students' Experiences and Cultural/Community Contexts

Dimension 1 (Table 1) captures teaching practices to support connections between students' experiences and the modeling process. Unlike some of the tool's dimensions, which focus on a

particular modeling phase, (see Table 2), Dimension 1 should be present throughout the cycle. Its descriptors address the degree to which the teacher sustains connections to students' experiences and cultural and community contexts. Key Terms and Ideas, listed below the descriptors, clarify the nature of the connections to be made and teacher moves that could support students' work.

Table 1: Connections to Students' Experiences and Cultural/Community Contexts

Not Present (0)	Emerging (1)	Proficient (2)	Advanced (3)
The teacher does not engage the class in making connections between the modeling context and their lives.	In one phase of the modeling process, teachers support multiple students to make connections to their experiences (lives, communities, cultures) to inform their work.	In two or more phases of the modeling process, teachers support multiple students to make connections to their experiences to inform their work.	In two or more phases of the modeling process, teachers support multiple students to make connections to their experiences to inform their work.
Key Terms and Ideas **Connections** include references to experiences related to the context or scenario, or references to understandings about a specific context, setting, scenario or activity. Connections can include connections to students' or teachers' experiences outside of school, and students'/teachers' shared experiences as members of the school community. **Teacher Support** includes teacher moves such as asking students to recall experiences related to the context to help them identify key quantities; reminding students to use what they know about a situation to help them evaluate their solution; asking students to draw on their experiences with the situation to propose revisions to their models; reminding students of a shared experience that might inform their work. Teacher support must involve prompting, probes and/or follow up and go beyond a single question, hook or statement, and beyond a teacher-provided connection that does not invite student input or response.			

Dimension 1 makes practices that advance equity while teaching mathematical modeling explicit and offers a trajectory for supporting students to connect to their experiences and cultural and community contexts. At the *emerging* level, teachers support connections in one phase of the modeling cycle, likely through a single prompt or statement. As teachers grow in their ability to support classroom modeling, they sustain connections to students' experiences throughout the cycle - perhaps by prompting students to draw on their experiences to identify important quantities, propose revisions to their model, or interpret their solution. The tool also makes clear that teachers support multiple students to connect to their backgrounds and experiences, increasing the likelihood that contributions of students from diverse backgrounds are honored.

Dimension 2: Posing Mathematical Problems

Dimension 2 (Table 2) captures practices that support students learning to pose mathematical problems. Its descriptors focus on the degree to which the teacher involves students in posing the question that drives the modeling task. Unlike typical mathematics lessons, where students must understand the problems posed by the teacher or the textbook, when students model they learn to focus on specific features of real-world problems and then translate those features into mathematical questions. Thus, supporting students in learning to pose mathematical problems requires teaching practices that are unique to mathematical modeling.

Table 2: Posing Mathematical Problems

Not Present (0)	Emerging (1)	Proficient (2)	Advanced (3)
The teacher poses the modeling problem. The problem is pre-determined by the teacher. There is no evidence of actions by the teacher to connect students' questions or wonders about the context to the problem posed.	The teacher poses the modeling problem, but the teacher makes connections between the problem posed and the questions/ wonders/ observations/ experiences that students posed about the context.	The teacher invites students to shape some components of the problem posed. Includes: supporting students to recognize math questions, inviting students to connect the problem with their wonders or to share insights about the problem posed.	The teacher consistently invites and builds on student ideas to pose and refine the modeling problem. The teacher may facilitate the discussion, but student ideas drive problem-posing.
Key Terms and Ideas **Posing problems** refers to formulating a mathematical question that can be invested through the modeling process. Problems may be descriptive, predictive, or evaluative in nature. **Teacher Connects** student ideas or wonders to the problem posed (by the teacher). The teacher might say, "You all asked about ……., that is very similar to the problem we are going to work on today. Here is our problem…" Or, the teacher may invite students to share an observation or experience related to the problem posed (by the teacher).			

Dimension 2 captures equity-oriented instruction by focusing on teaching practices that empower students to shape the mathematical questions that are asked. Teacher development in this trajectory moves from *emergent* practices that are limited to teachers making connections between students' ideas a pre-determined problem to *advanced* practices that consistently invite student contributions and allow student ideas to drive problem posing. When teachers support students in this way, they extend authority for what is legitimized as important and worthwhile mathematical work to students. Not only does this disrupt power structures present in most classrooms, it supports students in developing a critical mathematical modeling competency.

Discussion and Implications

Observation tools that combine equity-oriented teaching and a content focus advance the field by offering specific descriptions of practice and trajectories for developing effective, equity-focused mathematics teachers. Our next step is to test our tool on a larger set of modeling lessons from diverse grade levels and classroom contexts, and to conduct psychometric analysis of data generated from use of the tool (i.e., reliability analyses). We anticipate that mathematics teacher educators can use our tool to design mathematical modeling professional development experiences that center equity, and that researchers can use the tool to study potential shifts or growth in teacher practice of mathematical modeling over time. Teachers can use selected dimensions of our tool to reflect on their practice and to consider strengths and areas for growth.

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References

Aguirre, J.M., Anhalt, C., Cortez, R., Turner, E.E., & Simi-Muller, K., (2019). Engaging teachers in the powerful combination of mathematical modeling and social justice. *Mathematics Teacher Educator*. 7(2) 7-26.

Anhalt, C., Cortez, R. & Been Bennett, A. (2018). The emergence of mathematical modeling competencies: An investigation of prospective secondary mathematics teachers. *Mathematical Thinking and Learning International Journal,* 20(3) 1-20. DOI10.1080/10986065.2018.1474532

Bostic, J., Lesseig, K., Sherman, M. & Boston, M. (2021). Classroom observation and mathematics education research. *Journal of Mathematics Teacher Education, 24*, 5-31.

Bostic, J., Matney, G. & Sondergeld, T. (2019). A validation process for observation protocols: Using the Revised SMPs Look-for Protocol as a lens on teachers' promotion of the standards. *Investigations in Mathematical Learning, 11*(1), 69-82.

Boston, M., Bostic, J., Lesseig, K. & Sherman, M. (2015). A comparison of mathematics classroom observation protocols. *Mathematics Teacher Educator, 3*(2), 154-175.

Cai, J., Cirillo, M., Pelesko, J. A., Borromeo Ferri, R., Borba, M., Geiger, V., . . . Kwon, O. N. (2014). Mathematical modeling in school education: Mathematical, cognitive, curricular, instructional and teacher education perspectives. In P. Liljedahl, C. Nicol, S. Oesterle, & D. Allan (Eds.), *Proceedings of the joint meeting of PME 38 and PME-NA* 36 (pp. 145–172). Vancouver, Canada: PME.

Carlson, M.A., Wickstrom, M.H., Burroughs, B. & Fulton, E. (2016). A Case for Mathematical Modeling in the Elementary School Classroom. *Annual Perspectives in Mathematics Education (APME) 2016: Mathematical Modeling and Modeling Mathematics*, ed., Christian R. Hirsch, pp. 121–29. Reston, Va.: National Council of Teachers of Mathematics.

Cirillo, M., Bartell, T. G., & Wager, A. (2016). Teaching mathematics for social justice through mathematical modeling. In C. Hirsch & A. Roth McDuffie (Eds.), *Annual perspectives in mathematics education: Mathematical modeling and modeling with mathematics* (pp. 87-96). National Council of Teachers of Mathematics.

Foote, M.Q., Aguirre, J.M., Turner, E.E., & Roth McDuffie, A. (2020). M2C3-Scan: Mathematical Modeling with Cultural and Community Contexts Observation protocol. Unpublished tool.

Gleason, J., Livers, S. & Zelkowski, J. (2017). Mathematics Classroom Observation Protocol for Practices (MCOP2): A validation study. *Investigations in Mathematics Learning, 9*(2), 111-129.

Hwang, J. (2020). Instructional alignment observation protocol (IAOP) for implementing the CCSSM: Focus on the practice standard, "Model with mathematics". *Journal of Korean Society of Mathematics Education,* 23(3), 149-164.

Learning Mathematics for Teaching Project. (2011). Measuring the mathematical quality of instruction. *Journal of Mathematics Teacher Education, 14*, 25–47.

Lesh, R. A., & Doerr, H. M. (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. New York: Routledge.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester & National Council of Teachers of Mathematics (Eds.), *Second handbook of research on mathematics teaching and learning: a project of the National Council of Teachers of Mathematics* (pp. 763–804). Charlotte, NC: Information Age.

Nava, I., Park, J., Dockterman, D., Kawasaki, J., Schweig, J., Quartz, K. H. & Martinez, J. F. (2019). Measuring teacher quality of secondary mathematics and science residents: A classroom observation framework. *Journal of Teacher Education, 70*(2), 139-154.

Niss, M., Blum, W., & Galbraith, P. L. (2007) Introduction. In W. Blum, P. L. Galbraith, H. W. Henn, & M. Niss (Eds.) *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp. 3-32). New York: Springer.

Pianta, R. C., & Hamre, B. K. (2009). Conceptualization, measurement, and improvement of classroom processes: Standardized observation can leverage capacity. *Educational Researcher, 38*(2), 109–119.

Pollak, H. (2012). Introduction. In H. Gould, D. R. Murray, & Sanfratello (Eds.), *Mathematical modeling handbook* (pp. viii–xi). Bedford, MA: Consortium for Mathematics and Its Applications.

Salazar, M.d.C. (2018). Interrogating teacher evaluation: Unveiling Whitenessa as the normative center and moving the margins. *Journal of Teacher Education, 69*(5), 463-476.

Suh, J., Matson, K., Seshaiyer, P., Jamieson, S., & Tate, H. (2021). Mathematical Modeling as a Catalyst for Equitable Mathematics Instruction: Preparing Teachers and Young Learners with 21st Century Skills. Mathematics, 9(2), 162. MDPI AG. Retrieved from http://dx.doi.org/10.3390/math9020162

Suh et al (2018): Every penny counts: Promoting community engagement to engage students in mathematical modeling. In Goffney & Gutiérrez (Eds) *Rehumanizing mathematics for Black, Indigenous, and Latinx students*.

pp. 63-76. Reston, VA: NCTM
Turner, E. E., Bennett, A. B., Granillo, M., Ponnuru, N., Roth Mcduffie, A., Foote, M. Q., Aguirre, J.M. & McVicar, E. (2022). Authenticity of elementary teacher designed and implemented mathematical modeling tasks. *Mathematical Thinking and Learning*, 1-24.
Turner, E., Roth McDuffie, A., Aguirre, J., Foote, M. Q., Chapelle, C., Bennett, A., Granillo, M. & Ponnuru, N. (2021). Upcycling Plastic Bags to Make Jump Ropes: Elementary students leverage experiences and funds of knowledge as they engage in a relevant, community-oriented mathematical modeling task (pp. 235-266). In J. Suh, M. Wickstram, L. English (Eds). *Exploring Mathematical Modeling with Young Learners*. The Netherlands: Springer. DOI:10.1007/978-3-030-63900-6_11
Turner, E. & Bustillos, L. (2017). ¿Qué observamos aquí? ¿Qué preguntas tienen? Problem posing in Ms. Bustillos's Second-Grade Bilingual Classroom. (pp. 45-62.) In S. Celedón-Pattichis, D. White and M. Civil (Eds.) *Access and Equity: Promoting High Quality Mathematics in Grades PreK-2*! Reston, VA: National Council of Teachers of Mathematics.
Walkowiak, T., Berry, R., Meyer, J., Rimm-Kaufman, S., & Ottmar, E. (2014). Introducing an observational measure of standards-based mathematics teaching practices: Evidence of validity and score reliability. *Educational Studies in Mathematics, 85*, 109–128.