Optimal location of voltage sags monitors by determining the vulnerable area of network buses

Hamed Jalalat
Electrical engineering department
K. N. Toosi University of Technology
Tehran,Iran
HamedJalalat1991@gmail.com

Mohammad Tavakoli Bina
Electrical engineering department
K. N. Toosi University of Technology
Tehran,Iran
Tavakoli@eetd.kntu.ac.ir

Amir Shahirinia
Electrical engineering department
Univ.of the District of Columbia
Washington, USA
amir.shahirinia@udc.edu

***Abstract*— However, to analyze power quality parameters, it is necessary to record information through network buses but in order to save money, a limited number of buses are selected for this purpose. This paper presents the method of determining monitoring buses to observe the voltage sags in all parts of the network for the occurrence of any type of short circuit faults in the network. In this method, the number and location of monitoring buses are selected so that for the minimum number of monitoring buses, all network points are covered due to short circuit faults. For this purpose, first, by determining the vulnerable areas of network buses and then forming a monitor reach matrix for the network, the location and the minimum number of monitoring buses can be determined. The IEEE 14-bus system and 57 bus system are used to implement the proposed method.**

Keywords—voltage sag, power quality, area of vulnerability (AOV), monitoring buses.

I. INTRODUCTION

Voltage sag, which is a sudden decrease in the amount of rms voltage (0.1 to 0.9 of the nominal voltage) and duration is up to 1 min, if not resolved, can cause malfunction of electrical equipment and have a lot of financial losses, Therefore, in recent years, it has always been one of the topics studied by researchers[1].To study this phenomenon, various indices have been introduced[2, 3] Since to record and analyze network parameters and voltage sag indices, it is necessary to study the required information through network buses.

And although if this information can be obtained by studying all network buses, the resulting information will be accurate and correct, Since not all network buses can be selected due to the high cost of study, monitoring and maintenance, Therefore determining the optimal monitoring buses is one of the topics studied by researchers.

To obtain information of non-monitor buses, voltage sag estimation methods are used in references such as[4-6] where neural network methods, Bayes theory and genetic algorithm are used.

Obviously, if the monitoring buses are determined more accurately, the information of other buses that are obtained In recent years, papers in this field have been developed.in[7] integer programming-based method is proposed In this paper, the minimum number and location of monitors are shown, but the method used can only be used for one type of faults and does not cover all parts of the network.in [8]An Approach Based on Analytical Expressions is used for optimal placement The method used covers all points of the network and can be applied to all four types of faults (SLG-LL-LLG-3L), but a large number of points are required to apply the faults..

In[9] The fault position method is used to determine the fault locations and by forming a comprehensive database, the monitoring bus algorithm is applied. The formation of this database is time-consuming.

In[10] the fault position method is used in a distribution network and covers any type of faults (balance and unbalance) and all points of the network, a matrix called resulting binary observability matrix (BOMRes) Forms and by the method described reduces the dimensions of the matrix, which reduces the calculations. in[11] the types of fault variables (fault location, fault impedance and fault type) are considered by the hybrid method and to determine the optimal monitoring buses, the genetic algorithm method is used, which requires more time for solution convergence .in[12] simulated single-phase solid and the location of the fault and cost reduction are considered as the targets of the placement problem and the Bicriteria Discrete Optimization algorithm is used.

in[13] The p-Median model algorithm is used for optimal placement with full network observability in the distribution network. Monitoring equipment is prioritized based on the importance of the load that may be due to non-technical losses that cause voltage sag. In[14] The network impedance matrix (Z_bus) has been used to determine the monitoring buses. In this method, although the amount of calculations has been reduced, it has the same answer for all four types of fault.

In this paper, the vulnerable areas of network buses are used to determine the optimal monitoring buses.The method used in this paper to determine these areas requires less computation than other papers with the same high accuracy and is very effective in large networks[15-18]

To determine the critical points, the systematic numerical method is used, and to determine the monitoring buses by comparing the critical points of the lines with the threshold voltage, first form a comprehensive binary matrix of line status and solve the objective functions Monitoring buses are specified so that the entire network is completely covered.

The paper is organized into four-part: in section II, determining the vulnerable areas of the network buses for the considered threshold voltage, in section III comprehensive binary matrix of line status is obtained by comparing the critical points of vulnerable areas with the threshold voltage. in section IV, define the objective function and its constraints so that the entire network is

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covered and in section V, introduced methods are tested on IEEE 14-bus system and 57 bus system.
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II. IDENTIFY AREA OF VULNERABILITY DUE TO VOLTAGE SAG

To determine the AOV, critical points must be determined. These points for a bus are the points that if a fault occurs in those points, the voltage is recorded exactly equal to the threshold voltage specified in the bus.
To determine these points, assuming that the amount of voltage remaining in the buses due to the fault in the lines is quadratic, the method based on quadratic interpolation and secant method is used.
The aim is to determine whether the study line is completely or partially part of the AOV of the studied bus or not at all.

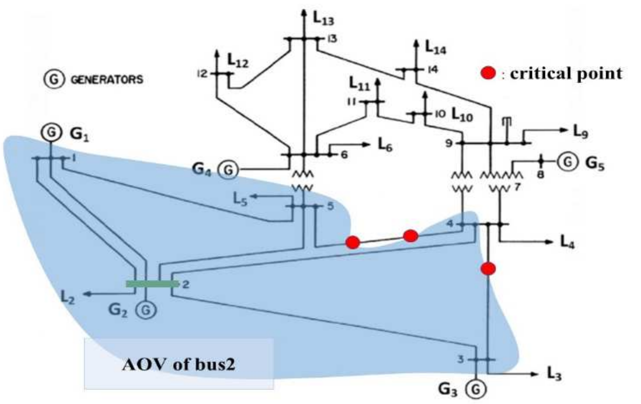


Fig. 1. area of vulnerability and critical points for specific threshold voltage

A. *Find critical point by secant method*

One of the types of numerical methods of rooting can be open methods. In these methods, you can start from anywhere, and unlike Bracketing methods, there is no limit. Among the open methods, the secant method has been used in this paper. In this method, two points are used to solve the problem[19]
In summary, in this method, a new point is created by connecting the values of these two selected points with a line and the collision of this line with the x-axis, then continue the same with the last two points. This operation is repeated until the desired answer, which is defined based on the convergence criterion.

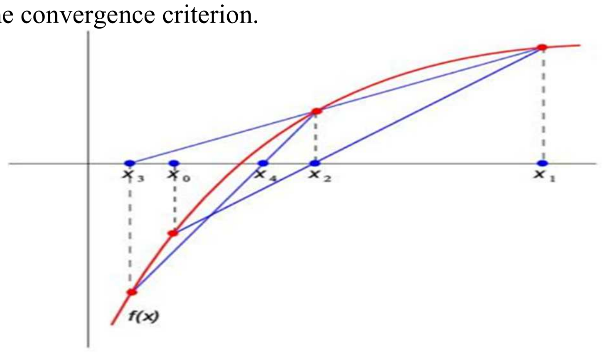


Fig. 2. The secant method

$$x_2=x_1-\frac{f(x_1)(x_1-x_0)}{f(x_1)-f(x_0)} \tag{1}$$

p_{from} and p_{end}, which are close to the roots of the quadratic equation, show the two initial points for solving the equation. The iteration loop and the convergence estimate of this method are as follows:

$$p_{new}=p_{end}-\frac{(|f(p_{end})|-v_{th})(p_{end}-p_{from})}{|f(p_{end})|-|f(p_{from})|} \tag{2}$$

$$p_{from}=p_{end} \tag{3}$$

$$p_{end}=p_{new} \tag{4}$$

$$\left||f(p_{new})|-v_{th}\right|<tolerance \tag{5}$$

B. *AOV determination process*

First, it forms a matrix that shows the magnitude of the voltage at bus k due to a fault in buses 1 to n, and then by subtracting the values of this matrix from the threshold voltage, a vulnerability index vector (BVI) is formed for bus k.

$$v_{mag}=\begin{bmatrix}|v_k^{fault\_1}|\\ \vdots \\ |v_k^{fault\_n}|\end{bmatrix} \tag{6}$$

$$\Delta v_{mag}=\begin{bmatrix}\Delta v_{mag,1}\\ \vdots \\ \Delta v_{mag,n}\end{bmatrix}=\begin{bmatrix}|v_k^{fault\_1}|\\ \vdots \\ |v_k^{fault\_n}|\end{bmatrix}-[v_{th}] \tag{7}$$

$$BVI=\begin{bmatrix}BVI_1\\ \vdots \\ BVI_n\end{bmatrix}=\begin{cases}1\ , & \Delta v_{mag}\le 0\\ 0\ , & \Delta v_{mag}>0\end{cases} \tag{8}$$

If ΔV_{mag} is negative or zero, the corresponding BVI becomes one, and if it is positive, it becomes zero. BVI is the vulnerability index of bus i(i=1,2,…,n).

BVI_i equal to 1 means that the magnitude of the voltage at bus k due to a fault in bus i is less than the threshold voltage, so bus i is part of the AOV of bus k.

Vulnerability index vector LVI is calculated for network lines according to Equation (9). LVI_j is the vulnerability index of line j between A_j and B_j (j=1,2,…,m) buses and m is the number of network lines. LVI of line j which is between A_j and B_j buses is obtained by adding the value of the BVI of these two buses.

$$LVI=\begin{bmatrix}LVI_1\\ \vdots \\ LVI_n\end{bmatrix}=\begin{bmatrix}BVI_{A1}\\ \vdots \\ BVI_{An}\end{bmatrix}+\begin{bmatrix}BVI_{B1}\\ \vdots \\ BVI_{Bn}\end{bmatrix} \tag{9}$$

If the value of BVI for one of the A_j and B_j buses is 1, the LVI_j is equal to 1, which means that a part of the line is AOV and the line has only one critical point.

In this case | f (0.5) | Calculated and using the value of three-point voltage $|v_k^{fault\_A_j}|$، $|v_k^{fault\_B_j}|$ و $|f(0.5)|$ Second degree interpolation is performed

Then the numbers close to the root of the quadratic equation are used as the starting point in the secant method and the critical point is obtained by solving the secant method.

And if LVI_j is zero, in this case, the study line is completely outside the bus AOV.

If LVI_j is equal to 2, in this case first $|f(p_{max})|$ Calculated using the golden section search method [20] if $|f(p_{max})|$ is less than threshold voltage the line is completely in the AOV of target bus

Otherwise with three values $\left|V_k^{fault\_A_j}\right|$, $\left|V_k^{fault\_B_j}\right|$ and $|f(p_{max})|$ second degree interpolation is performed and by solving the equation $v_{th}=ap_i^2+bp_i+c$ two points near the roots of the equation are selected as the starting points of the secant method and by solving the secant algorithm, two critical points are obtained. In the third case, two The end of the lines is part of the area that causes the voltage sag for the desired bus.

III. Comprehensive Binary Matrix of Line Status

Comprehensive binary matrix of line status (CBM) shows the observability of each part of the network at the network buses when the voltage sag occurs in the lines. Based on the data of this matrix, it can be determined that the occurrence of a fault in each segment of network lines will trigger which of the network buses, and the dimensions of this matrix depend on the amount of threshold voltage. To understand the method used to form a comprehensive binary matrix of line states, it is best to consider the L line. So that according section II, it is determined that the L line is completely or partially part of the AOV of assuming 4 buses m_1،m_2،m_3 و$m_4$or is not part of the AOV of these buses at all.

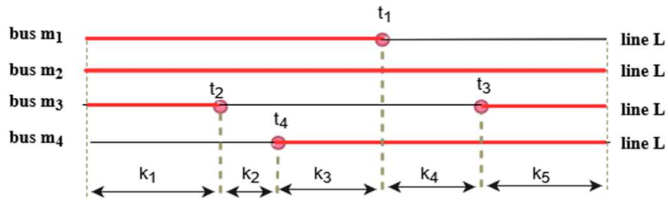


Fig. 3. the segment of line L for threshold voltage t with considering buses m_1 , m_2 , m_3 and m_4.

According to Figure 3, t_1 ،t_2 ،t_3 and $t_4$are the critical points of the line L for buses m_1،m_2،$m_3$andm_4, and the line L is completely part of the AOV of bus m_2. The L-line is divided into five segments k_1 ،k_2 ، k_3 ، k_4 and k_5. According to this figure, the occurrence of a fault in the segment k_1 causes the voltage sag in the buses m_1 , m_2 and m_3.

In the event of a fault in the $k_1$segment, by installing monitoring on the buses m_1،m_2 and m_3 the voltage sag with the considered threshold voltage can be recorded, but it is not visible on the bus m_4.

If a fault occurs in a segment of the line L and voltage sag occurs in the bus, the element associated with that segment and the bus in the binary matrix will be equal to 1 otherwise, it will be 0.

In segment k_4 of line L, elements corresponding for buses m_1،m_2،m_3 and m_4 are equal to 0,1,0 and 1, respectively, because in buses m_2 and m_4, the voltage sag can be observed and recorded, and in basses m_1 and m_3, the voltage sag is not recorded.

If the number of segments of line L for the threshold voltage t is equal to k and the number of buses is M, the dimensions of this matrix for line L will be equal to M * K.

$$B_L=b_l(m,k)$$

$$=\begin{cases} 0\ , & \text{if } V_m(k_x)>t \\ 1\ , & \text{if } V_m(k_x)\leq t\ , \quad \forall m,k \quad x=1,2,3,\ldots \end{cases} \tag{10}$$

Therefore, the values of the binary matrix elements of the L line state are as follows:

$$B_L=\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \tag{11}$$

This process must be performed for all network lines (N) and if these matrices are put together, the following comprehensive matrix can be achieved:

$$B=[\,B_1 \quad B_2 \quad \ldots \quad B_L \quad \ldots \quad B_N] \tag{12}$$

The number of rows of this matrix is equal to the total number of network buses and the number of columns of this matrix is equal to the sum of the segments of all network lines.

For each type of quadruple fault, the CBM matrix can be formed so that B_{3P} represents CBM for the case where the balance three-phase fault, B_{2P} and B_{2P-g} are related to the case where line to line and double line to ground faults and B_{1P} are related to the case where single line-to-ground in the network is considered for study.

IV. FORMULATION OF OPTIMIZATION AND PLACEMENT PROBLEM

The purpose of CBM is to determine the optimal number and location of monitoring buses so that while selecting the minimum number of monitoring buses, all points of the network will be covered by these monitoring buses.

The binary vector y, which contains M (equal to the number of network buses) elements, indicates the status of the network buses in terms of selection or non-selection as the monitoring bus. So that if the y (i) element is one, that is, the corresponding bus is selected as the monitoring bus, but if it is zero, the corresponding bus is not as the monitoring bus. The objective function for the optimization problem so that the least number of buses is selected is as follows:

$$\min \sum_{i=1}^{M} y(i) \tag{13}$$

as mentioned,one of the purposes of this paper is that if occures voltage sag because of the fault at any point of the network at least one bus can record it. In other words, the whole network is covered. Since each column of the CBM matrix represents one segment of network , so at least one entry of any columns must be 1, which indicates that the bus related to this entry is triggered if any kind of faults at that segment cause voltage sag.

Therefore, the following constraints must be considered for all four types of faults in Equation(13):

$$\sum_{i=1}^{M} b_{3p}(i,k)y(i)\geq 1\ , \qquad \forall k=1,2,\ldots,k_{3p}$$

$$\sum_{i=1}^{M} b_{2p}(i,k)y(i)\geq 1\ , \qquad \forall k=1,2,\ldots,k_{2p}$$

$$\sum_{i=1}^{M} b_{2p\text{-}g}(i,k)y(i)\geq 1\ , \qquad \forall k=1,2,\ldots,k_{2p\text{-}g}$$

$$\sum_{i=1}^{M} b_{1p}(i,k)y(i) \geq 1 \ , \qquad \forall k=1,2,...,k_{1p}$$

$$y(i) \qquad \text{binary} \qquad \forall i=1,2,...,N \qquad (14)$$

k_{1p} , $k_{2p\text{-}g}$ k_{2p}, and k_{3p} are the total number of segments or total number of columns of B_{1P}, $B_{2P\text{-}g}$, B_{2P} and B_{3P}.

to solve the (13) and (14) equations, the cplex solver is used in GAMS software[21]

V. CASE STUDY

In this part, the ieee 14-bus system and 57 bus system are used to implement the proposed method

First, with the method described in Section II, we can determine the vulnerable area of each bus or the vulnerability of each line to all buses(part of line or all of that is AOV of the bus under study or not part of the AOV at all)and CBM matrix is formed. . In this paper, to study the voltage sag, the lowest voltage value is considered in the phase with the highest amplitude drop. According to Figure 4, which shows the amount of voltage in three different phases, the amount of voltage at point A of the red curve is recorded as voltage sag.

And also, single-line to ground and three-phase faults are simulated, but the method is established for all four types of faults (SLG-LL-LLG and 3L). In this paper, for 14-bus system, three states, single-phase to ground fault with 0 and 4 ohm impedances and 3-phase fault with 0 impedance are considered, and for 57-bus network, single-phase to ground faults with 0 impedance is considered.

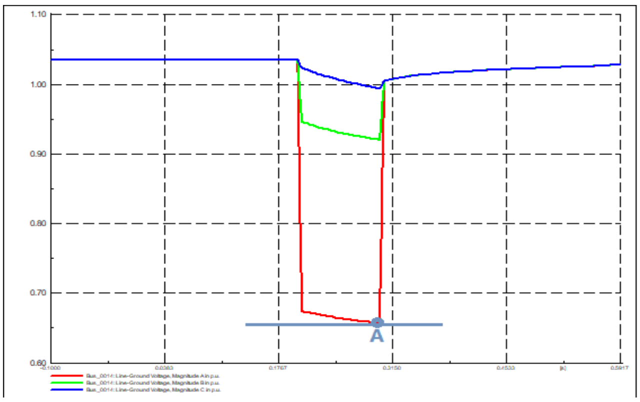
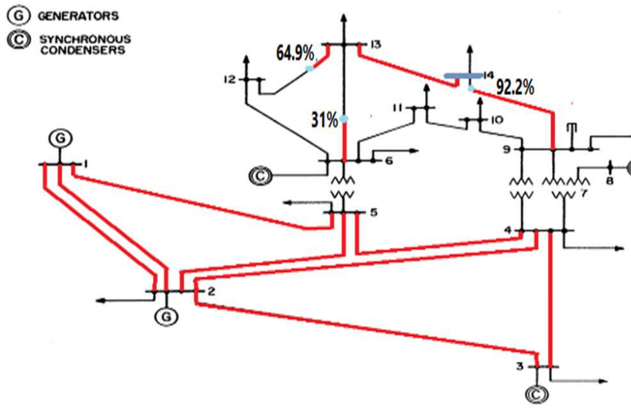
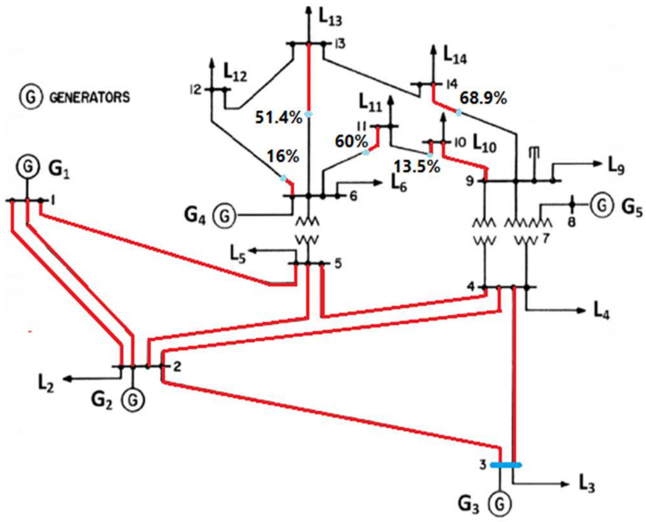


Fig. 4. voltage magnitude in different phases when a fault occurs.

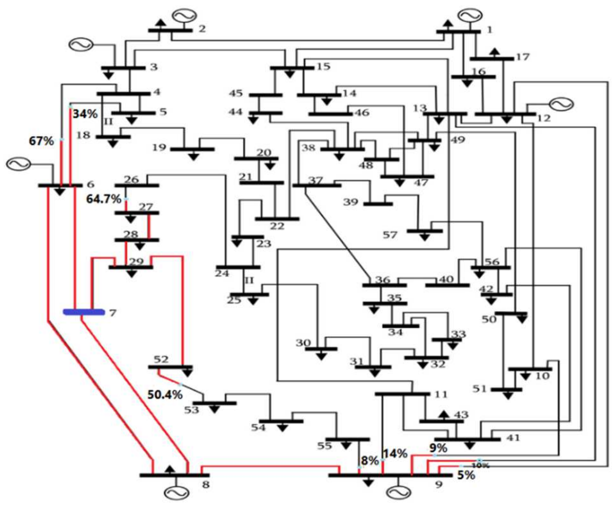
And the threshold voltage is assumed to be 0.9 pu.figure 5 shows the AOV of buses 14 and 3 in the ieee 14 bus system and AOV of bus 7 in the 57 bus system. For example, Figure 6 shows the condition of line 3-4 for a single line to ground fault with 4 ohm impedance in the 14-bus system relative to all network buses, indicating that line 3-4 is composed of 4 parts.



a. AOV for bus 14 in ieee 14-bus with SLG fault with R=0



b. AOV for bus 3 in ieee 14-bus with SLG fault with R=4



c. AOV for bus 7 in ieee 57-bus with SLG fault with R=0

Fig. 5. AOV for buses 3and 14 in the IEEE14-bus and bus 7 in the IEEE 57-bus

the line 4-3 matrix relative to all buses is as follows

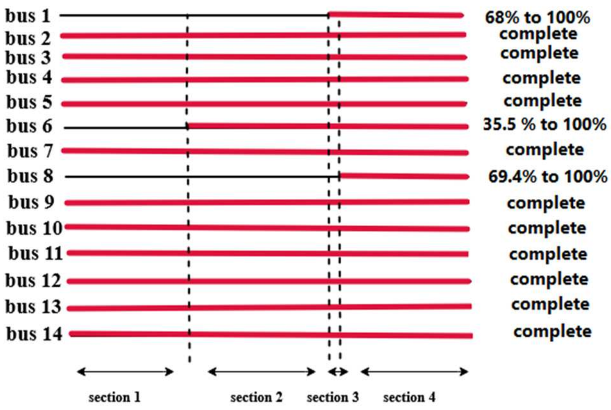


Fig.6. the segment of line 3-4 with threshold equal 0.9pu in ieee 14-bus with SLG fault

$$B_L=\begin{bmatrix}0&0&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\\0&1&1&1\\1&1&1&1\\0&0&0&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix} \qquad (15)$$

And Figure 7, which is related to the status of line 41-42 from the 57-bus network, shows that this line is divided into 8 parts.

If we do this for all network lines. By using the matrix of individual lines according to the formula (12), the CBM matrix is formed for these networks.

Table 1 shows the CBM matrix for single-line to ground fault with 4 ohm impedance for 14-bus network.

Finally, to determine the monitoring buses so that no specific buses is considered and all buses have the same priority, the CBM matrix is formed for each of the networks and scenarios and using the (13)and(14) formulas, the results shown in Table 2 are obtained.

As expected, as the amount of fault impedance increases, the vulnerable areas of the buses shrink, so more buses are needed for monitoring. With considering the three-phase fault instead of single-line to ground fault, As vulnerable areas grow, fewer buses are needed to monitor the network.

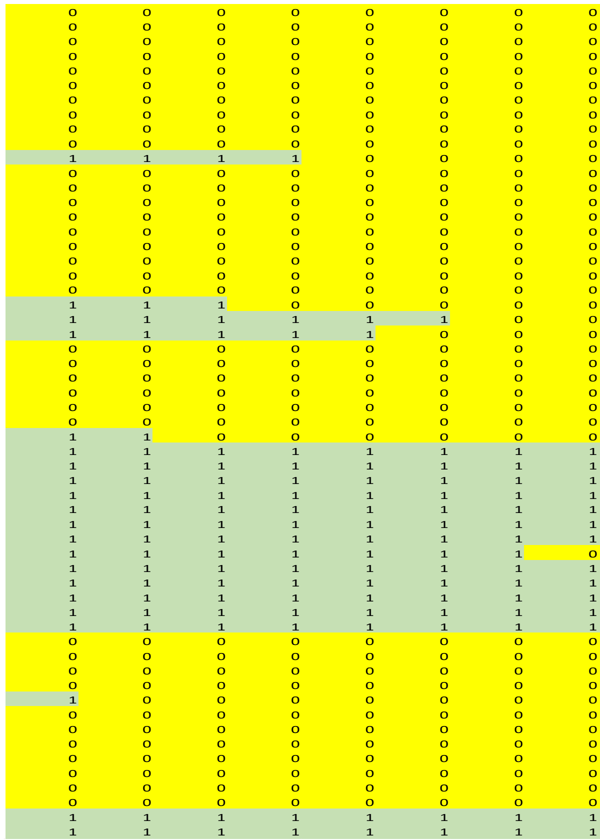


Fig . 7. Line 42-41 matrix with consideration of all network buses in 57 bus system.

Table 2. determine optimal monitoring buses to record voltage sag in different scenarios.

Network type	**Fault type**	**Optimal monitoring buses**	**Dimension of CBM**
14 bus	SLG	11,13,14	**14*77**
14 bus	SLG with R=4 ohm	10,11,12,13,14	**14*52**
14 bus	3L	12	**14*44**
57 bus	SLG	12,17,20,26,42,50,52	**57*871**

In Figure 8, the parts covered by each of the monitoring buses can be seen in SLG fault mode with impedance value of 4 ohms. areas covered by basses 10, 11, 12, 13, and 14 are shown in red, green, blue, yellow, and purple, respectively

Table 1. CBM matrix for single phase to ground fault with 4 ohm impedance for 14 bus network

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	0	0	0	1	1	0	0	1	1	1

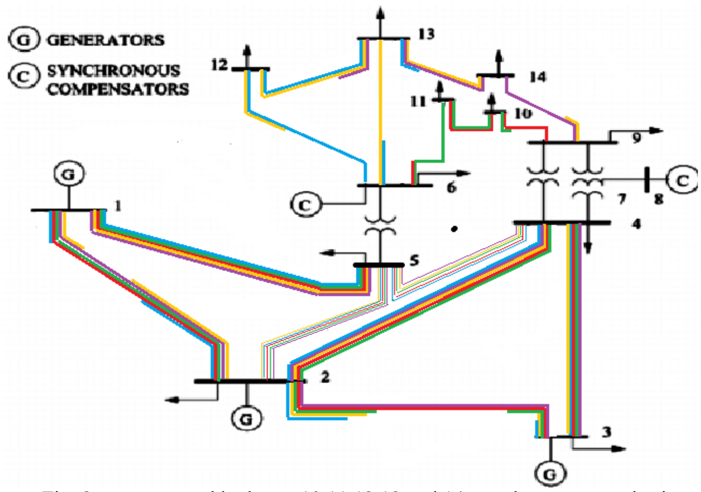

Fig. 8.. areas coverd by buses 10,11,12,13 and 14 to voltage sag monitoring

VI. CONCLUSION

Because it is not possible to use all the buses to monitor the entire network to record the voltage sag due to due to economic problems and high maintenance costs, so in this paper a method for optimal placement of monitoring buses is provided so that all parts of the network are covered by these buses. to determine these buses, first the vulnerable areas of the network buses for the threshold voltage of 0.9 were determined Then, by forming a comprehensive line status matrix (CBM) for the network, the formulas for determining the minimum and optimal monitoring buses can be applied to determine the desired buses. And as shown, the location and number of these buses is not fixed and depends on the impedance and the type of fault.
The proposed method, in addition to simplicity and low computational volume compared to other methods, has high accuracy in determining areas and consequently in determining monitoring buses

REFERENCES

[1] Bollen, M.H., Understanding power quality problems, in Voltage sags and Interruptions. 2000, IEEE press.

[2] Beaulieu, G., et al. Power quality indices and objectives. Ongoing activities in CIGRE WG 36-07. in IEEE Power Engineering Society Summer Meeting. 2002. IEEE.

[3] Sabin, D.D. and M.H. Bollen. Overview of IEEE Std 1564-2014 guide for voltage sag indices. in 2014 16th international conference on harmonics and quality of power (ICHQP). 2014. IEEE.

[4] Jalalat, H., S.G. Liasi, and M.T. Bina. Estimation of Number of Voltage Sags in the IEEE 14-Bus Network Using Bayesian and Artificial Neural Network: A Case Study. in 2020 28th Iranian Conference on Electrical Engineering (ICEE). 2020. IEEE.

[5] Lucio, J., E. Espinosa-Juarez, and A. Hernandez, Voltage sag state estimation in power systems by applying genetic algorithms. IET generation, transmission & distribution, 2011. **5**(2): p. 223-230.

[6] Zambrano, X., et al., Estimation of voltage sags from a limited set of monitors in power systems. IEEE transactions on power delivery, 2016. **32**(2): p. 656-665.

[7] Olguin, G., F. Vuinovich, and M.H. Bollen, An optimal monitoring program for obtaining voltage sag system indexes. IEEE Transactions on Power Systems, 2006. **21**(1): p. 378-384.

[8] Espinosa-Juarez, E., A. Hernandez, and G. Olguin, An approach based on analytical expressions for optimal location of voltage sags monitors. IEEE Transactions on Power Delivery, 2009. **24**(4): p. 2034-2042.

[9] Bertho, R., et al. Optimized power quality monitor placement based on a particle swarm optimization algorithm. in 2016 17th international conference on harmonics and quality of power (ICHQP). 2016. IEEE.

[10] Kempner, T.R., M. Oleskovicz, and D.P. Gomes, Optimal monitoring of voltage sags through simultaneous analysis of short-circuits in distribution systems. IET Generation, Transmission & Distribution, 2017. **11**(7): p. 1801-1808.

[11] Cebrian, J., C. Almeida, and N. Kagan. Genetic algorithms applied for the optimal allocation of power quality monitors in distribution networks. in Proceedings of 14th International Conference on Harmonics and Quality of Power-ICHQP 2010. 2010. IEEE.

[12] Martins, P.E.T., et al. Optimized allocation of power quality monitors in distribution systems considering fault location. in 2018 18th international conference on harmonics and quality of power (ICHQP). 2018. IEEE.

[13] Lucimario, G.d.O., A.A. da Silva, and A.T. de Almeida-Filho, Allocation of power-quality monitors using the P-median to identify nontechnical losses. IEEE Transactions on Power Delivery, 2016. **31**(5): p. 2242-2249.

[14] Hernández, A. and J.V. Milanović. Efficient Monitor Placement and Voltage Sag Estimation Using System Impedance Matrix. in 2019 IEEE Milan PowerTech. 2019. IEEE.

[15] Carpinelli, G., et al., Complete matrix formulation of fault-position method for voltage-dip characterisation. IET Generation, Transmission & Distribution, 2007. **1**(1): p. 56-64.

[16] Carpinelli, G., et al., Methods for assessing the robustness of electrical power systems against voltage dips. IEEE Transactions on Power Delivery, 2008. **24**(1): p. 43-51.

[17] Ceja-Espinosa, C., A. Ramos-Paz, and E. Espinosa-Juárez. Parallelization of the Fault Positions Method for stocastic assessment of voltage sags. in 2013 IEEE International Autumn Meeting on Power Electronics and Computing (ROPEC). 2013. IEEE.

[18] Goswami, A.K., C.P. Gupta, and G.K. Singh, Voltage sag assessment in a large chemical industry. IEEE Transactions on Industry Applications, 2012. **48**(5): p. 1739-1746.

[19] Bedford, E. and P. Frigge, The secant method for root finding, viewed as a dynamical system. Dolomites Research Notes on Approximation, 2018. **11**(4).

[20] Ray, S.S., Numerical analysis with algorithms and programming. 2018: Chapman and Hall/CRC.

[21] Soroudi, A., Simple Examples in GAMS, in Power System Optimization Modeling in GAMS. 2017, Springer. p. 33-63.