

Using Virtual Manipulatives to Conceptually Teach the Division of Fractions Using the Set Model

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Abstract:

Virtual manipulatives are a supportive tool to teaching fractions in a remote setting, as screens can be shared and problems can be explored as a class. For students who are new to dividing fractions, online, virtual two-sided chips are an adaptable tool used to facilitate student learning as they visualize the meaning of division using the set model to divide fractions. Preservice teachers explore the concept of dividing fractions using the virtual set model, moving beyond the traditional algorithm and the area model.

Keywords: virtual manipulatives, fractions, preservice elementary teachers, intermediate elementary mathematics

Overview

Oftentimes, teachers teach their students how to divide fractions using the traditional algorithm. However, one of the problems with teaching the traditional algorithm is that it does not help students make sense of the meaning of division in relation to fractions (Li, 2008). Models typically used to demonstrate conceptual understanding for fraction sense often focus on the area or linear model (Lo & Luo, 2012). The set model is not as common as the others. The set model can be defined as a set of objects that represent a fractional value and with the easy access to free virtual set model manipulatives, the possibilities for conceptual lessons increases.

Rationale and Significance in Teacher Preparation

Dividing can be challenging for intermediate elementary school children. They often struggle with the algorithm and making sense of divisional procedures (Squire & Bryant, 2002). Not only do elementary students have challenges when dividing, their teachers sometimes struggle as well. Isik and

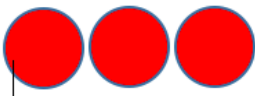
Kar (2012) studied fractional division error problems of preservice teachers ($n = 64$). They found that they made 311 errors regarding the division of fractions. The researchers observed that the conceptual dimension of fraction division is often ignored by teachers. Their findings indicate the importance of focusing on conceptual division of fractions with preservice teachers in their university preparation coursework. Lo and Luo (2012) found that elementary and middle school Taiwanese teachers often struggle with pictorial diagrams to represent dividing of fractions. There was an emphasis on flexible solving. However, they were more secure in using linear and area models versus other models like the set model. Pictorial representations were also lacking.

Conceptual Framework

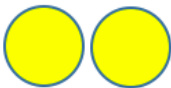
Defining the whole is an important component when using two-sided chips. In one example, 1, students must first build a whole. If the whole is built with just one two-sided chip, then division is not as obvious. However, building the whole with any number divisible by 3 (the denominator) will support the visualization. In **Figure 1**, there are 3 chips created using virtual manipulatives. This is the whole. What is $\frac{2}{3}$ of the whole? And how many groups of $\frac{2}{3}$ are there in the whole? There are 1 and $\frac{1}{2}$ groups of $\frac{2}{3}$ in 1. The whole could have been defined in a number of ways; if 18 is the whole, then 12 two-sided chips would be equivalent to $\frac{2}{3}$. There are 1 $\frac{1}{2}$ groups of 12 two-sided chips ($\frac{2}{3}$) in the whole (a whole is defined as 18 chips in this example). The whole can be defined using multiples of the least common multiple (LCM). A second example can be seen in **Figure 2**. $\frac{1}{4} \div \frac{5}{6}$, or how many groups of $\frac{5}{6}$ fit into $\frac{1}{4}$.

Figure 1. 1 using virtual manipulatives

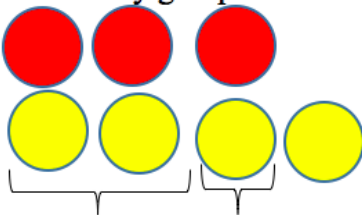
⊕ This 1 with a whole defined as three chips.



This is $\frac{2}{3}$ of the whole.



How many groups of $\frac{2}{3}$ fit into 1?

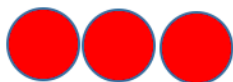


One group
of $\frac{2}{3}$ Half a
group of $\frac{2}{3}$

□

Figure 2. $\frac{1}{4} \div \frac{5}{6}$ using virtual manipulatives

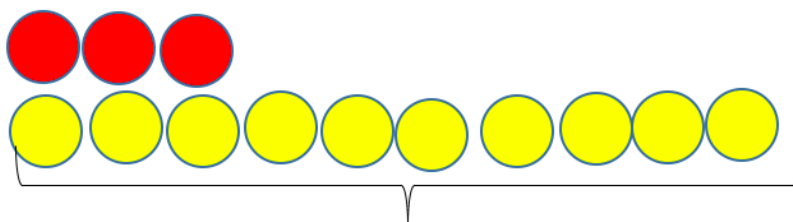
This is $\frac{1}{4}$ with the whole defined as 12 chips.



This is $\frac{5}{6}$ of the whole.



How many groups of $\frac{5}{6}$ fit into $\frac{1}{4}$?



$\frac{3}{10}$ of the $\frac{5}{6}$ fits into $\frac{1}{4}$

Participants worked in a classroom setting using virtual manipulatives as needed. In addition, the activities can be taught with learners through remote learning. Tasks can be presented to learners working in breakout rooms with virtual manipulatives, they can solve problems and then share screens. Free virtual manipulatives can be found on many websites using the term “virtual two-sided chips for math”.

Experiences in Practice

Preservice teachers were asked to explain $2\frac{3}{4} \div 1\frac{1}{2}$ using virtual manipulatives. A preservice teacher asked why the answer $1\frac{5}{6}$ does not have the whole (4 chips) in the denominator, nor is 6 a multiple of 4. The student could not make sense of it. This resulted in a discussion regarding the need (or lack of need) for common denominators when dividing fractions. While it may seem as a given that common denominators are not needed for division, preservice teachers were challenged by basic operational understanding. Another student explained, “The denominators are just part of the set, they do not need to be common like they do for adding and subtracting.” She continued, “The key is to find an LCM [least common multiple] that will work for both of the fractions.” She continued to give examples, “7 wouldn’t work, neither fraction can be made with a seven as a whole. If you make 7, then you can’t take $\frac{3}{4}$ of it.” Another preservice teacher responded, “See, 1 and $\frac{1}{2}$ is how many chips? It’s 6 chips. If a whole is 4, then half a whole is 2, and that makes 6. 6 chips is [equal to] $1\frac{1}{2}$. That’s where the denominator comes from, not from getting a common denominator.”

Scholarly Significance

Both elementary children and their teachers (both preservice and in-service) struggle with division of fractions. Lortie-Forgues, Tian, and Siegler (2015) found that fractional division can be very challenging for elementary school children. Their research showed that there was minimal instruction regarding fractional division and that oftentimes, instruction focused on memorization of procedures. In

order to improve elementary students' experiences, their teachers' experiences must also be improved to move beyond the memorization of procedures. More importantly, the virtual manipulatives aspect of the lesson is a productive way to engage in conceptual understanding of mathematical ideas through remote experiences if face-to-face meetings are not possible. While it can be particularly challenging to teach conceptual understanding in mathematics through remote learning, virtual manipulatives can provide a mechanism for learners to share their thinking and reasoning and see other people's perspectives.

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