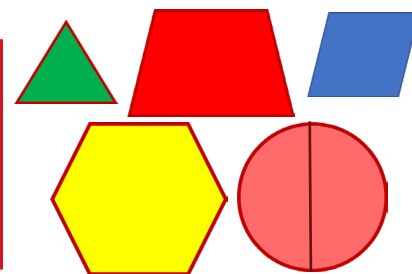


Visualizing Fraction Multiplication with Circles and Hexagons

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There are two common, traditional, algorithmically-based approaches to teaching the multiplication of fractions. In one method, students learn to convert all fractions to improper fractions, multiply the numerators, multiply the denominators, and then simplify. In a second method, students learn to cross-cancel after converting fractions to improper fractions, then multiply the numerators and multiply the denominators. Both of these approaches share a lack of focus on sense-making and visualization—students routinely follow these procedures without emphasizing sense-making.

But what happens when the students start to ask questions? Elementary students may ask: “What does multiplying fractions look like?” or “Why does the multiplication process (i.e., the algorithm) work?” teachers should be able to facilitate learning the answers to these questions for all students because they need to know the “why” of mathematics. This article presents two methods that use the area model to provide students with experiences that emphasize a visual approach to multiplying fractions. The first approach emphasizes fraction circles, and the second emphasizes pattern blocks (with the whole defined using hexagons).

Before multiplying fractions, teachers should provide students with an opportunity to explain what multiplication of whole numbers means, for example, to explain what 4×6 means. Students may say “four groups of six” or some version of that explanation. The word “of” is important when transitioning to fractions. For example, replacing the 6 in the original problem with a fraction can illustrate the transition such as: $4 \times \frac{1}{2}$. Students may say “four groups of one-half” equals 2 (four groups of $\frac{1}{2}$ can be written $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$). Through sense-making, it is not necessary to turn the 4 into an improper fraction, cross-cancel, and then multiply. For another example, replacing the 4 in the original problem with a fraction can support learning: $\frac{1}{2} \times 6$. Students may say “half of a group of six” equals 3. Students can take $\frac{1}{2}$ of a

group of 6; in this case, they do not need the iterations because the problem contains less than one group.

Using Fraction Circles or Pattern Blocks

Fraction circles are common manipulatives that help students visualize the multiplication of fractions. Most students are familiar with their values: whole, halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths. The fraction values might be less challenging for students because they generally think of a whole as a circle, which is more intuitive than the variations of other shapes defined as a whole. Students are also likely familiar with other common fraction values.

In contrast, pattern blocks can be more challenging since fractional values may not be as intuitive as they are with fraction circles. However, pattern blocks have a significant benefit over fraction circles because students can use them to define a whole more easily. For example, two hexagons can be the whole, one triangle can be the whole, or two blue rhombi can be a whole. In addition, pattern blocks generally require more thought and analysis when solving fractional problems. Learners must build and compare more frequently than with fraction circles.

Redefining wholes with fraction circles is more challenging. For example, fraction circles may come with pre-printed values, or students may struggle with the concept of $\frac{3}{8}$ (or any other fraction) being defined as the whole because the whole in this situation is not a circle.

To answer students’ conceptual questions, such as: “What does multiplying fractions look like?” or “Why does the multiplication process (i.e., the algorithm) work?” it is imperative that teachers provide opportunities for students to play, explore, and discover what the multiplication of fractions looks like. The activities described here can support students as they learn to understand mathematical concepts—moving beyond

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the standard algorithm. While the standard algorithm has great importance, so does helping students understand the operations of fractions before its introduction.

Using Fraction Circles to Multiply Fractions

Fraction circles are likely the most familiar models to help students make sense of multiplying fractions. For most commercially available fraction circles, the commonly available fractional values are a whole, halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths. For remote instruction, teachers can find free virtual manipulatives with a search using the term “free virtual manipulatives, fraction circles.” Not all multiplication problems are possible with manipulatives, so teachers must carefully select problems. All examples presented here are possible with the common fractions listed above, but some problems may require more than one fraction circle set.

Initially, ask students to solve $4 \times \frac{3}{5}$ or four groups of three-fifths (see *Figure 1*). Students can build the solution by making four groups of $\frac{3}{5}$, as seen in image *1a*. In image *1b*, the solution in *1a* is grouped into wholes. The solution is two wholes and $\frac{2}{5}$ of a whole.

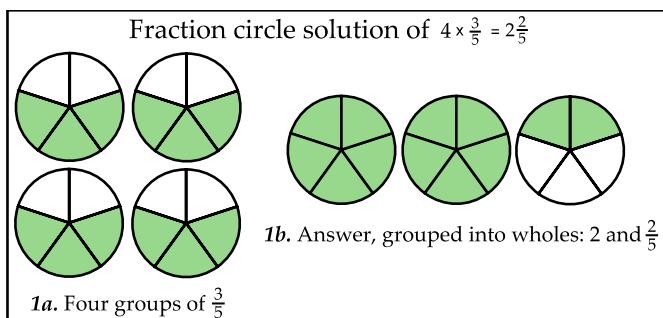


Figure 1

In the following example, as shown in *Figure 2*, the multiplier is a fraction, and the multiplicand is a whole number: $\frac{1}{4} \times 2$ (read this problem as one-fourth of a group of 2). *Figure 2a* shows the image of 2 wholes, and then one-

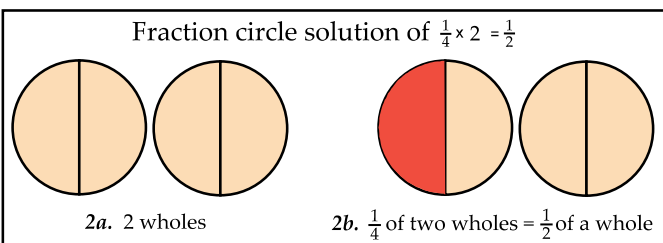


Figure 2

fourth of the two wholes are shown in *Figure 2b*.

It is more challenging when both fractions are less than one. For example, $\frac{2}{5} \times \frac{5}{6}$ can be read as two-fifths of a group of five-sixths. To visualize the solution and to stay consistent with the language, $\frac{5}{6}$ should be built first, and then $\frac{2}{5}$ of that group yields the visual solution of $\frac{2}{6}$ (see *Figure 3*). When conceptually teaching the multiplication of fractions, it does not always make sense to simplify the solutions.

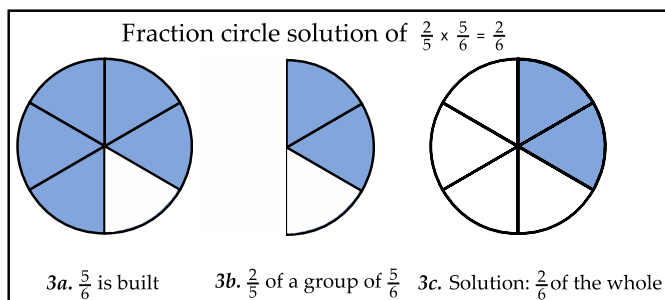


Figure 3

The final fraction circle example is for the problem $1\frac{1}{3} \times \frac{6}{8}$. Students can read the problem as 1 and $\frac{1}{3}$ of a group of $\frac{6}{8}$. The solution to this problem is 1 group of $\frac{6}{8}$ and of a group of $\frac{6}{8}$, as shown in *Figure 4*.

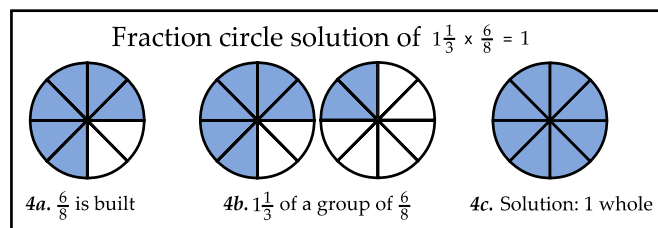


Figure 4

Multiplying Fractions Using Pattern Blocks

Pattern blocks are another manipulative used to support the visualization of multiplying fractions. When using pattern blocks for multiplying, only some of the manipulatives—hexagons, trapezoids, blue rhombi, and triangles—are helpful. The other pattern block shapes—squares and tan rhombi—are not helpful and should be removed. When using pattern blocks, the whole can be redefined; *Table 1* provides an overview of the values of the other manipulatives if a whole is defined using iterations of hexagons. Wholes can also be defined with other shapes, but only hexagons will be used as wholes in the activities we have included.

If a whole is defined as . . .	The value of a . . .			
	Hexagon	Trapezoid	Blue Rhombus	Triangle
one hexagon	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
two hexagons	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$
three hexagons	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$
four hexagons	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{24}$

Table 1

If one hexagon is a whole, the solution for $\frac{1}{2} \times \frac{2}{3}$ can be thought of as $\frac{1}{2}$ of a group of $\frac{2}{3}$. **Figure 5** shows what the solution would look like using pattern blocks. Problems with one hexagon are generally less challenging than those that redefine a whole with more than one hexagon.

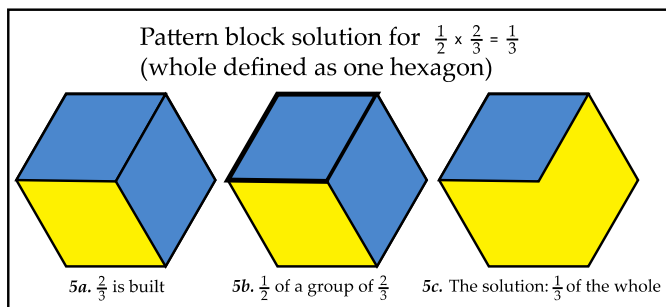


Figure 5

In the following example, a whole is defined as two hexagons. The two problems are similar: $\frac{3}{4} \times \frac{1}{3}$ (think of this as $\frac{3}{4}$ of $\frac{1}{3}$) and $\frac{1}{3} \times \frac{3}{4}$ (think of this as $\frac{1}{3}$ of $\frac{3}{4}$). While these problems have the same solution, the images are very different (see **Figure 6**).

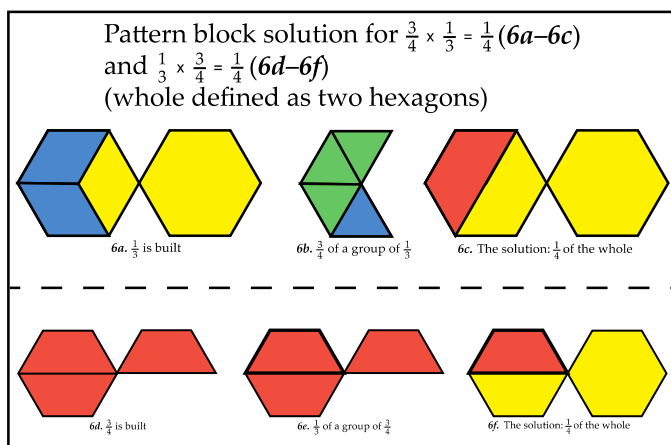


Figure 6

One final example defines a whole as four hexagons. The problem is $\frac{7}{15} \times \frac{1}{4}$; think of this as

$\frac{7}{15}$ of $\frac{1}{4}$. To build $\frac{1}{4}$ a whole is four hexagons, and one-fourth is one hexagon; so $\frac{1}{4} = 5$ hexagons (see **Figure 7**). Then, $\frac{1}{4}$ needs to be broken up into equal parts of 15 (see 7b).

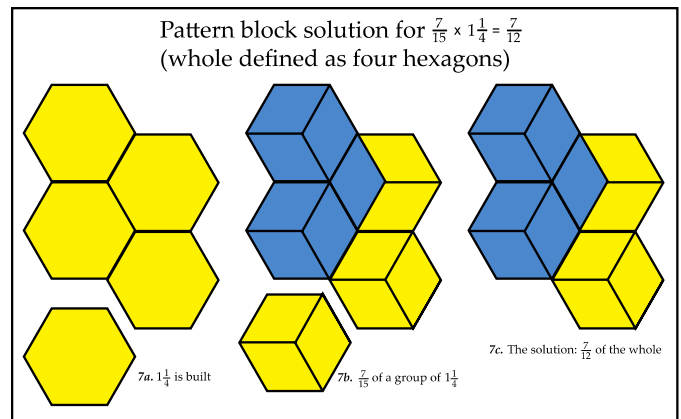


Figure 7

Class Discussion

Allowing students to discuss their observations will help their mathematical development. Since discussion prompts can help students make sense of their experiences, the teacher might ask the students:

- ✓ When using pattern blocks, if the whole is redefined, what changes? What stays the same?
- ✓ Can all fraction multiplication problems be solved using fraction circles and/or pattern blocks? Explain.
- ✓ What are some of the patterns you notice?
- ✓ Does the answer change if a problem is changed? In other words, does the multiplier become the multiplicand, or does the multiplicand become the multiplier? Does the image change? Try this problem $\frac{1}{3} \times \frac{3}{4}$ and see if your conjecture is correct. Use either fraction circles or pattern blocks with a whole defined as two hexagons.

Teachers might also ask students to create their own problems using either manipulative. There are challenges to writing problems because not all fraction multiplication problems are possible with these manipulatives. The task will be challenging and require some mathematical analysis.

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