

# Effects of initial consolidation on the Triggering of Static Liquefaction Considering Fabric Effects

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## ABSTRACT

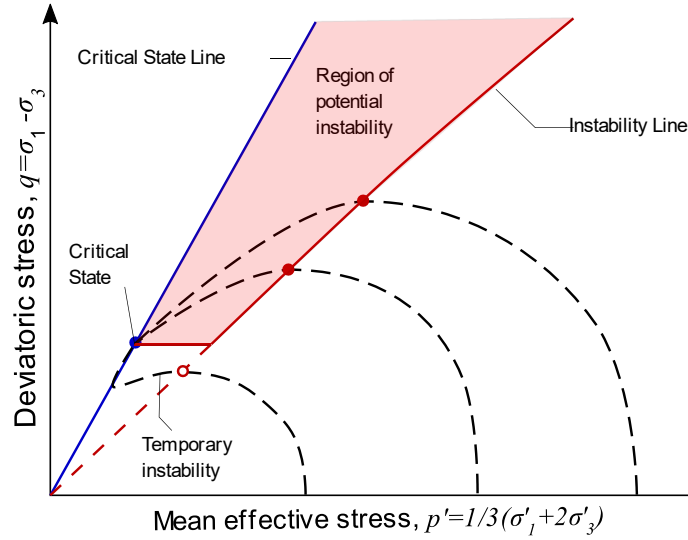
The onset of static liquefaction in anisotropically consolidated soils is of relevance in assessing the performance of geotechnical systems. Previous studies have also highlighted the role of inherent soil fabric. This study derives an analytical instability criterion for granular materials under undrained loading by using the relatively new anisotropic critical state theory (ACST). The criterion is established using the SANISAND-F model, and it is amenable to incorporating consolidation anisotropy and fabric effects. We assess different numerical strategies for simulating the instability onset on materials sheared from initially anisotropic conditions. Our assessments indicate that simulations that consider consolidation followed by shear better represent the response observed in laboratory tests. It is observed that the degree of anisotropic consolidation has no significant effect on the instability stress ratio, but a very high degree of anisotropic consolidation results in a spontaneous collapse. It is also observed that the anisotropic consolidated specimens have a higher instability stress ratio in triaxial compression than in triaxial extension, highlighting the effect of loading direction relative to the existing fabric.

**Keywords:** Flow liquefaction, SANISAND-F, Fabric Anisotropy, Anisotropic consolidation

## INTRODUCTION

The onset of instability under monotonic loading of granular materials, also referred to as static liquefaction or flow liquefaction, has caused numerous geotechnical failures in the past. Flow liquefaction is associated with a state of instability followed by sudden increases in strain and pore water pressure. It can occur in any saturated or near-saturated contractive soils, such as very loose sands, silts, as well as very sensitive clays. Previous research has suggested that the instability

onset is triggered at a characteristic stress ratio. This concept is illustrated in Figure 1 using undrained triaxial tests from Lade (1999). In particular, Lade (1999) highlighted that the stress state at the point of instability for samples with the same initial density but under different confining stresses are aligned on a unique line called the instability line. This so-called instability line represents the stress conditions in which flow liquefaction triggers, leading to the potential instability region shown in Figure 1.



**Figure 1 Illustration of Instability line in a  $p'$ - $q$  space, considering undrained triaxial compression tests. Modified from Lade (1999).**

Experimentally, flow liquefaction has been mostly explored under triaxial conditions. Previous research has shown that despite some small differences in the definitions adopted and conclusions drawn, physical interpretations are consistent across different studies, i.e., the instability onset specifies a yielding point where large plastic strains can develop (Najma and Latifi 2017). Majority of the studies on flow liquefaction focus on sand specimens subjected to shear under initial isotropic stress conditions. However, the in-situ stress condition for most common geo-structures (like slopes and dams) is not isotropic; an initial static driving shear stress exists on the soil element prior to any external loading and thus resulting in an anisotropic initial stress state.

The effect of initial anisotropy (induced during consolidation before shearing) under triaxial conditions has also been subject to different interpretations. For example, Najma & Latifi (2017) used undrained triaxial compression tests on Sacramento sands performed by Kramer (1996) and suggested that the higher the anisotropy during initial consolidation, the steeper the slope of the instability line. In contrast, Kato et al. (2001), using anisotropically consolidated specimens of Toyoura sand subjected to undrained triaxial compression loading, suggested that

the slope of the instability line did not vary significantly with respect to the initial anisotropic consolidation.

In this study, we derive an analytical instability criterion for granular materials under undrained loading by using the relatively new anisotropic critical state theory (ACST). The criterion is established using the SANISAND-F model, and it is amenable to incorporating consolidation anisotropy and fabric effects. We use the established criterion to numerically assess different modeling strategies to investigate the effects of initial consolidation in triggering instability. Specifically, we explore the effects of initial anisotropic consolidation and fabric anisotropy on the slope of the instability line estimated from the criterion.

**Table 1 SANISAND-F parameters**

| Description     | Symbol         | Values                                                                                           |
|-----------------|----------------|--------------------------------------------------------------------------------------------------|
| Elasticity      | $G_0$          | 125                                                                                              |
|                 | $\nu$          | 0.05                                                                                             |
| Critical state  | $e_{ref}$      | 0.934                                                                                            |
|                 | $\xi$          | 0.7                                                                                              |
|                 | $\lambda$      | 0.019                                                                                            |
|                 | $M_c$          | 1.25                                                                                             |
|                 | $c$            | 0.75                                                                                             |
| Plastic modulus | $h_1$          | 7.5                                                                                              |
|                 | $c_h$          | 0.85                                                                                             |
|                 | $n^b$          | 1.4                                                                                              |
| Yield surface   | $m$            | 0.01                                                                                             |
| Dilatancy       | $A_0$          | 0.704                                                                                            |
|                 | $n^d$          | 3.5                                                                                              |
| Fabric          | $e_A$          | 0.0818                                                                                           |
|                 | $F_{in}$       | 0.5                                                                                              |
|                 | $c_0$          | 5.2                                                                                              |
|                 | $h_2$          | 1.3                                                                                              |
|                 | $\mathbf{n}_F$ | $\begin{bmatrix} 2/\sqrt{6} & 0 & 0 \\ 0 & -1/\sqrt{6} & 0 \\ 0 & 0 & -1/\sqrt{6} \end{bmatrix}$ |

## THE SANISAND-F MODEL

The SANISAND-F model was recently proposed by Petalas et al. (2020). It is an extension of the critical two-surface plasticity model presented in Dafalias & Manzari (2004). The model is formulated within the ACST (Li and Dafalias 2012), which accounts for the effect of fabric

anisotropy on the mechanical behavior of granular soils. For a detailed presentation of the model and the ACST, the reader is referred to Li & Dafalias (2012) and Petalas et al. (2020).

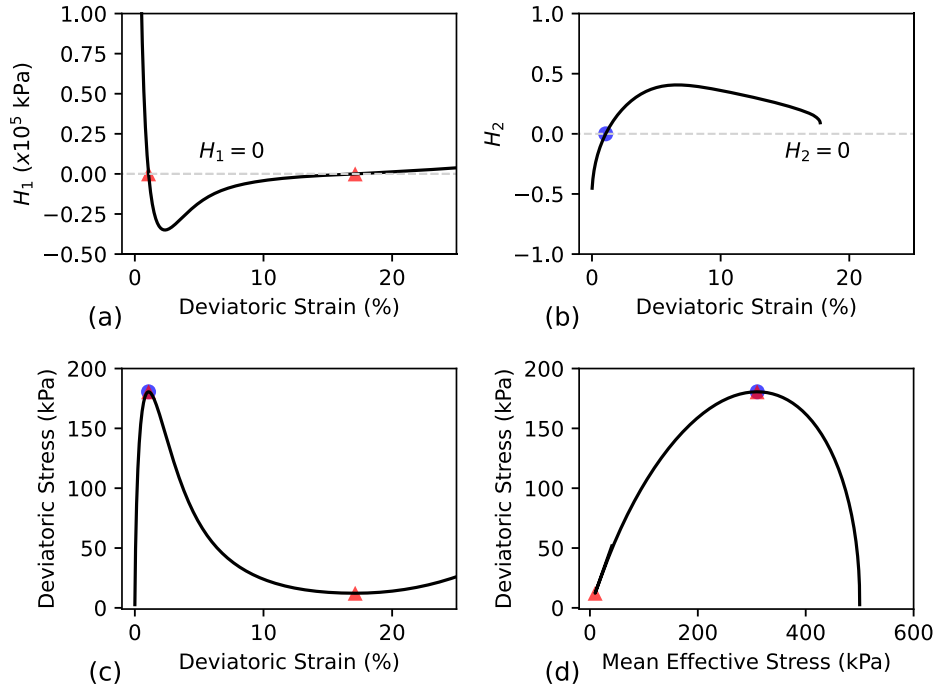
In addition to the features of the DM04 (Dafalias and Manzari 2004) model, the SANISAND-F model utilizes a deviatoric fabric tensor  $\mathbf{F}$  as an evolving state variable. A scalar-valued Fabric Anisotropy Variable,  $A = \mathbf{F}:\mathbf{n}'$ , is then introduced as a measure of relative orientation between loading ( $\mathbf{n}'$ ) and fabric directions ( $\mathbf{F}$ ). The original critical state conditions are enhanced as proposed in Li & Dafalias (2012) via  $A = A_c = 1$ , which denotes that at critical state the fabric and loading directions coincide. The isotropic state parameter  $\psi$  (Been and Jefferies 1985) is enhanced with the effect of fabric via the dilatancy state parameter  $\zeta$ . The new state parameter determines the estimated dilatancy, which depends on fabric anisotropy, and the model's response becomes more contractive as the difference in fabric and loading orientation increases. In this study, we used the material parameters suggested by Petalas et al. (2020) (see **Error! Reference source not found.**), who calibrated the model against the experimental results from Yoshimine et al. (1998).

## FLOW LIQUEFACTION INSTABILITY CRITERION

Analytical flow instability criteria can be derived using the fabric-dependent multi-axial SANISAND-F constitutive model for undrained loading following the procedure presented in Najma & Latifi (2017). Two main conditions can be used to derive the instability criterion from the SANISAND-F equations. The first condition is that in an undrained loading, the rate of volumetric strain can be considered zero, i.e.,  $d\epsilon_v = 0$ . The second condition is that the slope of an undrained stress path at the instability point on the stress space is zero, i.e.,  $\frac{dq}{dp} = 0$ . Using these two conditions, two instability criteria ( $H_1$ , and  $H_2$ ) can be derived, i.e.,  $H_1 = K_p - K_{p,f} = 0$  and  $H_2 = \beta - \beta_f = 0$ . Where  $K_p$  is the current plastic modulus and  $K_{p,f}$  is the plastic modulus at the instability point. Similarly,  $\beta$  is the current stress-ratio and  $\beta_f$  is the stress-ratio at the instability point. More details on the step-by-step procedure of the derivation of instability criterion are presented in Bokkisa et al. (2022).

Figure 2a and Figure 2b show the evolution of the instability criteria  $H_1$  and  $H_2$  while Figure 2c and Figure 2d show the stress-strain response of numerical simulations performed at an initial void ratio of 0.89, and confining stress of  $p_0 = 500 \text{ kPa}$  for a hollow cylinder loading path with constant principal stress direction,  $\alpha_\sigma = 45^\circ$ , and intermediate stress ratio,  $b = 0.5$ . It can be observed that the first criterion yields two instances where  $H_1 = 0$  (see Fig. Figure 2a): the first point marks the onset of flow liquefaction, while the second represents a change from contractive to dilative behavior, also known as the transformation point (Andrade et al. 2013). By contrast, the  $H_2 = 0$  criterion (see Fig. Figure 2b) predicts only the onset of flow liquefaction point. It is important to note that both  $H_1$  and  $H_2$  criteria are consistent in identifying the onset of flow

liquefaction, and from here on, unless specified, we use only the  $H_2$  criterion to predict flow liquefaction instability point.



**Figure 2 SANISAND-F simulation of a hollow cylinder test (a & b) the  $H_1$  and  $H_2$  criterion (c&d) stress-strain and stress path response**

## INFLUENCE OF INITIAL CONSOLIDATION AND FABRIC ANISOTROPY ON THE ONSET OF INSTABILITY

Assessing the instability conditions imposed by an undrained loading on a material that has been anisotropically consolidated is of interest, for example, for anisotropic consolidated triaxial tests (also known as  $K_0$  triaxial compression tests). When simulating an element test that involves anisotropic consolidation by the use of a constitutive driver, the method for initialization of the stress state and state variables after consolidation (before undrained shearing) affects the results. In this work, we consider two different methods, namely Method 1 and Method 2, for reference. In Method 1, the anisotropic stress state after consolidation is directly assigned as input to the algorithm, and state variables (e.g., void ratio, etc.) are equal to the ones in the element test after the process of consolidation. In Method 2, the simulation is performed in two steps. First, the anisotropic consolidation is simulated, and then the undrained shearing is imposed.

In bounding surface plasticity models for sands, like DM04 or SANISAND-F, the above-mentioned choices affect the simulated stiffness of the material during undrained shearing and thus the simulated flow instability stress ratio that is of interest in this work. More specifically, the

simulation results are sensitive to the determination of  $\alpha_{in}$  tensor because it controls the plastic modulus ( $K_p$ ) which is expressed as:

$$K_p = pH(\alpha_\theta^b - \alpha):n \quad (1)$$

$$H = \frac{2}{3} \frac{h(e, p, A)}{<(\alpha - \alpha_{in}):n>} \quad (2)$$

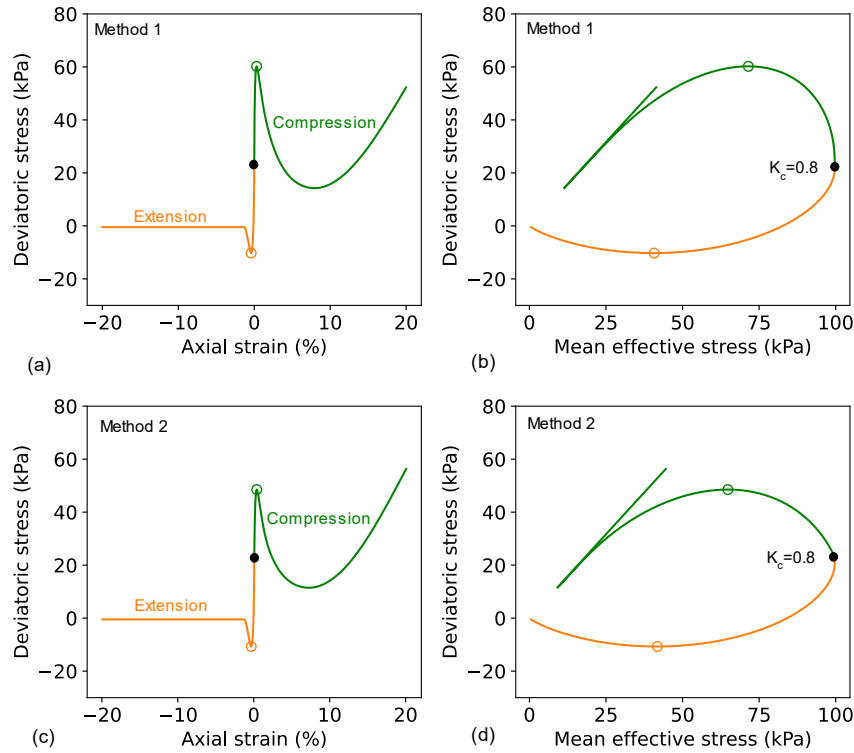
$\alpha_{in}$  is the value of  $\alpha$  at the initiation of a new loading process (e.g., unloading after loading), which is signified by the zero or negative value of the quantity  $(\alpha - \alpha_{in}):n$  within  $<>$  in the denominator. When  $(\alpha - \alpha_{in}):n \leq 0$ , i.e., a new loading process is determined,  $K_p \approx \infty$ ; thus, the model predicts a very small plastic strain increment (the step becomes elastic) and  $\alpha_{in}$  is updated to  $\alpha$ . Therefore, after the initiation of a new loading process, the behavior becomes elastic with very high stiffness for the first few steps, until  $K_p$  starts to decrease again due to elasto-plastic behavior.

We use the two methods (Method 1 and 2) to simulate undrained compression and extension shearing on Toyoura sand on anisotropically consolidated samples, considering an initial void ratio of 0.915,  $p'_0 = 100 \text{ kPa}$  and a consolidation ratio  $K_c = \sigma'_{hc}/\sigma'_{vc} = 0.8$ .  $\sigma'_{hc}$  and  $\sigma'_{vc}$  are the horizontal and vertical effective stresses before the undrained shearing. The discussion on the modeling strategy is relevant because, to our knowledge, previous numerical studies have explored the effects of anisotropic consolidation by using Method 1 (e.g., Najma & Latifi (2017)). Figure 3 shows the simulation results, highlighting the difference between the two methods. In Method 1, only the undrained shearing phase is simulated, and the initial back-stress ratio  $\alpha_{in} = \mathbf{0}$  is set to be equal with the stress ratio  $r = \alpha$  at the end of consolidation. On the other hand, in Method 2,  $\alpha_{in} = \mathbf{0}$ , due to the fact that the consolidation process is simulated starting from a zero stress and back-stress state, and there is no new loading process initiated (i.e., there is no reversal of loading direction) from the beginning until the end of the phase that updates  $\alpha_{in}$ .

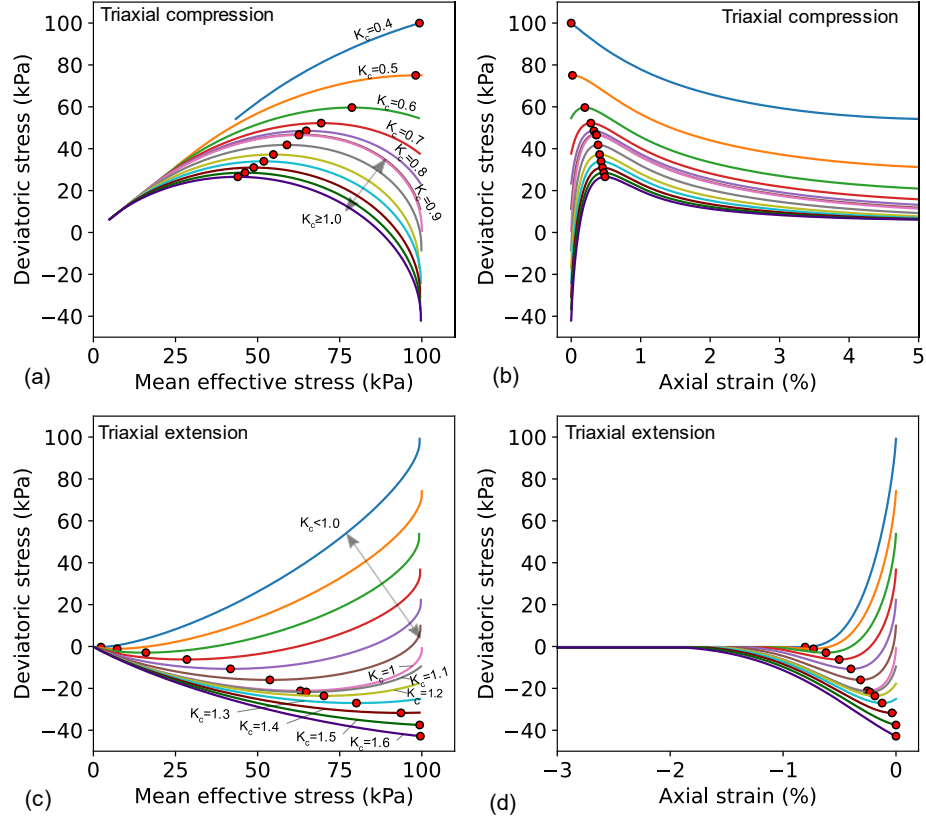
In Method 1,  $(\alpha - \alpha_{in}):n = 0$  during the initiation of both undrained compression and extension, which makes the denominator of the plastic modulus in Eq. 2 to infinity, and thus, the plastic part of the total strain negligible. Practically, during undrained shearing, the stiffness initially is elastic, as observed in Figure 3b, where the stress path initiates with a vertical orientation (no decrease in mean effective stress) for both cases. This leads to a larger peak deviatoric stress in triaxial compression compared to Method 2 (Figure 3b and d). This is due to the fact that in Method 2, during the consolidation simulation  $\alpha_{in}$  remains zero (no loading reversal), and when the undrained compression begins  $(\alpha - \alpha_{in}):n > 0$ , the plastic modulus  $K_p$  remains a positive and finite value, without minimizing the plastic strain increment during the first step. This is why in Method 2, during undrained compression, the slope of the stress path in Figure 3d starts immediately inclined, a decrease in mean effective stress initiates from the first step, and the simulated instability stress ratio is lower than the one predicted in Method 1. On the other hand, during undrained extension with Method 2,  $(\alpha - \alpha_{in}):n < 0$ ,  $\alpha_{in} = \mathbf{0}$ ,  $\alpha$  is a compression-like tensor due to a compression consolidation process and  $n$  is an extension-like tension. This

updates  $\alpha_{in}$ , a new loading process initiates and the response is elastic for the first few steps due to the very large plastic modulus.

Even though Method 1 has been used in the literature before (e.g., Najma & Latifi (2017)), the update in  $\alpha_{in}$  for both the compression and extension cases means that the loading history during consolidation affects the two cases equally by indicating that a new loading process begins. We believe that Method 2 should be preferred since it reflects more realistically the conditions experienced in the laboratory and takes into account the effect of loading history. Thus, we adopt Method 2 to investigate the effect of initial anisotropic consolidation and fabric anisotropy on the onset of instability. Figure 4 shows the results of simulations of triaxial compression (TC) and extension (TE) responses for several anisotropic consolidation ratios,  $K_c$  ranging from 0.4 to 1.6 at void ratio 0.915 and mean effective confining stress of  $100\text{kPa}$ . Note that consolidation ratios of  $K_c = 1.0$ ,  $K_c < 1.0$  and  $K_c > 1.0$  represent specimens that are isotropically, compressionally, and extensionally consolidated, respectively. The undrained triaxial compression tests (see Figure 4a,b) show that as  $K_c$  decreases, it is easier to trigger an instability. For instance, in the case of  $K_c = 0.4$ , there is a spontaneous collapse (that is, decreasing deviatoric stress with increasing axial strain), also referred to as “incipient instability” (Buscarnera and Whittle 2013). Similar behavior is observed in the triaxial extension tests (see Figure 4c,d), but now it is easier to trigger liquefaction as  $K_c$  increases. In this case, the incipient instability is observed at a consolidation ratio  $K_c$  of 1.5 or higher.



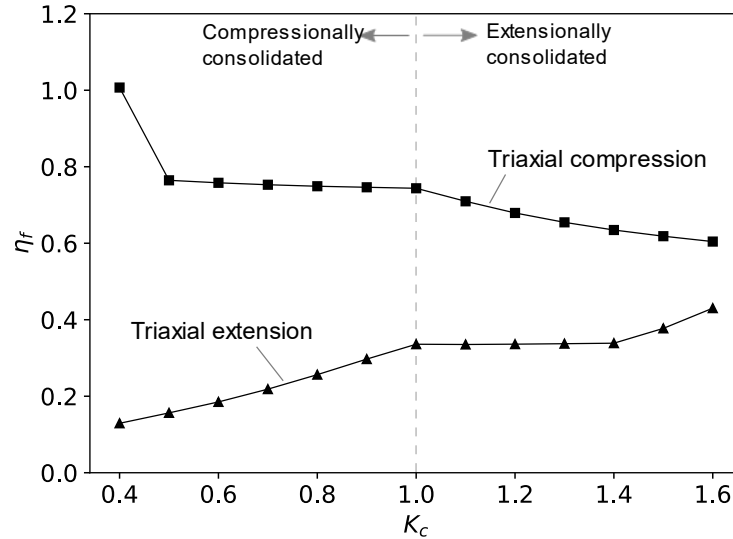
**Figure 3 Constitutive responses in triaxial compression and extension using Method 1 (a,b) and using Method 2 (c,d)**



**Figure 4 Constitutive responses for several anisotropically consolidated specimens with  $K_c$  ranging from 0.4 to 1.6 in triaxial compression (a, b) and triaxial extension (c, d)**

Using the results from Figure 4, Figure 5 shows that anisotropic consolidated specimens have a higher instability stress ratio ( $\eta_f$ ) in triaxial compression than in triaxial extension. In undrained triaxial compression tests on extensionally consolidated samples, it is observed that the instability stress-ratio gradually decreases as the anisotropy in consolidation increases (i.e., increase in  $K_c$ ). But, from the undrained triaxial compression tests on compressional consolidated specimens, it is observed that the initial consolidation ratio has no significant effect on the stress ratio at the instability point except for extreme  $K_c$  values; where due to the incipient instability behavior, there is an increase in instability stress ratio for  $K_c$  values  $< 0.5$ . Similar observations hold for the undrained triaxial extension tests where the compressional consolidated specimens show a decrease in instability stress ratio with the increase in anisotropic consolidation (i.e., decrease in  $K_c$ ) and extensionally consolidated specimens have no significant effect on the instability stress ratio except for extreme  $K_c$  values ( $> 1.4$ ). It is also observed that the anisotropic consolidated specimens have a higher instability stress ratio in triaxial compression than in triaxial extension. This highlights the effect of fabric anisotropy, i.e., as the degree of relative fabric-induced anisotropy increases (from TC to TE), the instability stress-ratio decreases. Of final note, the results presented for the triaxial compression tests on compressional consolidated specimens

in Figures Figure 4 and Figure 5 are consistent with previous experimental studies (Chu and Wanatowski 2008; Kato et al. 2001; Yang et al. 2021).



**Figure 5 Variation of instability stress ratio with respect to initial consolidation ratio,  $K_c$ , for triaxial compression and triaxial extension**

## CONCLUSION

In this study, we have used the SANISAND-F model formulated under the ACST framework to investigate the onset of static liquefaction under undrained loading conditions, considering anisotropic consolidation and fabric anisotropy. The different strategies for estimating instability stress ratios when the undrained loading is imposed after an initial anisotropic consolidation were discussed. Unless there is evidence of loading/unloading processes that update the value of  $\alpha_{in}$ , we recommend using a two-step process that consists of simulating anisotropic consolidation before imposing the undrained loading boundary conditions (i.e.,  $\alpha_{in}$  is updated only upon loading reversal). We showed that this strategy provides results that are consistent with previous experimental studies in triaxial compression conditions, where the instability stress ratio is not significantly affected by the initial anisotropic consolidation before loading. However, a very high degree of anisotropic consolidation resulted in a spontaneous collapse (i.e., decreasing deviatoric stress with increasing axial strain). Fabric anisotropy has a significant role in determining the onset of flow liquefaction where higher initial fabric anisotropy resulted in a lower instability stress ratio.

## ACKNOWLEDGMENTS

This study has been funded by the National Science Foundation (NSF) under the CMMI 2013947 project.

## REFERENCES

- Andrade, J. E., A. M. Ramos, and A. Lizcano. 2013. "Criterion for flow liquefaction instability." *Acta Geotechnica*, 8 (5): 525–535. Springer.
- Been, K., and M. G. Jefferies. 1985. "A state parameter for sands." *Géotechnique*, 35 (2): 99–112. Thomas Telford Ltd.
- Bokkisa, S. V., J. Macedo, A. L. Petalas, and C. Arson. 2022. "Assessing flow liquefaction triggering considering fabric anisotropy effects under the ACST framework." *Computers and Geotechnics*.
- Buscarnera, G., and A. J. Whittle. 2013. "Model prediction of static liquefaction: influence of the initial state on potential instabilities." *Journal of Geotechnical and Geoenvironmental Engineering*, 139 (3): 420–432. American Society of Civil Engineers.
- Chu, J., and D. Wanatowski. 2008. "Instability Conditions of Loose Sand in Plane Strain." *Journal of Geotechnical and Geoenvironmental Engineering*, 134 (1): 136–142.
- Dafalias, Y. F., and M. T. Manzari. 2004. "Simple Plasticity Sand Model Accounting for Fabric Change Effects." *Journal of Engineering Mechanics*, 130 (6): 622–634. American Society of Civil Engineers.
- Kato, S., K. Ishihara, and I. Towhata. 2001. "Undrained shear characteristics of saturated sand under anisotropic consolidation." *Soils and Foundations*, 41 (1): 1–11. The Japanese Geotechnical Society.
- Kramer, S. L. 1996. *Geotechnical earthquake engineering*. Pearson Education India.
- Lade, P. v. 1999. "Instability of granular materials." *Physics and mechanics of soil liquefaction*, 3–16. Rotterdam: Balkema.
- Li, X. S., and Y. F. Dafalias. 2012. "Anisotropic Critical State Theory: Role of Fabric." *Journal of Engineering Mechanics*, 138 (3): 263–275. [https://doi.org/10.1061/\(asce\)em.1943-7889.0000324](https://doi.org/10.1061/(asce)em.1943-7889.0000324).
- Najma, A., and M. Latifi. 2017. "Analytical definition of collapse surface in multiaxial space as a criterion for flow liquefaction occurrence." *Computers and Geotechnics*, 90: 120–132. Elsevier Ltd.
- Petalas, A. L., Y. F. Dafalias, and A. G. Papadimitriou. 2020. "SANISAND-F: Sand constitutive model with evolving fabric anisotropy." *International Journal of Solids and Structures*, 188–189: 12–31. Elsevier Ltd.
- Yang, J., L. B. Liang, and Y. Chen. 2021. "Instability and liquefaction flow slide of granular soils: the role of initial shear stress." *Acta Geotechnica*, 1–15. Springer.
- Yoshimine, M., K. Ishihara, and W. Vargas. 1998. "Effects of principal stress direction and intermediate principal stress on undrained shear behavior of sand." *Soils and Foundations*, 38 (3): 179–188. Elsevier.