Tractable Fragments of the Maximum Nash Welfare Problem

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We study the problem of maximizing Nash welfare (MNW) while allocating indivisible goods to asymmetric agents. The Nash welfare of an allocation is the weighted geometric mean of agents' utilities, and the allocation with maximum Nash welfare is known to satisfy several desirable fairness and efficiency properties. However, computing such an MNW allocation is NP-hard, even for two agents with identical, additive valuations. Hence, we aim to identify tractable classes that either admit a PTAS, an FPTAS, or an exact polynomial-time algorithm. To this end, we design a PTAS for finding an MNW allocation for the case of asymmetric agents with identical, additive valuations, thus generalizing a similar result for symmetric agents [2]. We also extend our PTAS to compute a nearly Nash-optimal allocation which also satisfies the best fairness guarantee offered by the optimal MNW allocation (a weighted relaxation of envy-freeness); showing we do not need to compromise fairness for tractability. Our techniques can also be adapted to give (i) a PTAS for the problem of computing the optimal p-mean welfare, and (ii) a polynomial time algorithm for computing an MNW allocation for identical agents with k-ary valuations when k is a constant, where every agent has at most k different values for the goods. Next, we consider the special case where every agent finds at most two goods valuable, and show that this class admits an efficient algorithm, even for general monotone valuations. In contrast, we note that when agents can value three or more goods, maximizing Nash welfare is NP-hard, even when agents are symmetric and have additive valuations, showing our algorithmic result is essentially tight. Finally, we show that for constantly many asymmetric agents with additive valuations, the MNW problem admits an FPTAS. The full version of the paper is available at [1].

Work supported by the NSF Grant CCF-1942321 and ERC Starting Grant ScaleOpt.

K. A. Hansen et al. (Eds.): WINE 2022, LNCS 13778, pp. 362–363, 2022.
https://doi.org/10.1007/978-3-031-22832-2

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