Towards An Operationalization of Mathematization

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Mathematization, the process of transforming a real-world situation into a mathematical model, is historically difficult for students, called horizontal mathematising by Freudenthal (2002). Literature on mathematizing typically frames the research problem as identifying some sort of "blockage" or that an action associated with mathematizing was "difficult" to perform (Brahmia, 2014; Galbraith & Stillman, 2006; Jankvist & Niss, 2020; Stillman & Brown, 2014). While these studies further our understanding of students' difficulties with the mathematizing step in modeling, there is more to be learned about how students' mathematical reasoning influences mathematizing a dynamic situation. Previous studies have indicated that quantitative reasoning promotes mathematization (Ellis, 2007; Ellis, Ozgur, Kulow, Williams, & Amidon, 2012; Mkhatshwa, 2020), and is a lens with which to understand students' reasoning while mathematizing during a modeling task (Carlson, Larsen, & Lesh, 2003; Czocher & Hardison, 2021; Larson, 2013). Given these arguments, it is appropriate to use a quantitative, covariational, and multivariational reasoning theories to describe students reasoning when mathematizing. The goal of this poster is to integrate and synthesize a definition of modeling (Lesh & Doerr, 2003), Thompson (2011)'s conception of quantities and quantitative reasoning, Carlson, Jacobs, Coe, Larsen, and Hsu (2002)'s conception of covariational reasoning, and Jones (2018)'s conception of multivariational reasoning to operationalize mathematization for future study of the mental processes students exhibit while performing a modeling task.

According to Lesh and Doerr (2003), a mathematical model is a conceptual system consisting of elements, the relationships between elements, operations, and rules of governing interactions. Because a mathematical model is a conceptual system, it is held, at least partially, internally and is expressed into the world through different representations. These different representations are dictated by a student's use of any external notation systems (Lesh & Doerr, 2003). The individual's mental representation of the real-world situation dictates the objects with attributes that the individual has the intention of measuring, and the rules governing interactions between objects. Through quantification, the act of conceptualizing the object with an attribute with a measure so that the measure has a proportional relationship with its unit, the objects with attributes are conceptualized as quantities. The quantities then define the elements of the model. The relationships between quantities can then be described by an individual's covariational reasoning, defined as "the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other" (Carlson et al. 2002 p 354), and multivariational reasoning, is the extension of covariational reasoning to two or more varying quantities (Jones, 2018). The operations on the quantities are determined by the schema of action employed by the students, where schema of action is defined as organized pattern of thoughts or behaviors (actions) that can be applied to different cognitive objects in different situations (Nunes & Bryant, 2021).

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