Title:

Neutrality, Ecofeminist Theory, and the Mathematical Analysis of Partisan Gerrymandering

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Abstract:

Mathematics is often positioned as either neutral or non-neutral by mathematicians. However, in practice, issues of neutrality arise in situated contexts, and the positioning of mathematics as either neutral or non-neutral is done for many purposes. We interpret positioning of mathematical work, with different degrees of neutrality, as a response to conflicts of interest and power dynamics. Using a framework from ecofeminist critical theory, we examine the ways that mathematical neutrality is positioned and communicated to different audiences in ways that can appear contradictory. Our goal is to demonstrate that the neutral/non-neutral dualism is insufficient for the analysis of neutrality in mathematics, which requires instead a robust analytical lens. We situate our discussion of these issues in the context of communication regarding the mathematical analysis of partisan gerrymandering in the United States. Through a study of communication to different audiences by mathematicians regarding mathematical techniques used to study partisan gerrymandering, we illustrate various ways in which dualistic views of neutrality are insufficient to describe and understand the complicated role of neutrality in this context.

Keywords:

Mathematics, Ecofeminism, Gerrymandering, Neutrality, Dualism

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1. The many aspects of neutrality in mathematics

The dualistic framing of mathematics as either strictly neutral or non-neutral is not representative of the complicated ways in which issues of neutrality arise in practice. One of the most common ways in which questions of neutrality arise is in regard to the utility of mathematics. A well-known example of this is G.H. Hardy's 1940 endorsement of the value of the uselessness of mathematics:

> *I have never done anything "useful." No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world. ...I have just one chance of escaping a verdict of complete triviality, that I may be judged to have created something worth creating. (Hardy 2012)*

In his *Apology*, Hardy actively endorsed the idea that his mathematical work had no impact on society, with an implied social neutrality. Hardy's assertion that his work had value both despite and because of this quality has had a lasting influence in mathematical culture, especially among practitioners of pure mathematics.

The dualism imposed on mathematical ideas as either having utility or not is closely related to the tensions between practitioners of "pure" and "applied" mathematics, a distinction that has recently been considered from a philosophical perspective (Pérez-Escobar & Sarikaya 2022). In practice, this distinction is typically made completely explicit in the mathematical community, and questions of value are often related to this distinction. For example, in the 2010 *International Review of Mathematical Science* from the UK Engineering and Physical Sciences Research Council (EPSRC), we find an acknowledgement that the categorification of different types of mathematical activity reflects and reinforces barriers and tensions among mathematicians:

> *Two of the most common distinctions are drawn between "pure" and "applied" mathematics, and between "mathematics" and "statistics." These and other categories can be useful to convey real differences in style, culture and methodology, but in the Panel's view, they have produced an increasingly negative effect… such distinctions can create unnecessary barriers and tensions within the mathematical sciences community by absorbing energy that might be expended more productively. (2010)*

As another example, in the United States (US) National Research Council Report *The Mathematical Sciences in 2025* (National Research Council 2013), a report funded by the US National Science Foundation and written by "a committee of mathematical scientists charged with examining the field now with an eye toward how it needs to evolve to produce the best value for the country by 2025" (National Research Council 2013), the authors wrote:

> *The committee members… believe that it is important to consider the mathematical sciences as a unified whole. Distinctions between "core" and "applied" mathematics increasingly appear artificial… It is true that some mathematical scientists primarily prove theorems, while others primarily create and solve models, and professional reward systems need to take that into account. But any given individual might move between these modes of research, and many areas of specialization can and do include both kinds of work. Overall, the array of mathematical sciences share a*

> *commonality of experience and thought processes, and there is a long history of insights from one area becoming useful in another.*
> *(National Research Council 2013)*

The assertion that pure mathematical work is disconnected from society, the opposite of applied mathematics work, implicitly asserts that pure mathematical work is neutral in its social impact. Mathematicians are frequently faced with the challenge of determining whose mathematical work is and is not important, and of determining the justifications for assignments of value. These justifications often involve debates about the importance and merit of the impact of mathematical work on society. The EPRSC and NRC reports demonstrate the intentional efforts by leaders in the mathematical community to mitigate this perceived conflict between pure and applied mathematics, to navigate conflicts of interest in determining what areas of mathematics should be provided with financial funding, and to resist the temptation of mathematicians to impose hierarchies of value on different domains of mathematics. When considering the tensions between pure and applied mathematical work, considerations of neutrality are never far beneath the surface.

In both the EPRSC and NRC reports, the tensions caused by the division between pure and applied math, and the attempt to generate perceptions of the unity of mathematics, are presented as new developments: note the use of the word “increasingly” in both quotes. In reality, these issues are not a new development, even in the narrow context of the history of mathematics in the US. As early as the 1920’s, when US mathematicians were first engaged in building a national funding network for mathematics research, leaders in the mathematical research community attempted to bridge the cultural divide between perceptions of the value of pure and applied work. Leaders in the American Mathematical Society (AMS) explicitly presented mathematics to the public and to potential funders as a unified discipline that would impact and advance civilization. Within the mathematical research community, the AMS launched the Gibbs Lectureship in 1924, with one of the major goals for this lecture series being to draw self-described pure mathematicians into greater contact with applications. The idea was that this would generate more evidence that mathematics should be funded as part of basic research in the sciences (Parshall 2022). Thus, in order to increase cohesiveness of public messaging and generate economic investment, leaders in the math community attempted to convince their own colleagues that mathematics was not neutral in social impact.

Vastly different examples of the role of neutrality in mathematics arise in mathematics education. For example, in 19th century Paris and Cambridge, mathematical education cultures developed that explicitly linked mathematical training and performance with physical strength and stamina, resulting in the exclusion of women from mathematical study (Barany 2021). In the late 19th and early 20th centuries in the US, mathematical culture was explicitly gendered as masculine and was closely intertwined with the concept of the self-made man (Abrams 2020). These cultural positions certainly did not reflect a conception of mathematics as a neutral discipline that was accessible to anyone. Rather, mathematics was positioned socially as having specific qualities and playing particular roles in social training for young men.

These examples illustrate some of the ways that different flavors of neutrality arise in diverse mathematical contexts. Three aspects of these examples are worthy of special scrutiny, as they underline themes of this article. First, the root of disputes about the nature and role of mathematics are quintessentially human conflicts of interest. The development and stewardship of human mathematical knowledge is subject to the whims and influences of human differences of opinion and emphasis. Second, questions of neutrality are not strictly either/or. There are degrees and dimensions to neutrality in mathematics, requiring a consideration of context in any study of this topic. Third, positions taken regarding neutrality are frequently embedded in discourse and communication regarding the nature, function, and purpose of mathematics, even if they are claimed as abstract principles. Thus, messaging regarding neutrality and mathematics is impacted by the intended audience and the purpose of such communication.

2. An ecofeminist approach to dualisms and conflicts of interest

As illustrated by our examples above, many narratives exist that assert what mathematics is and is not, what it should and should not be used for, who is or is not qualified to practice mathematics, or that take one side or another in similar dualisms. These narratives frequently arise in response to human conflicts involving beliefs, power, privilege, and authority. Thus, when considering disagreements regarding whether mathematics is or is not neutral, or to what degree such neutrality exists or applies, we should carefully consider our framework for interpreting and understanding conflicts of interest and dualisms.

In this article, I use a framework for understanding such conflicts inspired by ecofeminist philosophy. Ecofeminism is focused on the intersections among ecological health and destruction, women's rights and liberation, animal rights and liberation, disability rights and liberation, and closely related issues such as militarism, racism, colonialism, capitalism, and patriarchy (Adams 2021; Kheel 2008; Taylor 2017). One of the actions of ecofeminist theorists is to subvert and critique the dualistic structures prevalent in the Western philosophical tradition, e.g., masculine/feminine, nature/man, and emotion/reason. These dualities typically reflect and reinforce, either explicitly or implicitly, embedded hierarchical structures and exploitative power relationships, and they hinder moral reflection by providing an oversimplified perspective on complex issues. In ecofeminist ethics, there is also a strong focus on care and compassion, alongside rights and justice, as foundations for ethical deliberation and theorizing.

Adapted directly from Chapter 5 of Karen Emmerman's (2012) ecofeminist approach to analyzing inter-animal conflicts of interest, the approach taken in this work to conflicting interests regarding the neutrality of mathematics is non-hierarchical, pluralist about moral significance, and contextualized. Emmerman states that this approach "[moves] between these relevant features of the conflict to obtain as full a picture as possible of what is at stake for all parties" (2012 pp 168). In the context of mathematics and neutrality, this approach is non-hierarchical in that it does not assume that any one assertion regarding the neutrality or non-neutrality of mathematics is inherently privileged, and further that there is a full spectrum of possibilities that includes these two opposing positions. As Emmerman describes, "the approach is pluralist in that it recognizes that moral significance arises from a variety of sources" (2012 pp 169); while the moral dimension of assertions regarding neutrality in mathematics might not be immediately obvious, as we will see below these are indeed present and complex. Finally, this approach is contextualized in that we refuse to engage with discussions regarding neutrality and mathematics considered in an abstract, platonic setting. Rather, we insist that any engagement with these ideas involve a rich, multifaceted consideration of both general principles and situated contexts.

There are many connections and overlapping themes between ecofeminist critical frameworks and existing conceptions of mathematical practice and epistemology, for example Leone Burton's feminist epistemology of mathematics (1995), Suzanne Damarin's reflections on unifying feminism and mathematics (2008), Eugenie Hunsicker and Colin Jakob Rittberg's thick epistemologies of mathematics (2022), Val Plumwood's vision of feminist logic (1993), and Rochelle Gutierrez's frameworks for Rehumanizing Mathematics (2018) and Living Mathematx (2017). While a full exploration of these connections is beyond the scope of this article, I believe it is important to recognize that the ecofeminist-inspired frameworks used in this work are not completely new ideas in the philosophy of mathematics. It should also be remarked that a variety of misunderstandings and misconceptions regarding ecofeminist philosophy exist. This stems from a combination of both inaccurate "folklore" descriptions of ecofeminism and valid critiques of some ecofeminist works written in the 1970's and 1980's that endorsed essentialist ideas or philosophies. These essentialist ideas were present in only some of the ecofeminist works at that time, and contemporary ecofeminist theorists have addressed these criticisms (Gaard 2011).

3. A mathematical view of partisan gerrymandering

The situated context that will be considered in this work is that of the mathematical analysis of partisan gerrymandering in the United States. Gerrymandering is the term used to describe the process of creating the boundaries of a *districting plan*, i.e., a collection of voting districts within a particular state or region, with the intention of favoring some interest group or outcome. Partisan gerrymandering occurs when gerrymandering is conducted with the goal of favoring a specific political party. In this article, we will focus on partisan gerrymandering in the US (Duchin and Walch 2022).

It is important to recognize from the outset that gerrymandering is not inherently good or bad for society. For example, gerrymandering has been used to favor political parties who face opposition by a majority of voters and it has been used to provide political power to marginalized groups of voters (by concentrating votes within a district); both of these actions are viewed as virtuous by some but not others. Some actions that qualify as gerrymandering are required by law, while other types of gerrymandering are independent of legal mandates. Thus, gerrymandering can be used to concentrate or disperse power, for reasons that are dictated by or independent of legal requirements, and the ethical implications of gerrymandering are highly contextualized.

The topic of gerrymandering is situated within the broader topic of voting rights and access to participation in civic society, and thus hierarchical issues abound. For example, in the US, voting and civic participation has been granted and/or denied based on people's gender, ethnicity, race, ability or disability, incarceration record, immigration record, and more. Further, deliberations regarding voting rights and civic participation require pluralistic approaches to moral significance. Moral considerations regarding voting and participating in democratic governance have many sources, and there is not universal agreement in the US, either historically or today, regarding the moral or ethical foundations of voting and civic participation. Finally, voting and other civic issues must be considered outside of purely abstract settings. Every society, and certainly the US, has a unique and complex history of governance, determination of citizenship, and rules regarding voter (and general civic) participation. These contextualized considerations must be brought face-to-face with abstract principles and theories, not omitted.

Mathematics has been used to study gerrymandering in many ways. Mathematical models have been developed to measure "how fair" a particular districting plan is in the context of a specific election. These are often presented as a single numerical measure, and this quality has led to various criticisms and shortcomings. For example, Greg Warrington has conducted extensive experimental studies using many of these single numerical measures and found that almost all of them fail to meet certain desired criteria for measuring fairness and/or representation of the public will (Warrington 2019). An alternative approach, which has been developed and applied extensively over the past decade, is to analyze and detect partisan gerrymandering using techniques from Markov Chain Monte Carlo (MCMC) sampling. These MCMC techniques will be the subject of this work. In order to understand how these techniques impact questions of neutrality, it is important to understand some of the mathematical qualities underlying this approach.

The idea is this: suppose we have a known geographic distribution of votes in an election for a specific region (state, county, etc). Rather than measure fairness or representation of public will in this election for a single districting plan ("Plan A") considered in isolation, one instead considers the results of Plan A in comparison to all other possible districting plans. The goal is to determine whether the outcome of the election under districting Plan A is typical or an outlier among all possible results for that vote distribution using different districting plans. Where the mathematics becomes complicated is that it is completely infeasible to determine all possible districting plans, say for a given state in the US, because there are so many possible districting plans. Further, because of varying (sometimes vague) legal requirements for the structure of districting plans, it is necessary to consider only those plans that can be reasonably argued to satisfy the legal requirements. Even with these legal restrictions reducing the overall number of possible plans, there is a computationally infeasible number of valid plans to consider in any real-world scenario.

In situations where one wants to consider a universe of possibilities that is finite but too large to be determined, for example our unimaginably large number of legally allowed districting plans, mathematicians and scientists often apply sampling methods. In these methods, one generates a sample, or *ensemble*, of districting plans that is sufficiently random to be representative of the entire universe of allowed plans. A standard technique for generating such ensembles is MCMC sampling (Diaconis 2009, Duchin and Walch 2022). There is not a single MCMC algorithm to generate ensembles, but rather many different MCMC-based methods to randomly sample from legally valid districting plans. For relatively simple sampling problems, some MCMC algorithms can be used to generate an ensemble that is provably representative of the universe of possibilities. However, the universe of districting plans for any state in the US is too complex for current methods to admit such a precise analysis. Instead, various heuristic arguments are made justifying why the ensemble resulting from a particular MCMC algorithm is likely to be a representative sample, but without any provable guarantees.

Thus, we are led to one of the roots of the complexity regarding neutrality in the application of MCMC methods to partisan gerrymandering: the qualities of the ensembles that we observe are probably representative of the whole universe of plans, but not provably so. Thus, mathematicians and other practitioners of these methods are faced with two conflicting tasks if these methods are to be adopted and used. They must convince the public that these methods are reliable and unbiased, with a high degree of certainty that the ensembles represent the universe of districting plans. They also must convince experts in this type of sampling that they have developed tools that "correctly" bias the methods to generate an appropriate ensemble, making clear the limitations and deficiencies of the methods. This is a complicated tension, in which mathematicians must make seemingly conflicting arguments to different audiences.

Another root of complexity is that these techniques can be used both to test existing districting plans for evidence of partisan gerrymandering and to create gerrymandered districting plans. While the developers of these MCMC methods, as we will see below, are insistent that these methods should not be used to create districting plans, there is nothing prohibiting others from using these methods for that purpose. Thus, while many developers of these MCMC methods seek to detect and deter certain types of partisan gerrymandering, the resulting algorithms and software are freely available and can be used for other purposes by other individuals.

A third root of complexity is that this is an application of mathematics to politics and civic life, an area of human social life that is infused with power struggles, conflicts of interest, and clashing beliefs and values. Thus, these applications of mathematics are undertaken with the explicit goal of impacting society and public life. This stands in contrast to the common, though unrealistic and problematic, perception that applications of mathematics in engineering or physics are neutral applications in "pure" or deterministic science. In the political and civic setting, as we will see, mathematical methods are often positioned as a neutral arbiter of districting plans, but with the goal of having a non-neutral impact on public policy. Further, because of the necessary biases that are built in to the MCMC ensemble-generation algorithms, the methods are not strictly contained within the mathematics that is formally provable within a specified logical system. Claims of neutrality in mathematics often rely on the assertion that mathematics occurs within such a specified logical system, even though many mathematical actions and activities do not.

In the next two sections, I will elaborate on these observations by considering two contexts in which mathematicians communicate with others regarding these MCMC methods. The first of these contexts is communication directed outside the community of researchers in the mathematical sciences. Specifically, I will consider an amicus curiae brief filed with the US Supreme Court in the 2019 case Rucho v Common Cause. The second context is communication amongst mathematical science researchers. Here, I will consider the messaging in a published journal article regarding the "Recombination" algorithm for MCMC ensemble generation and a videorecording of a research colloquium given by Jonathan Mattingly at North Carolina State University.

4. Communication outside the community of mathematical sciences researchers

A US federal district court struck down the 2016 congressional map for the US state of North Carolina. This decision was appealed in 2018 by North Carolina Republicans to the Supreme Court of the United States, referred to as SCOTUS, resulting in the 2019 SCOTUS case Rucho v. Common Cause (Rucho v. Common Cause n.d.). A group of mathematicians, statisticians, and lawyers filed an amicus curiae, or "friend of the court," brief (Mathematicians 2019) in which they supported the argument that the North Carolina congressional map was unconstitutional. All quotes in this section are taken from this brief.

The main argument in the amicus brief is that partisan gerrymandering can be detected using MCMC ensemble-based analysis, which identified the struck district map as an outlier for ensembles generated using two different MCMC algorithms. One of these algorithms is the result of work by members of the Duke Quantifying Gerrymandering Group, which originated in an undergraduate research project supervised by Jonathan Mattingly. The other algorithm is the result of work by members of the Metric Geometry and Gerrymandering Group at Tufts University, founded by Moon Duchin. Jonathan Mattingly testified as a plaintiff's expert in the US district court case that led to the 2016 congressional map being struck, explaining that MCMC ensembles identified the 2016 map as a partisan gerrymander. Moon

Duchin is one of the authors of the amicus brief, along with another half-dozen mathematicians. Thus, we see that the mathematicians involved in developing these research tools and engaging in the scientific research process are and were directly involved in communicating the results of this research in legal settings and to the public.

The amicus brief consists of two parts. Part I is an argument that the federal courts can legally rule on issues related to partisan vote dilution, i.e., gerrymandering. Part II is an argument that MCMC methods are a "reliable and well-established computational method" (Mathematicians 2019) to evaluate claims regarding partisan gerrymandering. Thus, the goal in Part II of this brief is to convince the supreme court justices, other legal scholars, and the public that these mathematical methods are valid and trustworthy. The opening sentences of Part II are revealing:

> *In this section, we describe a powerful method to evaluate the districts in contested plans, setting a high bar to distinguish extreme outliers from those within the range of reasonable outcomes for that state. Unconstitutional vote dilution can be proved by showing that the manner in which the government drew the lines departed from a baseline of equal treatment by diminishing the weight, power, and value of an individual's vote. The district court in the North Carolina case framed matters similarly, observing that "there needs to be a baseline from which to measure to what degree a districting plan drawn on the basis of partisan favoritism deviates from the universe of 'fair and effective' plans."*

Note the authors' reference to the "powerful method" that "sets a high bar", allowing unconstitutional vote dilution to be "proved". The words "powerful" and "high bar" both implicitly invoke the privileged status of mathematics in society. The use of the word "proved" in a colloquial, but not mathematical, sense further invokes the social authority and certainty of mathematics. Even in the context of mathematical research, as we will discuss in the next section, the words "proof" or "proved" can be invoked in ambiguous but suggestive ways. In their quote from the district court ruling, the brief authors include the word "fair," which again invokes the common perception of mathematics as a neutral tool for decision-making. The authors of the brief go on to emphasize twice that a method is needed to generate districting plans that are neutral:

> *…we must have a reliable method to distinguish a normal, neutral, or non-gerrymandered district from an intentionally abusive, gerrymandered, dilutive district… We must therefore create a benchmark understanding of neutral districting plans in a state-specific setting.*
>
> *It is worth emphasizing that the technique we describe here is a method -- not a new score of partisan skew. The method of ensembles does not produce a number or score. Instead, it generates a neutral baseline that can be used to interpret scores for a challenged district plan.*

After implicitly invoking the need for neutral, fair, and powerful tools, the authors then present MCMC methods as the tool required for this situation:

> *For many decades, scientists, mathematicians, technologists, and government officials have used a technique known as Markov chain Monte Carlo ("MCMC") for prediction, modeling, and analysis of large data sets… MCMC permits us to carry out a comparative analysis of districting plans by generating a large and diverse sample of districting plans… Scientific consensus in the*

> *mathematics and statistic community increasingly endorses this approach to the problem of discriminatory redistricting.*

Note that the final sentence in the quote above invokes the authority of mathematical and statistical expertise as a justification for public trust in the methods being described. The amicus brief authors go on to assert that these methods can be used in a way that accurately represents the legal and geographical situation in any given state:

> *MCMC permits us to carry out a comparative analysis of districting plans by generating a large and diverse sample of districting plans… The search can be restricted to plans that comply with a given state's districting laws, and hold constant the state's geography and voting patterns.*

Despite this strong declaration that MCMC techniques can be trusted to produce neutral arbiters of partisan gerrymandering in districting plans, the authors then explicitly state that these methods should not be used to produce districting plans. Further, they state that despite the power of these methods, that power should not usurp the authority of the state:

> *We emphasize that the use of the method of ensembles for districting is proposed as an assessment technique, not proposed for optimization or map selection. This will never amount to usurping the state's authority to select a plan, because billions of substantially different plans remain viable, under any conception of outlier. This method does not choose a winner from among the abundance of options. This balances between state prerogatives and constitutional principles.*

What do we make of these invocations, both implicit and explicit, to neutrality, power, and expertise in the service of public policy? These arguments involve many responses to conflicts of interest, such as assertions that MCMC techniques are inherently trustworthy, assertions that votes should be non-diluted, assertions that it is appropriate to balance between state prerogatives and constitutional principles, and the stated goal of the authors of the brief to uphold the decision to strike the 2016 congressional map. To provide a more robust answer to this question, and a more insightful analysis of these conflicts of interest using Emmerman's ecofeminist framework, we must consider how messages regarding neutrality, power, expertise, certainty, and fairness in this setting are communicated to other audiences.

5. Communication within the community of mathematical sciences researchers

The MCMC algorithm for generating district plan ensembles developed by Daryl DeFord, Moon Duchin, and Justin Solomon is called the *Recombination* algorithm, or ReCom algorithm (DeFord, Duchin, and Solomon 2021). All quotes in the first part of this section are taken from an article by DeFord, Duchin, and Soloman (2021) introducing this algorithm, which I will refer to as the ReCom article. It is particularly interesting to consider the discussion of MCMC methods in this article, as all three of these authors are also authors of the 2019 amicus brief.

One quality that immediately stands out in the ReCom paper is the position taken regarding the certainty of MCMC methods for generating ensembles of districting plans, which appears to contradict the position taken in the amicus brief. For example, in the ReCom article, the authors clearly state:

Securing operational versions of rules and priorities governing the redistricting process requires a sequence of modeling decisions, with major consequences for the properties of the ensemble. Constitutional and statutory provisions governing redistricting are never precise enough to admit a single unambiguous mathematical interpretation.

The above statement immediately undercuts the certainty of the application of mathematical tools in this area. Note that this is different than undercutting the appropriateness or reasonableness of the application, instead undercutting the certainty of the results. What the authors are clarifying is that there is an important and noteworthy degree of ambiguity in these methods. This is in stark contrast to the amicus brief, in which there is no mention of ambiguity of these methods, rather repeated assertions of the certainty and reliability of these methods.

Another quality of mathematics that is typically associated with neutrality is the concept of mathematics as a proof-based discipline, where primitive axioms lead through deductive logic to “unassailable” truths. While some methods of mathematical proof rely more heavily on strict logical deduction from primitive axioms, for example automated theorem proving, in general mathematics is a socially and psychologically richer and complicated undertaking. This is robustly conceptualized by David Tall in his Three Worlds framework (Tall 2013) for mathematical thinking and learning. Further, even within the proof-driven formalist world of mathematics, the purpose of and standards for proof are more complicated than is commonly understood by many professional mathematicians (Tall et al. 2012). Despite this, the idea that mathematics provides ironclad proofs of true statements, separate from physical or social realities, is a widespread cultural belief among mathematicians.

The MCMC algorithms used to generate districting plan ensembles pose a challenge to this cultural belief, in that the outcomes that they generate are beyond our ability to analyze through formal mathematical proof. For some MCMC algorithms, it is possible to give a formal proof that sampling to a sufficient level will give a representative sample, which approximates the “steady state” distribution of the universe under consideration. And for a subset of these algorithms, it is possible to determine the value of the sufficient level of sampling required to realize this steady state distribution. However, the ReCom article authors write:

The number of steps that it takes to pass a threshold of closeness to the steady state is called the mixing time; in applications, it is extremely rare to be able to rigorously prove a bound on mixing time; instead, scientific authors often appeal to a suite of heuristic convergence tests and diagnostics, as we do here.

Thus, the authors state clearly that this mathematical work is based on, but outside of, the formalist world which conceives of mathematics as deductive proof within an isolated logical system. The authors go on to state that because of some special properties of their MCMC algorithms, there are additional mathematical results that support their use; however, as the following two quotes make clear, this is still not sufficient to guarantee that these techniques have the level of certainty that is often associated with a formal proof of a theorem:

…samples from reversible Markov chains admit conclusions about their likelihood of having been drawn from a stationary distribution π long before the sampling distribution approaches π. For redistricting, this theory enables what we might call local *search: While only sampling a relatively small neighborhood, we can draw conclusions about whether a plan has properties*

that are typical of random draws from π. Importantly, these techniques can circumvent the mixing and convergence issues, but they must still contend with issues of distributional design and sensitivity to user choice.

…the interactions between various choices of constraints and priorities are so far vastly underexplored.

Thus, we find that in the ReCom article, intended for research mathematicians engaged in MCMC modeling, the messages communicated regarding the MCMC ensemble-generation techniques are different in important ways from those found in the amicus brief. This is mirrored in the messaging from Jonathan Mattingly in a talk about his work on flip-based MCMC algorithms in gerrymandering analysis, from November 2018 at North Carolina State University. All quotes in the remainder of this section are taken from a YouTube video of Jonathan Mattingly's talk posted by the NCSU Mathematics Department (NCSU Mathematics 2018).

Mattingly begins by clearly stating that the MCMC ensemble methods are not intended to determine whether a map is fair:

When is a map fair?... Can we measure when something would have happened without any partisan bias, and how might we go about doing that?... What if we drew the districts randomly, without any political information? Without any explicit bias, in some ways? And then created a collection, an ensemble, of maps and used that as a normative standard? So, then we could replace "fair" with "expected behavior" of the collection. That's the little linguistic sleight of hand. I don't really understand what fair is, sometimes people think fair is equal, that doesn't seem fair. (NCSU Mathematics 10:00)

Note that here, speaking to an audience of mathematical scientists, Mattingly adds the caveat "in some ways" to the assertion that these are unbiased methods. Also observe that he reframes the mathematical modeling problem to avoid the issue of defining or evaluating fairness, even referring to this act as a "sleight of hand." In this way, he is clarifying to the mathematicians in the audience that there are choices being made in the mathematical model used to study partisan gerrymandering. This exactly reflects the sentiment expressed above in the ReCom article that the legal landscape of partisan gerrymandering has unavoidable ambiguity that is inherently imprecise, thus disallowing an unambiguous mathematical interpretation.

Mattingly also clearly conveys that there are concrete human choices being made that impact these models. For example, in describing the flip-based MCMC algorithm developed by his team, which is independent of and different from the ReCom algorithm, Mattingly states:

…you do things like this, standard tricks, in the dirt applied mathematical engineering where you just want to build a function that does what you want. (NCSU Mathematics 33:00)

Mattingly also makes clear that these mathematical analyses are not subject to the burden of mathematical proof, in alignment with the authors of the ReCom article. Mattingly justifies the correctness of the MCMC ensemble-based analysis through empirical methods, arguing that given reasonable attempts to falsify the results, the outcomes are robust:

We do a whole bunch of tests, we change all the parameters up and down 20%, we start from all different initial conditions, we get the same answer. We run, instead of making 24,000 maps we make 120,000 maps, and see if the answer changes -- doesn't change. So we do a whole bunch of kinda classic tests like this, and they don't seem to matter. We change our population threshold… none of these things seem to matter. (NCSU Mathematics, 35:00)

One way in which Mattingly is clearly aligned with the amicus brief authors is in his belief that these MCMC techniques should be used for analysis of districting plans, rather than for generating them:

I'm not advocating using an algorithm to draw redistrictings. It's not like we have some robot redistricter 2000 that spits out [inaudible] that tells you how to redistrict like a Dr. Who episode. (NCSU Mathematics 16:42)

Overall, in both the ReCom article and Mattingly's talk, we see that the messaging regarding MCMC methods within the community of mathematical sciences practitioners differs from the messaging in the amicus brief in clear and important ways.

6. Implications and Conclusion

One can attempt to define or conceptualize neutrality in mathematics in multiple ways, for example with neutrality being independent of human activity, or free from impact on human society, or free from the potential of human bias, etc. Regardless of how one decides to conceptualize neutrality, viewing neutrality in mathematics through a dualistic framework is insufficient.

What is the motivation behind the desire to determine in the abstract whether mathematics is neutral or not? Often, this motivation reflects a desire to arbitrate conflicts of interest among individuals or social groups, where the means of arbitration is sufficiently separate from interests or biases regarding the conflict at hand. If there were an abstract justification for mathematics as a neutral subject, this would provide a tool for resolving conflicts of interest in abstractly fair or unbiased ways. As the examples discussed throughout this article demonstrate, the practice of mathematics involves many fundamentally human choices regarding definitions of axiomatic systems, constructions of algorithms, and culturally acceptable standards for justification or proof, among others. These choices are inherent to the human practice of mathematical thought and activity, and they are not restricted to an abstract world.

Even if one accepts these choices as part of a mathematical practice that is deemed to be "sufficiently neutral" in scope, we are still left with the challenge of recognizing that these practices are carried out by humans, and thus subject to human whims, flaws, and unintended errors. Further, the desires and motivations underlying our mathematical actions fall along a varied spectrum of neutrality and bias, whether these actions are taken with the goal of impacting society or in the pursuit of abstract knowledge. These desires and motivations will necessarily differ and shift from person to person, and from group to group.

For example, my interpretation of the communication by the amicus brief and ReCom article authors, and by Jonathan Mattingly in his talk, is that these researchers are driven by an authentic desire to deeply understand the process and context regarding election systems in the US. Further, these researchers are communicating to different audiences in ways that are considered acceptable practice within the mathematical sciences community. The process of scientific and mathematical research is messy and nonlinear, and this process is successful when it results in reliable techniques whose justification meets the contemporary standards of the research community. Thus, within the standards of practice for this community, it is reasonable to tell one audience in the amicus brief that MCMC algorithms are powerful and neutral while telling another audience in the ReCom article and colloquium talk that these techniques are inherently imprecise and unavoidably biased to some degree. From this perspective, these researchers are working within well-understood community standards to develop tools that are as reliable and neutral as possible.

However, these MCMC techniques have been developed to address real and urgent conflicts of interest in society. Emmerman's ecofeminist framework forces us to avoid privileging the perspective of the researchers in a hierarchy of expertise, and consider as well the perspective of other stakeholders in this process. From the perspective of a non-expert who is genuinely interested in understanding the benefits and limitations of these MCMC methods, the communication in the amicus brief omits important information about the scientific process of developing these algorithms, about the standards for justification in the research community, and about the inherent uncertainties and biases in these algorithms. From the non-expert perspective, this omission of detail, even if well-intentioned, is not a neutral act. Also, the decisions that have been made by scientific researchers, even if made in an attempt to remain unbiased, are not neutral acts. Having said that, it is also not appropriate to claim that the scientific process and communication by the MCMC technique researchers is intentionally non-neutral. Instead, something more complicated is taking place in these processes and practices of mathematics.

Thinking about Emmerman's requirement that we be pluralistic about moral significance, we must consider the moral significance of these actions from multiple perspectives. From the perspective of the research teams, a good-faith effort is being made to inform the public using tools that are as unbiased and neutral as possible. One can certainly argue that this is a moral good. By developing these tools, one is also making available to everyone software and algorithms that can be used to create sophisticated gerrymanders, for reasons both well- and ill-intentioned with regard to the public good. From the perspective of an advocate for more transparent creation and use of districting plans, the level of communication in the amicus brief can be considered morally just: it accurately represents the consensus of the authors and their research teams, despite its' omissions. From the perspective of a citizen who is relying on the amicus brief for a completely honest and transparent record of both benefits and limitations of these methods, these omissions might not be morally justified. The varying degrees of neutrality in mathematics open the door to complicated considerations of moral significance, and it is important to recognize this complication.

The purpose of this article is not to reach a verdict on the moral actions of these mathematical scientists or their work on MCMC techniques. Also, my goal is not to draw a conclusion that their mathematical work, or their communication about that work, is either neutral or non-neutral. Rather, my goal is to use this context to illustrate some of the ways that a neutral/non-neutral dualism is insufficient for gaining insight in situations where issues of neutrality emerge in mathematics. Given the many complexities of this context, I believe that one cannot designate either the MCMC methods to analyze partisan gerrymandering or the communication about them as strictly neutral or non-neutral. A more nuanced approach is required, and this is true for general considerations of neutrality in mathematics.

To be clear, I am not issuing a call for moral relativism regarding neutrality and mathematics. Rather, I am arguing we must recognize that there are dimensions to the issue of neutrality in mathematics, and that these dimensions involve both abstract considerations and situated contexts. Further, because issues of neutrality in mathematics typically arise in the context of human conflicts of interest, I believe it is important to consider such issues from a non-hierarchical perspective, recognizing that moral virtue arises in different ways for different people. Any serious consideration of neutrality in mathematics must involve careful consideration of many factors, and the possible conclusions of such consideration must include more than simple dualistic statements.

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