Cooperative Driving between Autonomous Vehicles and Human-driven Vehicles Considering Stochastic Human Input and System Delay

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Abstract— **We investigate the coordination of autonomous vehicles (AVs) and intelligent human vehicles (IHVs) for merging on a two-lane road. An IHV is equipped with an automated system with advisory directives to the human driver to optimize its maneuver while communicating and collaborating with other vehicles. For optimal coordination of the two vehicles, modeling and incorporating stochasticity of the human driver's actions in the IHV is important. We introduce a method of cooperative driving that considers multiple stochastic human parameters in the IHV, such as human intent and human input transitions. We also model the system to account for computational delays and the driver's ability to follow advisory directives. The coordination actions for the AV and the IHV are generated in a stochastic model predictive control (sMPC) framework. Using simulated results, we demonstrate that the model considering stochastic effects of the human driver's actions performs better and can mitigate the effect due to the driver's inattentiveness while merging.**

I. INTRODUCTION

Autonomous vehicles (AVs) are entering our transportation system at a fast pace. In the United States, there are expected to be more than 2.1 million AVs operating by 2025 and 20.8 million by 2030 [1]. According to [2], more than 33 million AVs will be sold globally in the year 2040, with 7.4 million in the United States, 14.5 million in China, 5.5 million in European markets, and 6.3 million in other global markets. Although the number of AVs is expected to rise, AVs and human-driven cars are anticipated to coexist throughout the course of the following several decades. Cooperative driving between human-driven vehicles and AVs has great potential to increase transportation efficiency and safety in the near future. Modeling human behaviors and including stochastic factors of human actions are critical for effective vehicle coordination in a mixed-traffic scenario.

Intelligent vehicles and their interaction with human drivers have been a topic of interest to researchers over the past decade. The authors in [3] use a chance-constrained partially observable Markov decision process to generate risk-bounded motion policies of AVs considering uncertainty in the system from human intervention or road conditions. In that paper, human interaction with intelligent vehicles is modeled as a disturbance. In [4], the authors model the interaction between the driver and the vehicle in an assistive driving system using hidden mode stochastic hybrid systems. In the reference [5], authors employ partially observable Markov decision processes (POMDPs) as a unified framework for modeling the human behavior, the machine dynamics and the observation model in a human-in-the-loop control of semi-autonomous vehicles.

There have been studies concerning cooperative driving between connected vehicles. The authors in [6], formulate lane-changing decision-making of AVs in a mixed-traffic highway scenario using multi-agent reinforcement learning. In [7], connected AVs are controlled such that they are able to react properly to uncertain maneuvers of human-driven vehicles. Using the Discrete Hybrid Stochastic Automata (DHSA) [8] the authors in [9] develop a safe and efficient traffic system for connected vehicle platooning. None of these papers considers the scenario where the human driver can be influenced through advisory commands to coordinate with other AVs in a cooperative driving environment.

In this paper, we formulate cooperative driving between an IHV and an AV in a cooperative driving environment. Here an IHV is a human-driven vehicle equipped with an intelligent copilot that communicates with other AVs and IHVs and provides driving advice through voice. The objective is to optimize both the control inputs for the AV and the advisory commands for the driver in the IHV and coordinate their actions for lane merging. In [10], [11], we formulated optimized cooperative driving between the IHV and the AV without considering any uncertainties in human input or human intent. And in [12], we formulated and experimented with stochasticity in human state estimation. We improve the formulation in [10], [11], [12] and introduce a comprehensive system model to address various uncertainties that can arise in a driving scenario, including uncertainty in the driver's intent and stochastic transitions in the driver's actions. We also incorporate a system delay and a reaction constant into the formulation. The system delay addresses the lag in real-time operations, and the reaction constant models the human's capacity to follow advisory commands. We provide the simulation results of the method introduced and compare these results with the simulation results of a system without considering stochastic human input. With a Monte Carlo simulation, we show that by considering stochasticity in human actions, the two vehicles can achieve a shorter merge time while satisfying all safety constraints, when compared to the formulation without such stochasticity consideration.

The rest of this paper is organized as follows. The system modeling with incorporated delay is explained in Section II. The human behavior modeling approach used for the formu-

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lation is presented in Section III. In Section IV we present the system constraints for the dynamic modeling. The sMPC formulation is presented in Section V. An analysis of simulation results using our optimization approach and a comparative discussion are presented in Section VI. Conclusions and future work are given in Section VII.

II. SYSTEM MODELLING WITH DELAY INCORPORATION

We consider the lane merging coordination between IHVs and AVs. To enable a safe and effective merge, it is important to create the longitudinal gap between the two vehicles as quickly as possible. While the motion of an AV can be directly controlled, only the driver can have an impact on the motion of an IHV. The driver's action can be influenced through advisory directives. To coordinate the motion of the two vehicles, we propose a stochastic MPC (sMPC) problem with state and control constraints, whose solution provides the optimal advisory commands to the IHV and the optimal autonomous controls to the AV. The merging scenarios considered in this paper are illustrated in Figure 1.

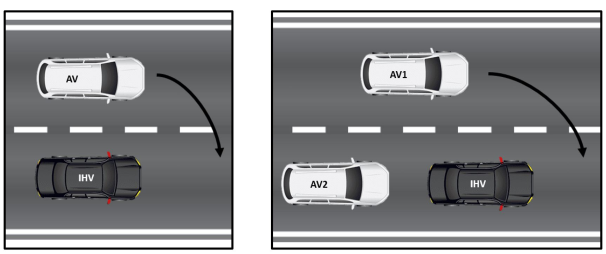

Fig. 1. Two vehicle merging scenarios.

In this section, we discuss the models of vehicle dynamics and the driver's actions. We consider the following linear dynamic model of an AV

$$x^r_{k+1} = A_r x^r_k + B_r u^r_k, \tag{1}$$

where the subscript $k \in \mathbb{Z}_+$ is the discrete-time index, the longitudinal position and velocity with respect to the origin are represented by $x^r_k \in \mathbb{R}^2$, A_r and B_r are matrices of suitable dimensions that define the AV dynamics, and $u^r \in \mathbb{R}$ is the input (acceleration) to the AV.

Whether the driver obeys the recommended commands determines the IHV behavior. As a result, depending on a binary decision variable $x^B_k \in \{0,1\}$, the dynamics of the IHV alternates between two dynamic systems: (1) following the advisory command when the IHV is under advisory control ($x^B_k = 1$), and (2) not following the advisory command, which denotes that the IHV is under human control ($x^B_k = 0$). We consider that the IHV's models are provided by

$$\text{IHV under human control:} \quad x^h_{k+1} = A_h x^h_k + B_h u^h_k, \tag{2}$$

$$s^a_{k+1} = u^h_k, \tag{3}$$

$$\text{IHV under advisory control:} \quad x^h_{k+1} = A_h x^h_k + B_h s^a_k, \tag{4}$$

$$s^a_{k+1} = \lambda s^a_k + (1-\lambda) u^a_k, \tag{5}$$

where A_h and B_h are matrices of suitable dimensions that define the IHV dynamics, and $u^h_k \in \mathbb{R}$ is the human input and $u^a_k \in \mathbb{R}$ is the advisory commands for the IHV.

Under advisory control, there is a delay as the human driver tries to follow an advisory command. As a result, the advisory command is implemented on the vehicle by the human gradually rather than instantaneously. In addition, the computation of the optimal commands after observing the states of the vehicles and the announcement of the advisory commands may cause further delays.

We use a first-order system (5) to account for such delay effects. The state s^a_k holds the input applied from the previous step to account for that delay. The reaction constant $\lambda \in [0,1)$ represents how fast the human in the IHV is adapting to the advisory action u^a_k after it is announced. If $\lambda = 0$, the driver applies the advised control command u^a_k exactly at the $(k+1)^{th}$ step. That is, the computation, the announcement, and the driver's tracking of the advisory commands are assumed to be completed within one step. As $\lambda \to 1$, the driver's response to the advisory action u^a_k is further slowed down. Note that (5) can be replaced by other human actuation dynamics, such as the second order dynamics in [13], [14].

Let $\bar{x}^h_k = [x^h_k; s^a_k]$. We rewrite the IHV dynamics as

$$\bar{x}^h_{k+1} = \begin{pmatrix} A_h & 0 \\ 0 & 0 \end{pmatrix} \bar{x}^h_k + \begin{pmatrix} B_h & 0 & B_h \\ 1 & 1-\lambda & \lambda \end{pmatrix} \begin{pmatrix} u^h_k(1-x^B_k) \\ x^B_k u^a_k \\ x^B_k s^a_k \end{pmatrix}. \tag{6}$$

Defining $\bar{z}^1_k = u^h_k(1-x^B_k)$, $\bar{z}^2_k = x^B_k u^a_k$, $\bar{z}^3_k = x^B_k s^a_k$, and the initial states as x^r_0 and x^h_0, one can obtain the states and s^a_k.

The stochastic transitions of the binary human state x^B_k are modeled by a stochastic finite state machine (sFSM). Denote by $u^B_k \in \{0,1\}$ the on/off action of an advisory control at time step k. We model the transition probability of x^B_{k+1} given x^B_k and u^B_k using first-order Markov assumption. There are 8 different possibilities for transitions.

The probability of an event w^i_k denoted by $p[w^i_k]$ needs to be estimated. A list of all possible transitions are presented in our previous work [10]. We let the probability of transitioning to an advisory action be denoted by $p[w^2_k] = p[x^B_{k+1} = 1 | x^B_k = 0, u^B_k = 1] = p_t$. Then the probability of not transitioning to advisory can be calculated as, $p[w^1_k] = p[x^B_{k+1} = 0 | x^B_k = 0, u^B_k = 1] = 1 - p_t$. We also let the probability of continuously following the advisory control be $p[w^6_k] = p[x^B_{k+1} = 1 | x^B_k = 1, u^B_k = 1] = p_f$. The p_t and p_f may be learned from human-in-the-loop experiments. Then the probability of not continuously following the advisory becomes, $p[w^5_k] = p[x^B_{k+1} = 0 | x^B_k = 1, u^B_k = 1] = 1 - p_f$. The rest of the possible transitions become deterministic.

Combining all the stochastic events, the following transition model can be constructed:

$$x^B_k = \sum_{j=0}^{k-1} u^B_j + C \sum_{j=0}^{k-1} w_j, \forall k \geq 1, \tag{7}$$

where $C = [-1 \; 0 \; -1 \; 0]$ and $w_j = [w^1_j \; w^2_j \; w^5_j \; w^6_j]^\top$.

III. STOCHASTIC HUMAN INPUT MODELING

Equations (1) and (6) model the state transitions of the AV and the IHV. To predict the motion of the IHV, a predictive model of the human actions u_k^h is needed. One can assume a constant model, i.e., $u_{k+1}^h = u_k^h$. However, the constant model does not take into consideration stochasticity of the human actions. By monitoring human actions using onboard sensors, we can construct a better model of u_h^k. In this section, we discuss an HMM approach to modeling the human driver input u_h^k and how to incorporate the HMM into our system model.

A. Human Behavior modeling using HMM

For a human driver driving forward, we classify the driver's actions into three categories through an HMM-based action recognition model, speeding up (s^u), slowing down (s^d), and normally driving (s^c). Define the current human action as $a_k \in \mathbb{A} = \{s^d,\ s^c,\ s^u\}$. The human action recognition model integrates three HMMs for the three aforementioned actions to generate a probability distribution $P(a_k)$.

Each HMM consists of five elements, transition probability matrix M^T, initial probability matrix M^I, emission probability matrix M^E, hidden states S, and observations O. The observations O are sequences of driving data including velocity and pedal percentage collected from a driving simulation testbed. The K-means algorithm is used for clustering the data into one dimensional symbols. The transition probability matrices M^T and emission probability matrices M^E are trained based on the Baum-Welch algorithm [15].

For the inference, the input sequence is converted into a formatted input sequence through the K-means clustering. The likelihood probabilities of the sequence fitting each HMM are calculated and normalized into a probability distribution matrix. The trained transition matrix $P(S_{k+1}|S_k, a_k)$ and the emission probability matrix $P(u_k^h|S_k, a_k)$ are used to calculate the human input transition probability $P(u_{k+1}^h|u_k^h)$, where u_k^h denotes the input acceleration at k^{th} time step. We include a detailed derivation of $P(u_{k+1}^h|u_k^h)$ in [16].

B. Incorporating the HMM

The HMM model described in the previous section gives us the human input transition model from one discretized input to another. Let the possible human actions be discretized as $[v^1, v^2, \dots, v^n]^\top \in \mathbb{R}^n$ and the current discretized human input be one hot encoded by $Q_k = [q_k^1, q_k^2, \dots, q_k^n]^\top$ where n is the number of discretized actions and each element $q_k^i \in \{0,1\},\ \forall\ i \in \{1, 2, \dots, n\}$. Here, only one element in q_k^h is 1, and $q_k^i = 1$ indicates $u_k^h = v^i$ where v^i is the corresponding discrete value of u^h. Thus, we have

$$u_k^h = \begin{bmatrix} v^1 & v^2 & \dots & v^n \end{bmatrix} Q_k. \tag{8}$$

We also define one hot encoded $E_k = [e_k^1, e_k^2, \dots, e_k^{n^2}]$, where each element event $e_k^s \in \{0,1\},\ \forall\ s \in \{1, 2, \dots, n^2\}$ indicates the event of transitioning from $q_k^i = 1 \rightarrow q_{k+1}^j = 1$ where $i, j \in \{1, 2, \dots, n\}$. This transition forms the following inequalities:

$$q_{k+1}^j \leq -q_k^i + 1 + e_k^s,\ \ e_k^s \leq q_k^i,\ \ e_k^s \leq q_{k+1}^j. \tag{9}$$

The transition probability of an event e_k^s is denoted by $P(e_k^s) = P(q_{k+1}^j = 1 | q_k^i = 1)$. Each such event will produce similar constraints to (9). Since only one event e_k^s occurs in the k^{th} time step, we have

$$e_k^1 + e_k^2 + \dots + e_k^{n^2} = 1. \tag{10}$$

IV. SYSTEM CONSTRAINTS

To coordinate the motions of the IHV and the AV, we consider the following four sets of constraints.

A. Merging constraints:

During the lane merging coordination, the longitudinal position between the two vehicles must be more than a threshold $d > 0$. The value of d is dependent on the relative position of the two coordinating vehicles. To determine the relative position between the two vehicles, binary variables f_k^{rh} and f_k^{hr} are defined, which denote whether a vehicle is in front or back. If AV is in front of IHV, then $f_k^{rh} = 1$ and $f_k^{hr} = 0$. Otherwise $f_k^{rh} = 0$ and $f_k^{hr} = 1$. This relationship forms the following inequalities.

$$x_{k,1}^r - x_{k,1}^h \geq -\bar{M} f_k^{hr},\ \ x_{k,1}^r - x_{k,1}^h \leq -\varepsilon + (\bar{M} + \varepsilon) f_k^{rh}, \tag{11}$$

$$and\ \ f_k^{rh} + f_k^{hr} = 1, \tag{12}$$

where $\bar{M}$ is a sufficiently large positive number and ε is a small positive number.

Define binary variables m_k^r and m_k^h to indicate the merging vehicle. If the AV is merging, $m_k^r = 1$ and $m_k^h = 0$. If the IHV is merging, then $m_k^r = 0$ and $m_k^h = 1$. We also define binary variables $l_k^{rh} \in \{0,1\}$ to indicate if the AV and IHV are in the same lane. If they are in the same lane then $l_k^{rh} = 1$ otherwise $l_k^{rh} = 0$. Let the safe following distance be d_f. For safe merging, we also consider additional rear clearance d_r while merging. The safe merging distance d can be formulated using the following equation:

$$d = (1 - l_k^{rh})(d_f + m_k^r f_k^{rh} d_r + m_k^h f_k^{hr} d_r) + l_k^{rh} d_f. \tag{13}$$

Here, d is the longitudinal distance threshold that the two vehicles should maintain right before a safe merging. If the AV is merging from behind, we have $m_k^r = 1$, $m_k^h = 0$, $l_k^{rh} = 0$, $f_k^{rh} = 0$, and $f_k^{hr} = 1$. From (13), $d = (1-0)(d_f + 1 \times 0 \times d_r) + 0 \times d_f = d_f$. If the AV is merging to the front of the IHV, $f_k^{rh} = 1$, $f_k^{hr} = 0$, and the rest would be the same. Then $d = d_f + d_r$. Similarly, if the IHV is merging from behind, we obtain from (13) $d = d_f$, and if the IHV is merging from the front, $d = d_f + d_r$. Thus, (13) allows incorporation of an additional clearance d_r while merging in front of a vehicle. If the two vehicles are in the same lane, they will keep a following distance as $d = d_f$.

To ensure the longitudinal distance between the two vehicles is larger than d, the following constraint is considered:

$$|x_{k,1}^r - x_{k,1}^h| \geq d \tag{14}$$

where $x_{k,1}$ denotes the position state. Two binary variables are defined, $b_{1,k}$ and $b_{2,k}$ which denote the satisfaction of the formula (14). The detailed formulation can be found in [10].

B. State constraints:

The equations defining the new states $\bar{z}_k^1$, $\bar{z}_k^2$, $\bar{z}_k^3$ and their relationships with the states and inputs lead to the state constraints. Using the mixed-logic dynamical (MLD) systems formulations presented in [17], the state constraints are formulated as inequalities.

C. sFSM transition constraints:

The state transitions defined in the sFSM are enforced in these constraints. Let $\delta_k^1 = x_k^B u_k^B$. Then the stochastic transitions form the following inequality constraints:

$$w_k^1 + w_k^2 \leq u_k^B - \delta_k^1, \quad w_k^1 + w_k^2 \geq u_k^B - \delta_k^1, \tag{15}$$

$$w_k^5 + w_k^6 \leq \delta_k^1, \quad w_k^5 + w_k^6 \geq \delta_k^1, \tag{16}$$

$$-x_k^B + \delta_k^1 \leq 0, \quad -u_k^B + \delta_k^1 \leq 0, \tag{17}$$

$$u_k^B + x_k^B \leq 1 + \delta_k^1, \tag{18}$$

D. Chance constraints:

Chance constraints are used to reject trajectories that only occur with a small probability from the set of possible solutions. The possible human input transition events are given by $E_k = [e_k^1 \; e_k^2 \; \dots \; e_k^{n^2}]^\top$ and the transition probabilities are $p = \begin{bmatrix} p[e_k^1] & p[e_k^2] & \dots & p[e_k^{n^2}] \end{bmatrix}^\top$. Thus, the chance constraint can be computed as

$$\sum_{k=0}^{K-1} \sum_{i=1,2,\dots,n^2} e_k^i \ln(p[e_k^i]) \geq \ln(\tilde{p}_e), \tag{19}$$

where $\tilde{p}_e \in [0,1]$ is the human input probability bound. This chance constraint (19) enforces that human input trajectory E realizes with at least $\tilde{p}_e$ probability.

Similarly, for the possible human state transition events, given by w_k and the chance constraint can be formulated as,

$$\sum_{k=0}^{K-1} \sum_{i=1,2,5,6} w_k^i \ln(p[w_k^i]) \geq \ln(\tilde{p}_w) \tag{20}$$

enforcing that w realizes with at least $\tilde{p}_w \in [0,1]$ probability.

V. STOCHASTIC MPC FORMULATION

The goal of the sMPC is to optimize the decision variables at each time step k to minimize a cost function designed by preference while satisfying all the constraints required to model the coordination of the IHV and the AV. The different components of the sMPC formulation are discussed below.

A. Decision variables

For a look ahead window of K, the decision variables are summarized as $\theta_k \in \mathbb{R}^{n_t}$ in the form $\theta_k = [\mathbf{u}_k^r \; \mathbf{u}_k^a \; \bar{\mathbf{z}}_k^1 \; \bar{\mathbf{z}}_k^2 \; \bar{\mathbf{z}}_k^3 \; \mathbf{u}_k^B \; \mathbf{w}_k \; \delta_k^1 \; \mathbf{b}_k \; \mathbf{q}_k^1 \; \mathbf{q}_k^2 \; \dots \; \mathbf{q}_k^n \; \mathbf{e}_k^1 \; \mathbf{e}_k^2 \; \dots \; \mathbf{e}_k^{n^2} \; \mathbf{f}_k^{rh} \; \mathbf{f}_k^{hr}]$ where $\mathbf{u}_k^r = \begin{bmatrix} u_k^r, & u_{k+1}^r, & \cdots, & u_{k+K-1}^r \end{bmatrix}$ and all the other decision variables are defined similarly. The total number of decision variables is denoted by n_t. The continuous variables are $\mathbf{u}_k^r$, $\mathbf{u}_k^a$, and $\mathbf{z}_k^u$ while the rest are binary. Note that among all these decision variables, the AV input $\mathbf{u}_k^r$ and the IHV advisory commands $\mathbf{u}_k^a$ along with the advisory state u_k^B are the control inputs entering the system dynamics.

B. Constraints

The equality and inequality constraints mentioned above are linear in decision variable θ_k, and can be formulated as

$$G_k \theta_k \leq g_k, \tag{21}$$

where $G_k \in \mathbb{R}^{n_c \times n_t}$ and $g_k \in \mathbb{R}^{n_c \times 1}$ where n_c is the total number of constraints.

C. Cost function

We take into account five goals in the cost function, which are, minimize the control inputs, minimize the time to reach the merging distance, maximize the speed of AV and IHV, minimize the number of advisory directives and maximize the probability of the stochastic events. The objective function of the sMPC is the sum of these five functions, which can be represented as

$$J(\theta_k) = \theta_k^\top Q \theta_k + c^\top \theta_k, \tag{22}$$

where $Q \in \mathbb{R}^{n_t \times n_t}$ and $c \in \mathbb{R}^{1 \times n_t}$ are the designed objective weights for the system.

D. Optimization problem

Assuming the human state x_k^B is a known parameter, the optimization problem can be formulated as

$$\min_{\theta_k} \; J(\theta_k), \quad s.t. \;\; \mathbf{G}_k \theta_k \leq \mathbf{g}_k. \tag{23}$$

However, whether the driver follows the advisory control may only be estimated, e.g., via monitoring the driver's performance. Thus, x_k^B may only be estimated by a probability distribution. To address the uncertainty in the estimated x_k^B, we formulate two sets of optimization problems and optimize the decision variables for two different conditions. If the human is following the advisory command, i.e., $x_k^B = 1$, the optimized decision variables are denoted as θ_k^1. If the human is not following the advisory command, i.e., $x_k^B = 0$, the optimized decision variables are denoted by θ_k^0. Let the probability of the IHV driver following the advisory command at time step k be $P_k^B = p[x_k^B = 1]$ and the probability of the driver not following, $p[x_k^B = 0]$, as $1 - P_k^B$. We then define the following optimization problem:

$$\min_{\theta_k^0, \theta_k^1} = P_k^B \left(J(\theta_k^1) \right) + (1 - P_k^B) \left(J(\theta_k^0) \right) \tag{24}$$

$$\text{s.t.} \;\; P_k^B \mathbf{G}_k \Big|_{x_k^B=1} \theta_k^1 \leq P_k^B \mathbf{g}_k \Big|_{x_k^B=1}, \tag{25}$$

$$(1 - P_k^B) \mathbf{G}_k \Big|_{x_k^B=0} \theta_k^0 \leq (1 - P_k^B) \mathbf{g}_k \Big|_{x_k^B=0}, \tag{26}$$

$$\begin{bmatrix} \mathbf{u}_k^r & \mathbf{u}_k^a & \mathbf{u}_k^B \end{bmatrix}^\top \Big|_{x_k^B=1} = \begin{bmatrix} \mathbf{u}_k^r & \mathbf{u}_k^a & \mathbf{u}_k^B \end{bmatrix}^\top \Big|_{x_k^B=0}. \tag{27}$$

The constraint (27) means that the control inputs to be implemented on the AV and the IHV must be the same for the two possible scenarios $x_k^B = 1$ and $x_k^B = 0$.

Applying this optimization algorithm in a receding horizon fashion with the constraints and defined objective, we get the sMPC solution, which consists of the control inputs applied to the AV and the advisory commands communicated to the IHV at each time step.

VI. SIMULATION RESULTS

We implement the aforementioned sMPC method in simulations. We use a double integrator model for the vehicle dynamics. The optimization produces the input for the AVs and the advisory command for the IHVs until the safe merging distance d is achieved. For all the simulations, the size of the look-ahead window is $K = 15$ and each time step is 0.8 seconds. The above-mentioned formulation is programmed as a mixed integer optimization problem in Python. Then the problem is solved using the Gurobi optimizer [18]. The simulations were run in Python on a computer with Intel(R) Core i7-3770 CPU @ 3.40GHz with 16GB RAM and NVIDIA GeForce GT 630 graphics card.

We use the learned emission and transition probability matrices mentioned in Section III-A to create a human input probability model $p(u^h_{k+1}|u^h_k)$ where the number of discretized inputs is 59 with a range of $[-3.9, 1.9]$ m/s^2. We denote that model as the original transition model $T_o \in \mathbb{R}^{59\times59}$. For faster calculation in the sMPC we compress T_o from $\mathbb{R}^{59\times59}$ to $\mathbb{R}^{12\times12}$ and denote it as the compressed transition model $T_c \in \mathbb{R}^{12\times12}$. To imitate the behavior of the real driver, we use the T_o as the *true* human driver model. In the sMPC, we use the model T_c to optimize and predict the merging of the vehicles.

A. Coordination of two vehicles

We simulate the coordination for the scenario in Fig. 1 with the following initial conditions: the initial longitudinal distance between the two vehicles $dif = 0\ m$, and the probability of transitioning to advisory action $p_t = 0.99$. The initial probability of human following the advisory with increasing time steps is $P^B_0 = [0.0, 0.9, 0.9, \dots]$. The d_f is set to 7m, and for the two vehicle scenario we set d_r to 0m.

Figure 2 shows the simulated optimization results for different initial IHV inputs. The predicted and actual human behaviors are plotted together for comparison. The prediction has some error but the optimization calculates the best merging action satisfying all the constraints. In both of these simulations, $\lambda = 0.0$, which means the human exactly follows the advisory command with one-step delay.

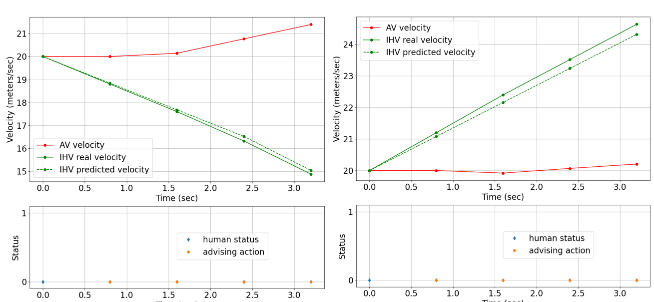

Fig. 2. Simulated optimization results for initial IHV acceleration -1.5m/s^2 (left column) for initial IHV acceleration 1.5m/s^2(right column).

In a real-life scenario, $\lambda \neq 0$ due to latency in human adaptation to the advisory command. The effect of λ is demonstrated in Figure 3. In this figure, the initial IHV acceleration is 0 m/s^2, and the rest of the initial conditions are kept the same as in Figure 2. When $\lambda = 0.0$ the optimization ends at 4.8 sec while when $\lambda = 0.2$ the optimization ends at 5.6 sec. The merge happens faster with $\lambda = 0.0$. With increasing λ, the human driver's capacity to follow the advisory command is reduced. As a result, the optimization takes longer to reach the merging condition.

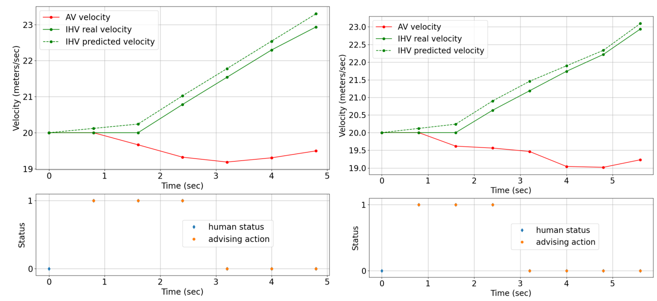

Fig. 3. Simulated optimization results for initial IHV acceleration 0m/s^2 with $\lambda = 0.0$ (left column) with $\lambda = 0.2$ (right column).

In our previous work [10] we used a constant velocity (CV) model for human action prediction in the sMPC algorithm. To assess the performance of the HMM model for human action prediction, we conduct Monte Carlo simulations of 20 random variations of human inputs. For the same initial conditions, the performances of the optimization with human input prediction using the compressed HMM (cHMM) model and the CV model are presented in Table I. The average time for merging is denoted by t_{avg}.

TABLE I
MONTE CARLO SIMULATION RESULTS FOR CV AND CHMM MODELS.

Model	λ	p_t	p^B_0	t_{avg}
CV	0.0	0.99	[0,0.9,0.9,...]	5.6
		0.5	[0,0.6,0.6,...]	5.558
	0.7	0.99	[0,0.9,0.9,...]	6.358
		0.5	[0,0.6,0.6,...]	7.073
cHMM	0.0	0.99	[0,0.9,0.9,...]	5.432
		0.5	[0,0.6,0.6,...]	5.642
	0.7	0.99	[0,0.9,0.9,...]	6.189
		0.5	[0,0.6,0.6,...]	5.095

In Table I, for each model in the Monte-Carlo simulation, λ is increased and p_t and p^B_0 are decreased to simulate the effect of the human driver's decreasing attention and willingness to follow advisory commands. We observe that, with $\lambda = 0.0$, the average time to merge for the CV model and the cHMM model does not differ much. However, with $\lambda = 0.7$ and reduced p_t and p^B_0, the average merging time increases for the CV model while the average merging time for the cHMM model decreases. This is because the cHMM predicts human actions more accurately than the CV model, which is then incorporated in the optimization process. With a higher λ, the less attentive to the advisory commands the human gets, the more effective the cHMM model becomes at optimizing the merge compared to the CV model. This indicates that the inclusion of human action prediction using cHMM helps the optimization process to minimize the merging time for human drivers who pay less attention to advisory commands and have a lower willingness to follow advisory commands.

B. Coordination of three vehicles

The sMPC method is also implemented to simulate the three vehicles' coordination in Figure 1 to demonstrate its effectiveness. The probability of transitioning to advisory action is $p_t = 0.99$. The initial probability of human following the advisory with increasing time steps is $P_0^B = [0.0, 0.9, 0.9, \dots]$. The λ is set as 0.0 and the initial IHV acceleration $0m/s^2$. For safe merging d_f is 7m, and d_r is 3m. The initial longitudinal distance between the IHV and AV1 is dif_1 and between the IHV and AV2 is dif_2.

We illustrate two cases in Figure 4, where AV1 is merging. In the left figure, the distance between the IHV and AV1 is −4m, and the distance between the IHV and AV2 is −9m. From the velocities of the vehicles, it is seen that the IHV accelerates, the AV2 keeps a steady velocity and the AV1 slows down to merge from behind. In the right figure, the distance between the IHV and AV1 is 0m and the distance between the IHV and AV2 is −4m. Here we see that the IHV and the AV2 both accelerate, and the AV1 decelerates to merge from behind.

The advisory in the left scenario is not turned on. This is because the distance between AV1 and the IHV is higher, and the predicted action of the IHV is sufficient for the optimal merging. Thus, there is no advisory given to the IHV. However, in the right scenario, where the distance between AV1 and the IHV is 0m, the IHV's predicted action is not sufficient. Therefore, in the initial steps, advisory inputs are given to the IHV for faster acceleration.

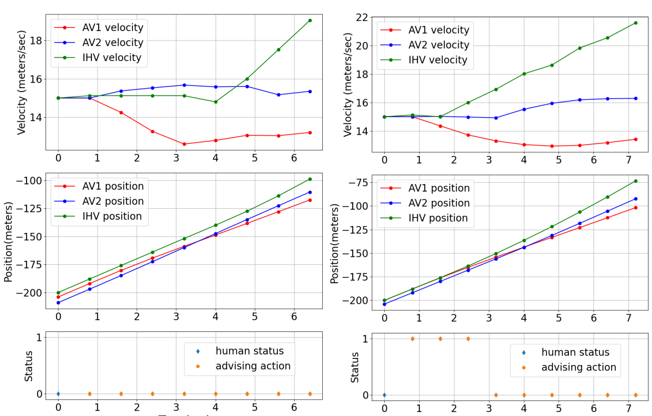

Fig. 4. Simulated optimization results with $dif_1 = -4$m and $dif_2 = -9$m (left column) with $dif_1 = 0$m and $dif_2 = -4$m (right column).

VII. CONCLUSIONS

We present an sMPC formulation for coordinating the motion of IHVs and AVs for optimal merging. The formulation incorporates stochastic elements of a human driver, such as uncertainty in human's intent and stochastic human actions. Additionally, it accounts for both system and human delays. The solution to the sMPC provides optimal inputs for the AVs and optimal advisory commands for the IHVs. Our simulations demonstrate the effectiveness of the proposed formulation and illustrate improved merging actions when compared to a system without human input prediction. Future work includes experimenting with more complex driving scenarios and improving the models of human driver behaviors.

REFERENCES

[1] Statista, "Projected number of autonomous vehicles in operation in the United States in 2025 and 2030," 2020.

[2] "Autonomous Vehicle Sales Forecast 2018 - AutoTechInsight." [Online]. Available: https://autotechinsight.ihsmarkit.com/shop/product/5001816/autonomous-vehicle-sales-forecast-2018

[3] X. Huang, A. Jasour, M. Deyo, A. Hofmann, and B. C. Williams, "Hybrid risk-aware conditional planning with applications in autonomous vehicles," in *2018 IEEE Conference on Decision and Control (CDC)*. IEEE, 2018, pp. 3608–3614.

[4] C.-P. Lam, A. Y. Yang, K. Driggs-Campbell, R. Bajcsy, and S. S. Sastry, "Improving human-in-the-loop decision making in multi-mode driver assistance systems using hidden mode stochastic hybrid systems," in *2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2015, pp. 5776–5783.

[5] C.-P. Lam and S. S. Sastry, "A POMDP framework for human-in-the-loop system," in *53rd IEEE Conference on Decision and Control*, 2014, pp. 6031–6036.

[6] W. Zhou, D. Chen, J. Yan, Z. Li, H. Yin, and W. Ge, "Multi-agent reinforcement learning for cooperative lane changing of connected and autonomous vehicles in mixed traffic," *Autonomous Intelligent Systems*, vol. 2, no. 1, pp. 1–11, 2022.

[7] S. Mosharafian and J. M. Velni, "Cooperative adaptive cruise control in a mixed-autonomy traffic system: A hybrid stochastic predictive approach incorporating lane change," *IEEE Transactions on Vehicular Technology*, pp. 1–13, 2022.

[8] A. Bemporad and S. Di Cairano, "Model-predictive control of discrete hybrid stochastic automata," *IEEE Transactions on Automatic Control*, vol. 56, no. 6, pp. 1307–1321, 2010.

[9] S. Mosharafian and J. Mohammadpour Velni, "A hybrid stochastic model predictive design approach for cooperative adaptive cruise control in connected vehicle applications," *Control Engineering Practice*, vol. 130, p. 105383, 2023. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0967066122002143

[10] S. Hossain, J. Lu, H. Bai, and W. Sheng, "Stochastic model predictive control for coordination of autonomous and human-driven vehicles," *IFAC-PapersOnLine*, vol. 55, no. 41, pp. 142–147, 2022.

[11] J. Lu, R. Stracener, W. Sheng, H. Bai, and S. Hossain, "Development of a research testbed for cooperative driving in mixed traffic of human-driven and autonomous vehicles," in *2022 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2022, pp. 11 403–11 408.

[12] J. Lu, S. Hossain, W. Sheng, and H. Bai, "Cooperative driving in mixed traffic of manned and unmanned vehicles based on human driving behavior understanding," 2023, accepted for publication at The 40th International Conference on Robotics and Automation (ICRA).

[13] C. C. Macadam, "Understanding and modeling the human driver," *Vehicle system dynamics*, vol. 40, no. 1-3, pp. 101–134, 2003.

[14] W. Hoult and D. J. Cole, "A neuromuscular model featuring co-activation for use in driver simulation," *Vehicle System Dynamics*, vol. 46, no. S1, pp. 175–189, 2008.

[15] L. E. Baum and J. A. Eagon, "An inequality with applications to statistical estimation for probabilistic functions of markov processes and to a model for ecology," *Bulletin of the American Mathematical Society 73*, no. 3, pp. 360–3, 1967.

[16] S. Hossain, J. Lu, H. Bai, and W. Sheng, "Additional document: Cooperative driving between autonomous vehicles and human-driven vehicles considering stochastic human input and system delay," 2023. [Online]. Available: https://coral-osu.github.io/assets/pdf/ECC2023_additional_doc.pdf

[17] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints," *Automatica*, vol. 35, no. 3, pp. 407–427, 1999.

[18] Gurobi Optimization, LLC, "Gurobi Optimizer Reference Manual," 2022.