PROBLEM SOLVING AND PERSEVERANCE IN GEOMETRY: REVELATIONS FROM THINK ALOUDS WITH MIDDLE GRADES STUDENTS

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Problem solving is a very important skill for students to learn (e.g., Bonilla-Rius, 2020; NGA, 2010), and part of developing problem solving skills is learning to persevere. One strategy for learning how to persevere is by providing students with materials that allow them the opportunity to engage with challenging problems (e.g., Kapur, 2010; Middleton et al., 2015). This study of the Volume unit of the AC²inG materials analyzes students' strategies for problem solving and persevering. Findings from these think-aloud interviews indicate that different students will utilize one or more methods for solving challenging problems, such as asking clarifying questions, talking themselves through the problem, and attempting various mathematical approaches.

Keywords: Problem solving, middle school education, geometry and spatial reasoning, curriculum

Theoretical Framework

The first Standard for Mathematical Practice outlined in the Common Core State Standards (NGA, 2010) states that students should be able to "make sense of problems and persevere in solving them" (SMP.1). Perseverance and problem solving are also emphasized in standards outside the United States (e.g., Canada (Ontario, 2020); Mexico (Bonilla-Rius, 2020)). The ability to problem solve and persevere are highlighted as part of the skillset required to be successful in our modern society. According to Bonilla-Rius (2020), the recently re-envisioned national standards in Mexico outline eleven skills and characteristics necessary to be a successful citizen in the 21st century, including critical thinking and problem solving.

Students also recognize the importance of perseverance when faced with a challenging problem. When asked what it takes to solve problems, one group of elementary students offered the following responses: "Be open minded."; "Be able to defend your thinking."; and "Don't give up – persevere!" (Costello, 2020). However, while many stakeholders understand the importance of perseverance and problem solving abilities, researchers understand that these crucial skills are not always innate for students and, thus, must be modeled for them (Colgan, 2020). One way to model these skills is through the materials and resources teachers provide for their students (Kapur, 2010; Middleton et al., 2015). Materials that utilize contrasting cases are one such set of resources that provide opportunities for students to practice problem solving (Loibl et al., 2020), an important part of which is persevering.

The Animated Contrasting Cases in Geometry (AC²inG) Materials

The Animated Contrasting Cases in Geometry (AC²inG) project has recently developed a set of materials that utilize contrasting cases to offer middle-grades students the chance to reason about various geometric topics in a relatively novel way. Contrasting cases are materials that present two or more methods for solving the same or similar problems, and research has shown that these types of activities can be effective for developing both procedural fluency and conceptual understanding (Rittle-Johnson & Star, 2009). Covering topics across the CCSS-M

Grade 8 standards, including angles, transformations, the Pythagorean theorem, and volume, the AC^{2}inG materials are organized into four units, each covering one of the aforementioned topics, that include five or six Worked Example Pairs (WEPs) per unit. Each WEP presents students with two fictitious characters' methods for solving a problem and allows students to explore the reasoning of these two characters side by side, providing them the opportunity to compare and contrast each character's method. Furthermore, each WEP consists of five parts: (1) the first character's method alone; (2) the second character's method alone; (3) both methods side by side; (4) a series of discussion questions and practice problems about the concept covered in the WEP; and (5) a Thought Bubble page where one of the characters shares a revelatory idea about the concept. This design presents students with myriad opportunities to critique the reasoning of others (another of the Standards for Mathematical Practice, SMP. 3 (NGA, 2010)) while practicing their problem solving skills and challenging themselves to persevere.

Methods

Due to the constraints of the current pandemic, our team conducted individual think aloud interviews (Piaget, 1976) with 42 students in lieu of piloting these materials in classrooms. In each interview, students worked through one or more of the WEPs from a particular unit while discussing their thoughts about the methods of each character as well as how the students themselves would solve the problems and answer the discussion questions. These interviews were transcribed and independently coded by two members of our research team. After one researcher coded an interview, a second researcher then coded the interview to determine agreement. In the case of disagreements, a third researcher made a final determination about the code(s) in question.

This report focuses on codes from the Volume unit pertaining to students' original geometric thinking, particularly those where students struggled and persevered. The code 'Persevered' was a subcode of the code 'Struggle,' which was used when a student showed us they were having trouble working through a problem. We analyzed quotes tagged with the 'Struggle' code for similarities and differences and realized that, when students were confused, they took one of two possible pathways. First, the student could stop working and give up, an option students exercised approximately one-third of the time. Second, the student could choose to continue working through the problem, an option we observed the other two-thirds of the time and coded as 'Persevered.' These codes were further analyzed for similarities and grouped into two broad approaches, as outlined below.

Findings

Throughout the 14 think alouds covering WEPs from the Volume unit, students worked to solve a variety of problems. Many of these problems caused them to struggle, as indicated by the 41 times we used the code 'Struggle' when analyzing transcripts from this unit. Given that perseverance was observed in eight of the 13 students who completed WEPs from the Volume unit, it is no surprise that this trait manifested itself in several different ways. Primarily, we noticed two broad approaches to persevering when problem solving: (1) the student tried to make sense of the problem; or (2) the student tried to use an alternate strategy.

Making Sense of Problems

Persevering through a challenging problem was seen when students attempted to make sense of the problem before proceeding. They did this in three distinct ways: (1) asked clarifying questions; (2) talked themselves through the problem; and (3) tried to recall something they had previously learned.

Ask clarifying questions. When students encountered difficult problems while working through the materials, one strategy they used to persevere was to ask the researcher clarifying questions about the problem. For example, when posed with the question, "Can two cylinders with different dimensions have the same volume?", one student asked about the word dimensions. "Is that like height or depth or something?" Rather than give up on this problem due to a vocabulary issue, this student chose to ask a clarifying question and was able to craft a cogent, correct response.

Other students asked clarifying questions and were able to correct their misconceptions through the act of asking their question aloud. One WEP prompt asked how much paint is necessary to paint the walls of Rachel's room. Two of her friends, Damien and Sydney, offered to help her solve this problem. Damien used surface area to calculate his answer, and Sydney used volume. One student originally said that volume would be best because "Rachel painted the entire room, and it said that she used [volume]." This student had misassigned Sydney's method to the fictitious character, Rachel, who was the friend in the scenario who needed help. This student asked the researcher to clarify what the question was asking and realized that they had made a mistake when interpreting the methods and solutions. They then changed their answer to surface area, realizing that volume would fill the room with paint.

Talk through the problem. Another strategy students used was to talk themselves through the problem at hand. One student initially struggled to understand the method employed by one of the characters when attempting to find the volume of a cylindrical container. "It seems weird to me," said the student. "I kept running through it. I was wondering why it was like that." Then revelation struck. "Oh wait! I see now. This is B [the area of the base] - you're trying to find the circle." By walking through the problem and provided method several times, this student was able to understand the reasoning of the character in the WEP on their own.

Recall previous learning. A third strategy we observed students using was to dig into their memories to recall concepts and information they had previously learned. Sometimes they were successful in remembering. One student knew that they had learned how to calculate the volume of a cylinder previously and thought their teacher would be "so angry with [them] if [they] forget these things." Consequently, the student tried to think back to their previous work and finally recalled the correct strategy. Another student initially misread a problem about calculating the volume of composite figures and struggled to remember a specific formula before remembering that the figure could be separated into more common figures, the formulas for which the student was able to recall.

Other students attempted to remember what they had learned but were not as successful. One such student stated that they "kind of forgot how to do the area of a cylinder" and cited this as the reason they were confused when trying to reason through the characters' methods. Even though this student could not recall the knowledge necessary to solve the problem, we identified this as perseverance because they attempted and did not give up on the problem until they had attempted several times.

Trying an Alternate Strategy

When students would get stuck in the middle of a problem, we noticed that some of them: (1) tried a different mathematical approach; or (2) took an educated guess.

Tried a new mathematical approach. Students sometimes decided to take a different mathematical approach than the one they originally thought of or tried. One student was confused by a problem that asked them to find the volume of a cylinder if the radius were scaled by a factor of three. The first thing they attempted was to find the height of the cylinder so they

could use the volume formula. When they realized the problem did not give them the height, they attempted to plug in what they were given and solve for the height. This strategy was more complex than they were expecting, and even though they did not find the correct answer, they persevered through their selected strategy and found a solution, albeit one that was incorrect.

Took an educated guess. After exhausting other efforts, some students settled on offering a best guess. After talking through the problem of scaling the radius of a cylinder, one student offered an incorrect solution but was able to justify their guess and the steps they took to arrive at their guess. Another student, when trying to use one of the characters' methods to find the volume of a cylinder, admitted that they did not fully understand the fictitious character's method, and while they did not come to the correct solution, they did offer an explanation of their thinking and a guess as to what they thought it might be.

Several times, students initially did not have any idea what to do with a problem, but when the researcher asked what they were thinking, they were able to offer a guess. When thinking about why scaling the radius by some number n causes the volume to scale by n^2, one student tried to reason through the problem, after initially stating that they did not know how to answer it. "Um, I mean, you didn't change the height, I guess, so that's why the volume multiplied. I don't know." Even though this student was not able to completely articulate their thinking, they persevered and demonstrated a preliminary understanding of the relationship between the radius and the volume.

Discussion and Future Research

The math education community places great importance on problem solving and perseverance, as witnessed by the standards we set for our students (e.g., CCSS Standards for Mathematical Practice (NGA, 2010)). These skills are essential in the 21st century (Bonilla-Rius, 2020), and students themselves recognize the importance of persevering when faced with challenging problems (Costello, 2020). In this study, we witnessed students using a variety of problem solving strategies and persevering to overcome questions that caused them confusion. They asked clarifying questions, talked through the problem, dug into their memories, tried multiple strategies, and even took educated guesses when necessary.

The think aloud interviews allowed us to uncover ways in which students persevered when working with middle-grades volume concepts. Future research might analyze what type of materials best encourage students to persevere. It also might be pertinent to compare the students who persevered with those who did not and try to understand the differences between these students.

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