“Where is the z-axis?”: Negotiating Understanding of Servo Rotation through Gestures and Tools

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Abstract: Understanding abstract concepts in mathematics has continuously presented as a challenge, but the use of directed and spontaneous gestures has shown to support learning and ground higher-order thought. Within embodied learning, gesture has been investigated as part of a multimodal assemblage with speech and movement, centering the body in interaction with the environment. We present a case study of one dyad’s undertaking of a robotic arm activity, targeting learning outcomes in matrix algebra, robotics, and spatial thinking. Through a body syntonicity lens and drawing on video and pre- and post-assessment data, we evaluate learning gains and investigate the multimodal processes contributing to them. We found gesture, speech, and body movement grounded understanding of vector and matrix operations, spatial reasoning, and robotics, as anchored by the physical robotic arm, with implications for the design of learning environments that employ directed gestures.

Introduction

Within the growing literature on embodied learning in mathematics, gesture has been investigated as part of a multimodal assemblage with speech and movement (e.g., Alibali & Nathan, 2012), centering the role of the body in interaction with the environment. The use of directed and spontaneous gestures has shown to support learning and has been especially useful for grounding abstract concepts, which have continually presented as a challenge (e.g., Pier et al., 2019). Though extant work has focused on children’s use of gesture and its implications for mathematics learning (e.g., Abdu et al., 2021), few studies have explored students’ use of gesture as part of their sense-making process toward understanding of specific domain knowledge in mathematics within higher education. Engaging the body along with and beyond the use of gesture and speech, in interaction with physical artifacts, is another aspect of existing research that has been left underexplored, yet holds the potential to reveal much about students’ learning processes in mathematics (Robutti et al., 2022).

This paper is a case study of one dyad’s undertaking of a robotic arm activity targeting learning outcomes in matrix algebra, robotics, and spatial thinking. We designed the activity to assess participants' growing understanding of vector and matrix operations, spatial reasoning, and robotics, as they manipulated the robotic arm and calculated its end coordinates. This exploratory pilot study adopts a constructionist learning theory lens of body syntonicity and takes on a mixed methods approach to address the following research questions: *To what extent did the robotic arm activity inform student learning gains in mathematics? How did gesture, speech, and the body contribute to these learning gains across dimensions (i.e., vector and matrix operations, spatial reasoning, robotics)?* Two out of the four participating groups who observed the largest learning gains from pre- to post- test engaged gesture and the body in their sense-making. We selected one of the groups to theorize the processes that contributed to the observed learning gains, with implications for the design of learning environments that employ directed use of gesture.

Background

Within the literature on embodied learning in mathematics, extant work has considered the role of multimodal dialogue of small groups for learning mathematics (Abdu et al., 2021), the role of teachers’ gestural use in improving math understanding (e.g., Nathan et al., 2017) and increasing learners’ visual engagement (Farsani et al., 2021), and the design of technology-integrated tools to support embodied interactions in math learning (Abrahamson, 2009). With mathematical ideas inherently metaphorical (Lakoff & Núñez, 2000), gesture has been found to be especially helpful for grounding abstract concepts calling for higher-order thinking. For instance, Pier and colleagues (2019) investigated undergraduate students’ multimodal use of gesture and speech in mathematical reasoning, finding that the use of dynamic gestures (i.e., gestures "that depict and transform mathematical objects," p.3) correlated with the construction of valid mathematical proofs. Gesture has also been found to support problem solving reliant on spatial reasoning skills in particular (e.g., Chu & Kita, 2011).

To understand students' use of gesture in an embodied learning task in matrix algebra, we drew on constructionist learning theory (Papert, 1980), which puts forth the notion of "objects-to-think-with," that is objects in the physical or digital world that help ground mental models and help advance understanding of abstract ideas as learners interact with these objects. Papert further puts forth the notion of body syntonic learning, or a form of embodied learning, aligned with constructionism, where learners draw on their experience as a person with a body to imagine themselves in place of or in relation to the object they are interacting with. Learning thus emerges as a result of these processes of interaction between the learner and the physical object. Adopting a body syntonic lens supported our investigation of students' gestural use in interaction with the robotic arm and motivated our analysis of the ways in which specific gestures and speech grounded abstract concepts across the four dimensions previously discussed.

Methods

Set in the context of a graduate elective class at a 4-year minority-serving public institution in Southern California, this paper zooms in on a hands-on activity, which contextualizes a matrix math application within robotics. Before the activity, students assembled a robotic arm kit at home and viewed a pre-recorded 90-min lecture, introducing them to foundational concepts in matrix algebra and robot kinematics (e.g., degrees of freedom; vector and frame notation; matrix operations). For the activity, we positioned the arm in front of a grid and students selected how to rotate each of the three middle servos, marking the position of the end effector (i.e., the position of the arm's claw) and determining its coordinates. Students then created a diagram to visualize the servos' rotations and calculated the coordinates of the end effector, with facilitators available on Zoom and on site for guidance.

Participants were PhD or MS students in Education, Informatics, Engineering, or Computer Science and formed 2 dyads and 2 triads (Latinx = 3, Asian = 2, Multiracial = 2, and White = 3). We selected one of the dyads as an instrumental case study (Stake, 1995) to understand how students negotiate understanding of servo rotation, the z-axis, and their abstract mathematical representations. We selected this dyad because of their engagement in episodes of independent problem-solving and the high density of gestural content related to the discussed mathematical content. One student in the dyad, Miguel, had taken a linear algebra course as an undergraduate, while the other student in the dyad, Alex, had not (all names are pseudonyms).

We drew on pre- and post- assessment data to evaluate student learning gains, conducted before and after completion of the activity. The assessment consisted of 13 items, covering (a) vector operations, (b) spatial reasoning, (c) matrix operations, and (d) robotics fundamentals, which we evaluated on a 0–3-point scale. The first and third author established IRR with 20% of the assessment data (n=52 items, ɑ=.97), before proceeding to code the rest of the data. We conducted paired-sample t-tests to evaluate the significance of the learning gains, and calculated effect sizes. After an initial pass of the focal dyad's video data from a 360-degree camera (a total of 1.5 hours), we selected 7 short clips with low facilitator involvement and high-density of pertinent gestural content for more in-depth analysis. In our iterative analysis of these segments, we identified and grouped student gestures, developing coding rules around them, and qualitatively analyzing representative moments.

Findings

Assessing learning gains and effect sizes

On average, participants increased their total scores on all four components, with our focal dyad observing learning gains well-above average for most categories (see Table 1).

Table 1. Average learning gains and effect sizes

	Max score	Range at pre-test	Range at post-test	Learning gains (avg)	Learning gains (focal dyad)	Cohen's *d*
Total score	30	1-16	4-21	4.11*	7	0.79
Vector operations	9	1-7	1-6	0.33	0	0.38
Spatial reasoning	9	0-6	0-7	1.78	2	0.65
Matrix operations	6	0-3	0-4	0.44	1.5	0.29
Robotics	6	0-1	0-5	1.56*	3.5	0.86

* $p < .05$

Hand gesture and talk ground concepts in spatial reasoning and vector operations

Purposeful use of hand gestures grounded the dyad's developing understanding of key concepts in spatial reasoning and vector operations (e.g., concept of a plane, vector components). The *drawing-in-space* gesture was a one or two-handed gesture aimed at the physical representation of an abstract construct through "drawing" and bounding the construct in space. For instance, Miguel used the drawing-in-space gesture to situate the Cartesian plane in the physical space through two cutting hand motions, one from top to bottom to represent the y-axis and one from left to right to represent the x-axis (see Figure 1). He named the vector components in the x- and y-direction concurrently, capturing the expanse of the Cartesian plane as constituted at the intersection of the two axes, *"That perfectly straight line is* n_y*. And that perfectly horizontal is* n_x*."* Alex followed what Miguel was doing, interrupting his notetaking to observe his gestures. The hand gestures Miguel performed were key in establishing common ground with Alex about the properties of the physical space. Miguel naming the vector components further signaled an understanding of a vector as being able to be broken into its respective components, in the horizontal x-direction and the vertical y-direction, and the importance of doing so in this context for calculating the end coordinates. Altogether, the drawing-in-space gesture established common referents for the dyad moving forward in a way that could be replicated when necessary, and established a sense of direction, needed when discussing servo rotation and the axis of rotation.

Figure 1. Miguel (left) using a drawing-in-space gesture to represent the x-axis, with a start in (a) and end in (b)

(a) (b)

Another type of hand gesture, *rotational gesture,* most directly informed understanding of servo rotation, what it looks and feels like, drawing on one's spatial reasoning to envision the rotation – and its specifics, such as the axis of rotation – in space. For instance, following the drawing-in-space gestures from above, Miguel modeled the rotation of the first servo by bringing his hands together at an angle and turning them counterclockwise, noting that the rotation is completed around the first angle, *"So this is rotated on* $_1$ *[froomph]."* Difficult to describe it in words, Miguel used utterances (i.e., "froomph") to signify when the gesture was completed. This rotational gesture closely followed the drawing-in-space gesture, so Miguel's right hand was still in the direction of the x-axis while his left hand was in the direction of the y-axis. Though the axis of rotation was not brought up until later, through the specific choice of how to orient one's hands and the direction and angle of rotation, Miguel represented the z-axis as the axis of rotation (see Figure 2). As it was not discussed explicitly, we termed this representation of the z-axis, "the invisible z-axis". Essential to understanding the servo rotation and its properties, the z-axis emerged as an inherent albeit not explicit component of rotational gestures, whose understanding, we theorize, contributed to the development of one's spatial reasoning competencies.

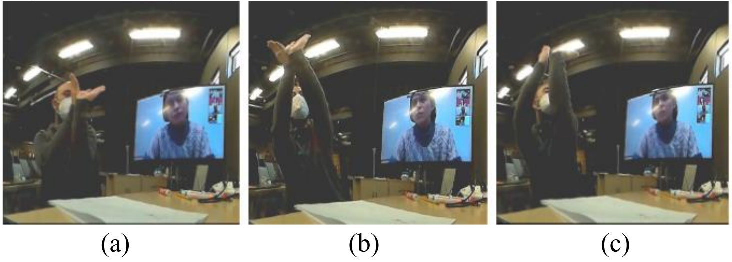
Figure 2. Miguel enacting a rotational gesture to represent a servo's rotation, with a start in (a) and end in (b)

(a) (b)

Body movement and utterances highlight nuances in robotic arm movement

Extending the traditional hand gestures, *augmented gestures* incorporated sound or shifts in body movement to emphasize the size of angle of rotation or the change in frame. For example, Miguel overlaid rotational gestures with sound, which increased in volume with the greater angle of rotation. He explained to Alex, *"The second part – this one - is rotated – [FROOMPH] in comparison."* To underscore the magnitude of the angle of rotation for the second servo, in relation to the angle of rotation for the first servo, Miguel augmented the rotational hand gesture with sound where varying volume levels correlated with the angle's magnitude. In the activity, it was especially important to understand servo rotation as a relative rotation with respect to a specific frame and we found that shifts in full body movement helped articulate the change in frame. To trace the movement of the first, second, and third servo, Miguel used rotational gestures, augmented with shifts in head and body movement and extending the gestures in space to capture the change in frame. When modeling the rotation of the second servo, we observed Miguel turn his body in the direction specified by the new frame (see Figure 3b) and yet again, when modeling the rotation of the third servo, with respect to another, new frame (see Figure 3c). Involvement of full body movement enhanced and grounded understanding of what is meant by a change in frame, otherwise difficult to grasp through hand gesture alone.

Figure 3. Miguel modeling the shift in frame for the first (a), second (b), and third servo rotation (c)



(a) (b) (c)

Discussion and Implications

Drawing on constructionism and body syntonicity, in this paper, we presented an application of matrix algebra within robotics and investigated how gesture, speech, and the body contribute to learning gains in the case of the focal dyad. Though we observed statistically significant learning gains in total scores, the small sample size could have produced larger effect sizes, which is a limitation of the study. Qualitative analysis of the focal dyad's interactions with the robotic arm showed that gesture and the body complemented speech by providing a visualization of what is otherwise abstract and difficult to articulate through language alone (e.g., z-axis, servo rotation). Collectively, this study contributes to the literature on learners' unprompted use of gesture and the body to ground understanding in a specific domain (matrix algebra in context), as anchored in the physical "object-to-think-with". The paper has implications for the design of learning environments that use directed gestures to support specific learning outcomes as learners negotiate more challenging and abstract concepts in mathematics.

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Acknowledgments

This work was supported by a grant from the National Science Foundation (#2100401).