FEASIBILITY/FLEXIBILITY-BASED OPTIMIZATION FOR PROCESS DESIGN AND OPERATIONS

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Abstract

In this paper, we attempt to provide a brief overview of methods and approaches that deal with the evaluation of feasibility/flexibility and how these concepts can be used to optimize process design or process operations. We focus on the description of process feasibility and the feasibility-based optimization problem as a way to efficiently incorporate multiple constraints and avoid exploring infeasible space. The ideas of process flexibility and robust optimization are also highlighted to illustrate how to treat uncertainty within process design and operations. Applications on pharmaceutical design and process scheduling problems are used to provide context in the utilization of the presented work.

Keywords

Feasibility, Flexibility, Feasibility-based Optimization, Robust Optimization.

Introduction

Feasibility is a fundamental concept in mathematical optimization, which generally uses equality and inequality constraints to define a set that contains all feasible solutions, also referred to as the feasible region. When optimizing the design and operation of chemical processes, an accurate description of the feasible region is of crucial importance due to strict safety considerations. This has led to the development of advanced feasibility analysis methods with important applications in, for example, chemical and pharmaceutical manufacturing. Surrogate-based feasibility analysis methods can also be incorporated into simulation-based optimization, providing an efficient approach to optimizing over expensive black-box models.

A concept closely related to feasibility is flexibility, which broadly refers to the ability of a process to handle varying process conditions, disturbances, and uncertainty. Flexibility is becoming increasingly important in the process industries as the production plants use more intermittent renewable resources, are subject to stricter safety and environmental requirements, and face higher levels of uncertainty due to various factors, including unexpected extreme weather events and supply chain disruptions. Flexibility analysis approaches have been proposed to quantify the level of operational flexibility of a given process design. The resulting metrics generally represent the size of the set of feasible parameters of interest; as such, feasibility analysis plays a key role in flexibility analysis. The flexibility concept can be further extended to address optimization problems with given flexibility requirements. Flexibility analysis problems can also be formulated as robust optimization problems, establishing a connection to robust optimization, a research area that has gained tremendous popularity in recent years, and allowing us to leverage methods developed in that area. This relationship has only been formally recognized more recently, but it is probably not a surprise as one can view robustness as the “pessimistic” interpretation of flexibility.

In this work, we present an overview of feasibility analysis, feasibility-based optimization, flexibility analysis, and its relationship to robust optimization, with a focus on applications in process design and operations. We outline the major concepts as well as current capabilities of existing methods, and provide a brief perspective on future opportunities in these research areas.

Feasible Region Evaluation

A feasible region describes the design space within which a process can satisfy all production, operating, safety, and quality constraints. The feasibility function is used to describe whether for a fixed value of an uncertain parameter, a process can be feasible by adjusting the control

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variables. The mathematical expression for the feasibility function ($\psi(d,\theta)$) is shown in Eq. (1), where d represents the set of design variables, θ represents the set of uncertain parameters, z represents the control variables, and g_j represents a set of J constraints (Grossmann et al., 2014).

$$\psi(d,\theta) = \min_{z} \max_{j \in J} g_j(d,z,\theta) \tag{1}$$

The purpose of feasibility analysis is to identify the feasible region where $\psi(d,\theta) \leq 0$. The boundary between feasible and infeasible regions is indicated by $\psi(d,\theta) = 0$. The feasible region of an example problem is shown in Figure 1, where the green region indicates feasible operation.

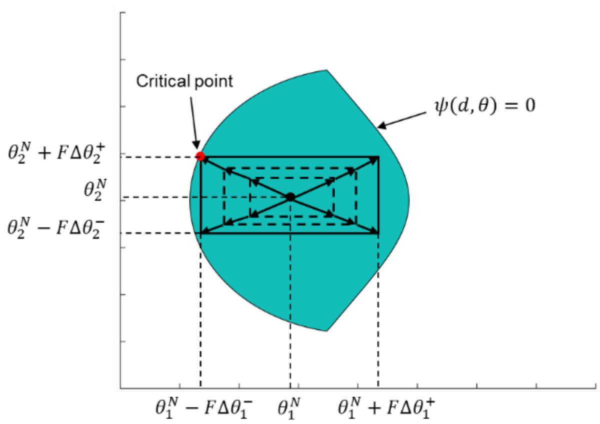

Figure 1. Illustration for feasibility and flexibility of an example problem

Different approaches have been developed to evaluate the design space, including geometry-based and surrogate-based methods. For geometry-based methods, Goyal and Ierapetritou (2004) developed a simplicial approximation approach, which is not applicable for nonconvex feasible regions. Based on this idea, Ipsita Banerjee and Ierapetritou (2005) proposed a surface reconstruction approach that can be extended to disjoint and nonconvex feasible regions. The limitation of geometry-based methods is that they require a large number of function evaluations, which is not applicable for problems involving computationally expensive models. Surrogate-based methods are developed to fit an inexpensive surrogate model to the feasibility function and use this surrogate to predict the feasible region. The sample points used to build the surrogate model need to be carefully chosen to reduce sampling cost, which motivates the use of adaptive sampling strategies to identify the samples that provide the most important information regarding the feasible region boundary. Different types of surrogate models and adaptive sampling strategies have been investigated, which are summarized in Table 1.

Table 1. Surrogate models and adaptive sampling approaches for feasibility analysis

Deterministic feasibility

Surrogate models	Adaptive sampling approaches	Ref.
High dimensional model representation (HDMR)	-	Ipsita Banerjee et al. (2010)
Gaussian process (kriging)	EI_{feas}	Boukouvala and Ierapetritou (2014)
Radial basis function (RBF)	EI_{feas}	Wang and Ierapetritou (2017)
Support vector machine (SVM)	Probability of feasibility	Basudhar et al. (2012)
Artificial neural network (ANN)	EI_{feas}	Metta et al. (2020)
Decision tree	-	Dias and Ierapetritou (2019)

Stochastic feasibility

Surrogate models	Adaptive sampling approaches	Ref.
High dimensional model representation (HDMR)	Filter using methodological constraints	Kucherenko et al. (2020)
Stochastic kriging	EI_{feas}, AEI_{feas}, EQI_{feas}	Wang and Ierapetritou (2018b)

Dynamic feasibility

Surrogate models	Adaptive sampling approaches	Ref.
Gaussian process (kriging)	EI_{feas}	Rogers and Ierapetritou (2015)

There have been many applications of the feasible region evaluation approaches, for example, the feasible region of solid-based continuous pharmaceutical manufacturing processes (Wang, Escotet-Espinoza, & Ierapetritou, 2017), continuous chromatography (Ding & Ierapetritou, 2021), and integrated planning and scheduling problems (Badejo & Ierapetritou, 2022; Dias & Ierapetritou, 2019). As an illustration, two plots showing the feasible operation conditions for a direct compaction (DC) pharmaceutical manufacturing process are shown in Figure 2 due to limited space. Figure 2A indicates that the value of fill depth in the tablet press unit needs to be carefully maintained to assure product quality whereas plot 2B shows that the whole input space for active pharmaceutical ingredient (API) flowrate and blender blade speed is feasible.

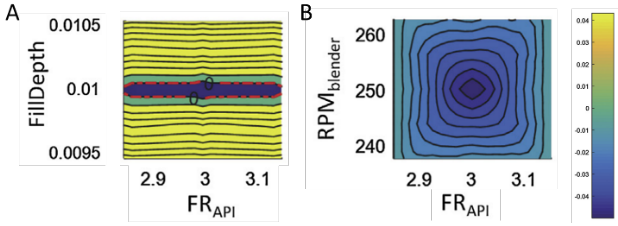

Figure 2. Feasible region of the DC line flowsheet model (Wang, Escotet-Espinoza, & Ierapetritou, 2017)

Feasibility-Based Optimization

Simulation-based optimization has been widely studied in the field of operations research for the optimization of complex systems. A general problem formulation can be represented by (P1), where $f(x)$ represents the objective function which depends on an expensive simulation model, $x \in R^K$ represents a set of decision variables, x_k^{lb} and x_k^{ub} form the lower and upper bounds for the decision variables, $g_i(x)$ are a set of I constraints that are also dependent on the simulation, and there may be J constraints that are analytically available.

$$\begin{aligned} &min\ f(x) && \text{(P1)} \\ &s.t.\ \ g_i(x) \leq 0, i \in \{1,2,\dots,I\} \\ &\qquad g_j(x) \leq 0, j \in \{1,2,\dots,J\} \\ &\qquad x_k^{lb} \leq x_k \leq x_k^{ub}, k \in \{1,2,\dots,K\} \end{aligned}$$

The solution of simulation-based optimization has many challenges. First, the simulation is only available as a black box for the evaluation of objective function and constraints. Second, many simulations are computationally expensive to run, limiting the number of function evaluations that can be performed in search of the optimal solution. Third, the derivative information is usually unavailable or hard to estimate due to the computational burden and output noise. Thus, it becomes challenging to utilize the conventional optimization approaches such as derivative-based and random search methods. Surrogate models have been proposed to approximate the expensive function evaluations and facilitate optimization (Bhosekar & Ierapetritou, 2018). The surrogate-based optimization framework has also been extended to feasibility analysis, as mentioned in the feasible region evaluation section.

In this section, we discuss simulation-based optimization approaches, where the concept of feasibility is integrated as black-box constraints. This strategy is shown to be beneficial as it does not require any assumptions regarding the form of the underlying feasible region (Boukouvala & Ierapetritou, 2014). However, the method depends on the collected samples, which should be placed at regions, according to certain infill criteria, that provide maximum information regarding the feasible region boundary. This needs to be balanced with the sampling requirement for optimization, in which case samples are needed to improve the objective function towards the optimal solution.

The first type of strategy is to consider both feasibility and optimization aspects together in one infill criterion. There have been both unconstrained and constrained formulations for the infill criteria to extend the efficient global optimization (EGO) framework (Jones et al., 1998). Bagheri et al. (2017) surveyed the existing constraint handling methods for EGO and modified the unconstrained method by introducing a newly defined probability of feasibility.

The second type of strategy is to design separate stages that focus on feasibility and optimization, respectively. Basudhar et al. (2012) used the first stage to drive the optimization based on expected improvement (EI) function and probability of feasibility indicated by probabilistic SVM models, and the second stage to refine the constraint boundary approximation by selecting samples in sparse regions with a high probability of misclassification. Boukouvala and Ierapetritou (2014) proposed a novel expected improvement function for locating feasibility boundaries in the feasibility stage, followed by an optional global search stage and trust region-based local search stage.

The one-stage approach is expected to be more economical in terms of sampling cost as every sample is added with the consideration of both aspects. However, when the original function is complex, for example, when feasibility function is hard to approximate with a small number of samples, it will be beneficial to focus on feasibility refinement first and then perform optimization within the feasible space. Therefore there is a trade-off between sequential and simultaneous considerations of the two objectives depending on the nature of the problem.

Note that both approaches can be used to address the integration of uncertainty in the design and process operations problems and have found different applications. Wang and Ierapetritou (2018a) proposed a feasibility-enhanced EI function that considered objective and feasibility simultaneously for a stochastic direct compaction pharmaceutical manufacturing process. Wang, Escotet-Espinoza, Singh, et al. (2017) used penalized feasibility EI function and penalized EI function for the feasibility and optimization stage, respectively, for a deterministic pharmaceutical manufacturing process.

Flexibility Analysis and Robust Optimization

In flexibility analysis, the concept of the feasibility function is further extended to derive a metric that quantifies the degree of flexibility of a given design d. The flexibility index, denoted by F, is defined as follows (Swaney & Grossmann, 1985a):

$$F(d) = \max_{\delta \in \mathbb{R}_+} \left\{ \delta : \max_{\theta \in T(\delta)} \psi(d,\theta) \leq 0 \right\}, \tag{2}$$

where $T(\delta)$, which is a function of a nonnegative scalar δ, denotes the set of all allowed realizations of the uncertain parameters θ. Traditionally, $T(\delta)$ is assumed to be a hyperbox defined as

$$T(\delta) = \{ \theta: \theta^N - \delta\Delta\theta^- \leq \theta \leq \theta^N + \delta\Delta\theta^+\}, \tag{3}$$

where θ^N is a nominal point, and $\Delta\theta^-$ and $\Delta\theta^+$ are incremental negative and positive deviations from θ^N, respectively. The flexibility index problem is to find $F(d)$, i.e. the largest δ such that the design d is feasible for all $\theta \in T(\delta)$. An example of the largest possible $T(\delta)$ is shown in Figure 1 as the rectangle inscribed in the projection of the feasible region onto the θ-space; it illustrates how the flexibility index is representative of the degree of operational flexibility of the given design. Earlier works have proposed vertex-exploration (Swaney & Grossmann, 1985b) and active-set (Grossmann & Floudas, 1987) strategies to solve the flexibility index problem.

There is a close relationship between flexibility analysis and robust optimization (Ben-Tal et al., 2009), which was independently developed in the operations research community. This relationship was first formally established for linear optimization problems in (Zhang, Grossmann, et al., 2016). It turns out that the flexibility index problem can be formulated as a two-stage robust optimization problem, where $T(\delta)$ represents the so-called uncertainty set. Applying modern robust optimization techniques, the flexibility index problem can often be solved more efficiently than using traditional approaches. Recently, alternative definitions of the flexibility index based on uncertainty sets with shapes other than a hyperbox have been proposed (Pulsipher et al., 2019), many of which are drawn from the robust optimization literature. Defining the feasibility space using alternative shapes and data-based ideas is also described in the Feasibility section. Also, note that the robust optimization formulation of the flexibility index problem involves endogenous uncertainty (Lappas & Gounaris, 2018; Zhang & Feng, 2020) since the uncertainty set $T(\delta)$ depends on the decision variable δ. While Zhang et al. (2016) show that the problem can be reformulated with a fixed uncertainty set if $T(\delta)$ is a hyperbox, more involved solution methods are needed for more complex uncertainty sets.

The feasibility analysis and flexibility index problems are solved to investigate a given design. Often, we also wish to optimize the design itself subject to some flexibility requirements. Such a design problem with flexibility constraints was first formulated by Halemane and Grossmann (1983) as follows:

$$\begin{aligned} \min_{d,\bar{z}} \quad & f(d) + \textstyle\sum_{s\in S} \varphi_s \, \bar{f}(d, \bar{z}_s) \\ \text{s.t.} \quad & g_j(d, \bar{z}_s, \bar{\theta}_s) \leq 0 \quad \forall\, j \in J, s \in S \\ & \max_{\theta\in T} \min_{z} \max_{j\in J} g_j(d, z, \theta) \leq 0, \end{aligned} \tag{P2}$$

where the objective is to minimize the expected cost approximated using a set of scenarios S. Each scenario $s \in S$ is defined by a specific realization of the uncertainty $\bar{\theta}_s$ and the corresponding probability φ_s. Apart from the constraints for each scenario, the additional flexibility constraints further enforce feasibility for all θ over a given uncertainty set T. This problem can also be naturally formulated as a two-stage robust optimization problem. Halemane and Grossmann (1983) developed a solution algorithm that iterates between a master problem in which the flexibility constraints are replaced by constraints over a finite set of critical points C, i.e.

$$g_j(d, z_c, \theta_c) \leq 0 \quad \forall\, j \in J, c \in C, \tag{4}$$

and subproblems that check whether the design proposed by the master problem satisfies the original flexibility constraints and, if it is not yet feasible, generates new critical points to be added to C. Interestingly, this is essentially the same approach as the column-and-constraint generation algorithm proposed for two-stage robust optimization three decades later (Zeng & Zhao, 2013). Note that design problems of the form (P2) can also be solved using feasibility-based optimization.

Flexibility analysis gives rise to two-stage robust optimization problems where the control variables z are the second-stage decisions. In recent years, there has also been a growing interest, including in process systems engineering (Lappas & Gounaris, 2016; Shang & You, 2019; Zhang, Morari, et al., 2016), in multistage robust optimization that can consider problems in which uncertainty is realized and recourse decisions can be taken at multiple time points, as it is often the case in planning, scheduling, and control. Here, the decision rule approach (Georghiou et al., 2018) has proven to be very effective, where the recourse variables are explicitly stated as functions of the uncertain parameters. For example, the following decision rule defines the recourse variables at time t, z_t, as an affine function of all uncertain parameters realized up to time t, i.e. θ_k where $k \leq t$:

$$z_t = p_t + \textstyle\sum_{k=1}^{t} Q_{tk}\theta_k. \tag{5}$$

Such a decision rule inherently satisfies nonanticipativity and can be applied to problems with arbitrarily many stages. Importantly, the recourse decisions are now specified by the variables p and Q, which can be chosen before the actual realization of the uncertainty. As such, all methods for static (i.e. single-stage) robust optimization can be applied to solve decision-rule-based multistage robust optimization problems. Note that in general, decision rules only approximate the fully adjustable recourse decisions; however, they are highly effective as they typically lead to solutions that are very close to the true optimum while being much more tractable.

Figure 3 shows some results from a multistage robust optimization problem that considers the scheduling of a cryogenic air separation plant over a one-week time horizon with an hourly time discretization (Zhang, Morari, et al., 2016). The plant operates under time-sensitive electricity prices and provides an interruptible load to the power grid; however, it is uncertain when and how much load reduction will be requested by the grid operator. One must decide at the beginning of the scheduling horizon how much interruptible load the plant will provide. Then, in each time period, the plant operation can be adjusted depending on the realized load reduction request. Therefore, recourse can be taken in every time period, resulting in a problem with 169 stages. Affine decision rules were applied to solve this problem, and Figure 3 shows the target liquid oxygen flows and inventory profile, where "target" refers to the case in which no load reduction is ever requested during the scheduling horizon. In addition, the green columns in Figure 3 represent the cumulative recourse actions in terms of changes in production and purchase rates. Negative production recourse indicates time periods in which interruptible load is provided. One can see that most of the lost production is made up by increasing production after load reduction (positive production recourse).

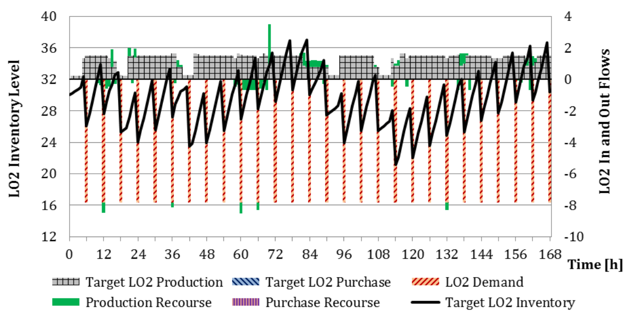

Figure 3. Target and recourse liquid oxygen flows, and target inventory profile. Reproduced from (Zhang, Morari, et al., 2016).

Flexibility analysis and robust optimization remain very active areas of research. Recent efforts focus on, for example, new variants of the flexibility index problem (Ochoa & Grossmann, 2020; Zhao et al., 2021), mixed-integer recourse (Feng et al., 2021; Nasab & Li, 2021), and endogenous uncertainty (Zhang & Feng, 2020).

Conclusions

The concepts of process feasibility and flexibility are highlighted, and ideas of evaluating and integrating those within process design optimization and process operations are described. Some applications are outlined on how to efficiently utilize those methods and effectively address uncertainty within process design optimization.

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References

Badejo, O., & Ierapetritou, M. (2022). Integrating tactical planning, operational planning and scheduling using data-driven feasibility analysis. *Computers & Chemical Engineering, 161*. doi:10.1016/j.compchemeng.2022.107759

Bagheri, S., Konen, W., Allmendinger, R., Branke, J., Deb, K., Fieldsend, J., . . . Sindhya, K. (2017). *Constraint handling in efficient global optimization*. Paper presented at the Proceedings of the Genetic and Evolutionary Computation Conference.

Banerjee, I., & Ierapetritou, M. (2005). Feasibility Evaluation of Nonconvex Systems Using Shape Reconstruction Techniques. *Ind. Eng. Chem. Res., 44*, 3638-3647.

Banerjee, I., Pal, S., & Maiti, S. (2010). Computationally efficient black-box modeling for feasibility analysis. *Computers & Chemical Engineering, 34*(9), 1515-1521. doi:10.1016/j.compchemeng.2010.02.016

Basudhar, A., Dribusch, C., Lacaze, S., & Missoum, S. (2012). Constrained efficient global optimization with support vector machines. *Structural and Multidisciplinary Optimization, 46*(2), 201-221. doi:10.1007/s00158-011-0745-5

Ben-Tal, A., El Ghaoui, L., & Nemirovski, A. (2009). *Robust optimization* (Vol. 28): Princeton University Press.

Bhosekar, A., & Ierapetritou, M. (2018). Advances in surrogate based modeling, feasibility analysis, and optimization: A review. *Computers & Chemical Engineering, 108*, 250-267. doi:10.1016/j.compchemeng.2017.09.017

Boukouvala, F., & Ierapetritou, M. G. (2014). Derivative-free optimization for expensive constrained problems using a novel expected improvement objective function. *AIChE Journal, 60*(7), 2462-2474. doi:10.1002/aic.14442

Dias, L. S., & Ierapetritou, M. G. (2019). Data-driven feasibility analysis for the integration of planning and scheduling problems. *Optimization and Engineering, 20*(4), 1029-1066. doi:10.1007/s11081-019-09459-w

Ding, C., & Ierapetritou, M. (2021). A novel framework of surrogate-based feasibility analysis for establishing design space of twin-column continuous chromatography. *International Journal of Pharmaceutics, 609*, 121161. doi:10.1016/j.ijpharm.2021.121161

Feng, W., Feng, Y., & Zhang, Q. (2021). Multistage robust mixed-integer optimization under endogenous uncertainty. *European Journal of Operational Research, 294*(2), 460-475. doi:10.1016/j.ejor.2021.01.048

Georghiou, A., Kuhn, D., & Wiesemann, W. (2018). The decision rule approach to optimization under uncertainty: methodology and applications. *Computational Management Science, 16*(4), 545-576. doi:10.1007/s10287-018-0338-5

Goyal, V., & Ierapetritou, M. (2004). Determination of Operability Limits Using Simplicial Approximation. *AIChE Journal, 48*(12), 2902-2909.

Grossmann, I. E., Calfa, B. A., & Garcia-Herreros, P. (2014). Evolution of concepts and models for quantifying resiliency and flexibility of chemical processes. *Computers & Chemical Engineering, 70*, 22-34. doi:10.1016/j.compchemeng.2013.12.013

Grossmann, I. E., & Floudas, C. A. (1987). Active constraint strategy for flexibility analysis in chemical processes. *Computers & Chemical Engineering, 11*(6), 675-693.

Halemane, K. P., & Grossmann, I. E. (1983). Optimal process design under uncertainty. *AIChE Journal, 29*(3), 425-433.

Jones, D. R., Schonlau, M., & Welch, W. J. (1998). Efficient Global Optimization of Expensive Black-Box Functions. *Journal of Global Optimization, 13*(4), 455-492. doi:10.1023/A:1008306431147

Kucherenko, S., Giamalakis, D., Shah, N., & García-Muñoz, S. (2020). Computationally efficient identification of probabilistic design spaces through application of metamodeling and adaptive sampling. *Computers & Chemical Engineering, 132*. doi:10.1016/j.compchemeng.2019.106608

Lappas, N. H., & Gounaris, C. E. (2016). Multi‐stage adjustable robust optimization for process scheduling under uncertainty. *AIChE Journal, 62*(5), 1646-1667. doi:10.1002/aic.15183

Lappas, N. H., & Gounaris, C. E. (2018). Robust optimization for decision-making under endogenous uncertainty. *Computers & Chemical Engineering, 111*, 252-266. doi:10.1016/j.compchemeng.2018.01.006

Metta, N., Ramachandran, R., & Ierapetritou, M. (2020). A novel adaptive sampling based methodology for feasible region identification of compute intensive models using artificial neural network. *AIChE Journal, 67*(2). doi:10.1002/aic.17095

Nasab, F. M., & Li, Z. (2021). Multistage adaptive stochastic mixed integer optimization through piecewise decision rule approximation. *Computers & Chemical Engineering, 149*, 107286. doi:10.1016/j.compchemeng.2021.107286

Ochoa, M. P., & Grossmann, I. E. (2020). Novel MINLP formulations for flexibility analysis for measured and unmeasured uncertain parameters. *Computers & Chemical Engineering, 135*. doi:10.1016/j.compchemeng.2020.106727

Pulsipher, J. L., Rios, D., & Zavala, V. M. (2019). A Computational Framework for Quantifying and Analyzing System Flexibility. *Computers & Chemical Engineering, 126*, 342-355.

Rogers, A., & Ierapetritou, M. (2015). Feasibility and flexibility analysis of black-box processes Part 1: Surrogate-based feasibility analysis. *Chemical Engineering Science, 137*, 986-1004. doi:10.1016/j.ces.2015.06.014

Shang, C., & You, F. (2019). A data-driven robust optimization approach to scenario-based stochastic model predictive control. *Journal of Process Control, 75*, 24-39. doi:10.1016/j.jprocont.2018.12.013

Swaney, R. E., & Grossmann, I. E. (1985a). An index for operational flexibility in chemical process design. Part I: Formulation and theory. *AIChE Journal, 31*(4), 621-630.

Swaney, R. E., & Grossmann, I. E. (1985b). An index for operational flexibility in chemical process design. Part II: Computational algorithms. *AIChE Journal, 31*(4), 631-641.

Wang, Z., Escotet-Espinoza, M. S., & Ierapetritou, M. (2017). Process analysis and optimization of continuous pharmaceutical manufacturing using flowsheet models. *Computers & Chemical Engineering, 107*, 77-91. doi:10.1016/j.compchemeng.2017.02.030

Wang, Z., Escotet-Espinoza, S., Singh, R., & Ierapetritou, M. (2017). Surrogate-based Optimization for Pharmaceutical Manufacturing Processes. In *27th European Symposium on Computer Aided Process Engineering* (pp. 2797-2802).

Wang, Z., & Ierapetritou, M. (2017). A novel feasibility analysis method for black-box processes using a radial basis function adaptive sampling approach. *AIChE Journal, 63*(2), 532-550. doi:10.1002/aic.15362

Wang, Z., & Ierapetritou, M. (2018a). Constrained optimization of black-box stochastic systems using a novel feasibility enhanced Kriging-based method. *Computers & Chemical Engineering, 118*, 210-223. doi:10.1016/j.compchemeng.2018.07.016

Wang, Z., & Ierapetritou, M. (2018b). Surrogate-based feasibility analysis for black-box stochastic simulations with heteroscedastic noise. *Journal of Global Optimization, 71*(4), 957-985. doi:10.1007/s10898-018-0615-4

Zeng, B., & Zhao, L. (2013). Solving two-stage robust optimization problems using a column-and-constraint generation method. *Operations Research Letters, 41*(5), 457-461. doi:10.1016/j.orl.2013.05.003

Zhang, Q., & Feng, W. (2020). A unified framework for adjustable robust optimization with endogenous uncertainty. *AIChE Journal, 66*(12). doi:10.1002/aic.17047

Zhang, Q., Grossmann, I. E., & Lima, R. M. (2016). On the relation between flexibility analysis and robust optimization for linear systems. *AIChE Journal, 62*(9), 3109-3123. doi:10.1002/aic.15221

Zhang, Q., Morari, M. F., Grossmann, I. E., Sundaramoorthy, A., & Pinto, J. M. (2016). An adjustable robust optimization approach to scheduling of continuous industrial processes providing interruptible load. *Computers & Chemical Engineering, 86*, 106-119. doi:10.1016/j.compchemeng.2015.12.018

Zhao, F., Grossmann, I. E., García‐Muñoz, S., & Stamatis, S. D. (2021). Flexibility index of black‐box models with parameter uncertainty through derivative‐free optimization. *AIChE Journal, 67*(5). doi:10.1002/aic.17189