Spectral Clustering Aided User Grouping and Scheduling in Wideband MU-MIMO Systems

Chih-Ho Hsu, *Student Member, IEEE*, Carlos Feres, *Member, IEEE*, Zhi Ding, *Fellow, IEEE*
Department of Electrical and Computer Engineering
University of California, Davis, CA, USA
Email: {lchhsu, cjferes, zding}@ucdavis.edu

***Abstract*—Multiuser MIMO (MU-MIMO) technologies can help provide rapidly growing needs for high data rates in modern wireless networks. Co-channel interference (CCI) among users in the same resource-sharing group (RSG) presents a serious user scheduling challenge to achieve high overall MU-MIMO capacity. Since CCI is closely related to correlation among spatial user channels, it would be natural to schedule co-channel user groups with low inter-user channel correlation. Yet, establishing RSGs with low co-channel correlations for large user populations is an NP-hard problem. More practically, user scheduling for wideband channels exhibiting distinct channel characteristics in each frequency band remains an open question. In this work, we proposed a novel wideband user grouping and scheduling algorithm named SC-MS. The proposed SC-MS algorithm first leverages spectral clustering to obtain a preliminary set of user groups. Next, we apply a post-processing step to identify user cliques from the preliminary groups to further mitigate CCI. Our last step groups users into RSGs for scheduling such that the sum of user clique sizes across the multiple frequency bands is maximized. Simulation results demonstrate network performance gain over benchmark methods in terms of sum rate and fairness.**

***Index Terms*—Multiuser MIMO (MU-MIMO), maximal clique, spectral clustering, user grouping, wideband user scheduling.**

I. Introduction

Multiple-Input-Multiple-Output (MIMO) technologies, through which a base station (BS) with a large number of antennas serves multiple devices simultaneously in a shared channel, provide promising solutions to meet the increasing demand for high-speed data services and high spectrum efficiency in modern wireless systems [1]. By exploiting spatial diversity through multiple antennas, MIMO coverage delivers improved capacity and spectrum efficiency [2].

Multiuser MIMO (MU-MIMO) allows multiple users on the same time-frequency resource [3] served by a BS. One major obstacle of MU-MIMO lies in the co-channel interference (CCI) among co-channel users in the same resource-sharing group (RSG) despite spatial channel diversity. Specifically, the network capacity and spectrum efficiency depends on the severity of CCI among the multiple RSGs each occupying one of N frequency bands (channels). Since strong CCI corresponds to co-channels users with large spatial channel correlation [4], MU-MIMO capacity and spectrum efficiency can be significantly improved by scheduling users with low channel correlation to the same RSG. Thus, MU-MIMO performance critically relies on user scheduling algorithms [4].

Desirable scheduling algorithms should divide users into RSGs across all frequency bands such that within each RSG, users enjoy low channel correlation [5]. However, due to the combinatorial nature of the problem, the number of possible RSG choices grows exponentially with the number of users. Exhaustive search is prohibitively costly even for a moderate number of users [6]. Moreover, given the ultra-short transmission time interval in 5G and future wireless systems [1], it is urgent to develop low-complexity algorithms that yield good sub-optimum scheduling decisions dynamically in real-time.

To mitigate the complexity of finding decent RSGs, existing works have exploited user channel correlation in a heuristic or greedy manner. The authors of [7] designed a greedy algorithm to form RSGs by iteratively separating two users into different RSGs if their channel correlation is greater than a pre-set threshold. Another work [8] characterized interference degrees between users as the weight of edges in a graph and adopted a heuristic RSG formation by sequentially assigning users to RSGs with the minimum sum increase of graph weights.

However, heuristic methods tend to fall victim to local optimum and suffer globally. To overcome such drawbacks, several studies adopt unsupervised learning to extract common features of user channel correlations [9]. Instead of directly assigning users to RSGs based on heuristic criteria, the authors of [10], [11], [12] have adopted two-step strategies to form RSGs based on user channel correlations. Specifically, the first step identifies clusters of users with highly similar channels (i.e. channels with high correlations) by leveraging unsupervised clustering techniques such as the well-known K-means. The second step forms RSGs by greedily scheduling users from the same cluster into different RSGs. The motivation behind these two-step unsupervised strategies is that it is easier to cluster users with similar channels than to cluster users of dissimilar channels, e.g., users with low channel correlation. Although these strategies do not directly minimize user CCI within each RSG, they provide good performance guarantees by rejecting RSGs with significant CCI users.

Although clustering-based MU-MIMO scheduling schemes can effectively form RSGs with mild complexity, to our best knowledge, none of the existing methods considers wideband multi-channel systems. In fact, it is highly challenging to extend existing works to wideband multi-channel systems,

This material is based upon work supported by the National Science Foundation under Grants No.2029027 and No. 2002937.

such as the multi-subcarrier (multi-subband) physical layers of 4G-LTE and 5G-NR, particularly because practical channel characteristics are frequency-selective due to multi-path effects. Other traditional algorithms also do not apply to such problems. Hence, how to improve the sum rate and spectral efficiency of MU-MIMO systems by forming RSGs in consideration of user channel state information (CSI) in different subbands remains an open but vital question.

In this paper, we aim to address two joint and crucial aforementioned challenges: (1) forming RSGs by clustering users with dissimilar channels into the same RSG, and (2) forming RSGs in a multi-band (or multi-channel) system based on user CSI knowledge. Specifically, we tackle the first challenge by forming user graphs with edges representing channel dissimilarity, considering user grouping as a k-way normalized cut (NCut) problem in graph theory, and adopting spectral clustering (SC) to obtain a preliminary grouping decision. We then post-process results from SC to determine its maximal cliques, so as to further reduce the CCI. We tackle the second challenge by formulating the multi-band scheduling problem to maximize the sum size of user cliques in all subbands, and developing a heuristic procedure accordingly. Integrating these steps, we propose a novel SC-aided multi-band user grouping and scheduling (SC-MS) algorithm and demonstrate its efficacy through extensive numerical simulations.

Notations: Throughout this paper, we use small bold letters for vectors, capital bold letters for matrices and N-dimensional (ND) arrays, calligraphic capital letters to denote sets and graphs, and non-bold font for scalars and functions. We use $(\cdot)^{\mathsf{T}}$, $\overline{(\cdot)}$ and $(\cdot)^{\mathsf{H}}$ to denote transpose, conjugate and conjugate transpose, respectively. Finally, $\|\cdot\|$ denotes ℓ_2 norm.

II. System Model and Problem Formulation

We consider a single-cell MU-MIMO OFDM system where a BS equipped with Q antennas serves S single-antenna users. The system spectrum is equally divided into N subbands and each subband consists of M timeslots, which are shorter than the channel coherence time. A time-frequency resource block (RB)[1] is each combination of timeslot and subband, and hence there are NM RBs in total. We further denote RB(f,t) the RB of subband f and timeslot t, $\forall f \in \mathcal{N} = \{1,\dots,N\}$ and $\forall t \in \mathcal{M} = \{1,\dots,M\}$. Without loss of generality, the BS is assumed to have perfect knowledge of users' CSI. Let $\boldsymbol{H} = \{h_{i,q}^{f}\}_{i,q,f} \in \mathbb{C}^{Q\times S\times N}$ be the CSI 3D-array, where each $h_{i,q}^{f} \in \mathbb{C}$, $\forall i \in \mathcal{S} = \{1,\dots,S\}$, $\forall q \in \mathcal{Q} = \{1,\dots,Q\}$, $\forall f \in \mathcal{N}$ denotes the CSI between user i and BS's antenna q on subband f. Note that user CSIs can follow any arbitrary channel model. Here, we consider a general case, where CSIs are random and independent between users, subbands and BS antennas. We further assume quasi-static block fading channels, such that CSIs are constant during the M time slots, less than channel coherence time.

To represent the mapping between users and RBs, we introduce an indicator 3D-array $\boldsymbol{X} = \{x_i^{f,t}\}_{i,f,t}$, where each $x_i^{f,t} \in \{0,1\}, \forall i \in \mathcal{S}, \forall f \in \mathcal{N}, \forall t \in \mathcal{M}$ denotes whether user i is allocated to RB(f,t) or not. Also, each RB can be shared by multiple users and thus CCI exists among users allocated to the same RSG. Based on the above elaboration, the signal-to-interference-plus-noise ratio (SINR) of user i on RB(f,t) can be obtained by

[1]Note that our definition of RB may not match the PRB in LTE or 5G. In this work, RB is defined as a resource unit occupied by users in a RSG.

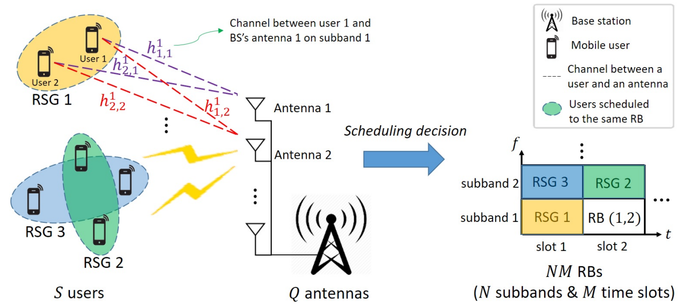

Fig. 1: Illustration of the system model and objective. Note that user grouping is decided based on users' channels rather than geographical locations.

$$\gamma_i^{f,t} = \frac{p_i\left|(\boldsymbol{h}_i^f)^{\mathsf{H}}\boldsymbol{z}_i^f\right|^2}{\sigma^2 + \sum\limits_{j\in\mathcal{S},j\neq i} x_j^{f,t}\cdot p_j\left|(\boldsymbol{h}_i^f)^{\mathsf{H}}\boldsymbol{z}_j^f\right|^2}, \tag{1}$$

where $\boldsymbol{h}_i^f = [h_{i,1}^f,\dots,h_{i,Q}^f]^{\mathsf{T}} \in \mathbb{C}^Q$ is the CSI vector of user i at subband f, p_i is the transmission power allocated to user i, σ^2 is the variance of additive white Gaussian noise (AWGN), and $\boldsymbol{z}_i^f$ is the beamforming precoder of user i at subband f, which can be selected as the zero-forcing (ZF) [13], weighted minimum mean squared error (MMSE) [14] or maximum ratio transmitter (MRT) [15]. Without loss of generality and for the sake of exposition, we use the MRT precoders so that

$$\boldsymbol{z}_i^f = \frac{\overline{\boldsymbol{h}_i^f}}{\|\boldsymbol{h}_i^f\|}, \forall i \in \mathcal{S}, f \in \mathcal{N}. \tag{2}$$

After applying (2) into (1), the SINR can be reformulated as

$$\gamma_i^{f,t} = \frac{p_i}{\frac{\sigma^2}{\|\boldsymbol{h}_i\|^2} + \sum\limits_{j\in\mathcal{S},j\neq i} x_j^{f,t}\cdot p_j\rho^2(\boldsymbol{h}_i^f,\boldsymbol{h}_j^f)}, \tag{3}$$

where $\rho(\boldsymbol{h}_i^f,\boldsymbol{h}_j^f)$ is the spatial correlation between $\boldsymbol{h}_i^f$ and $\boldsymbol{h}_j^f$:

$$\rho(\boldsymbol{h}_i^f,\boldsymbol{h}_j^f) = \frac{\left|(\boldsymbol{h}_i^f)^{\mathsf{H}}\boldsymbol{h}_j^f\right|}{\|\boldsymbol{h}_i^f\|\|\boldsymbol{h}_j^f\|}, \forall i \in \mathcal{S}, f \in \mathcal{N}. \tag{4}$$

Clearly, the CCI experienced by user i increases with the sum of squares of CSI correlations between user i and other users in the same RSG. Since we are interested in exploiting channel correlations of users at the scheduling problem and aim to develop simple solutions, we assume equal power allocation (i.e. $p_i = p_j, \forall i,j \in \mathcal{S}, i \neq j$).

As shown in Fig. 1, the goal of this paper is to design an user grouping and scheduling algorithm that allocates a given set of RBs to S users so that the sum rate of users is maximized, which is formulated as follows:

$$\begin{aligned} \max_{\boldsymbol{X}} & \sum_{f\in\mathcal{N}}\sum_{t\in\mathcal{M}}\sum_{i\in\mathcal{S}} x_i^{f,t} R_i^{f,t} \\ \text{s.t.} \quad & \text{C1}: x_i^{f,t} \in \{0,1\}, \quad \forall i, f, t, \\ & \text{C2}: \sum_{f\in\mathcal{N}}\sum_{t\in\mathcal{M}} x_i^{f,t} = 1, \quad \forall i, \end{aligned} \tag{5}$$

where $R_i^{f,t} = \log_2\left(1+\gamma_i^{f,t}\right)$ denotes the data rate of user i if allocated to RB(f,t) and constraint C2 requires all users to be assigned to exactly one RB and constraint. Note that (5) is a nonlinear integer programming problem, which is NP-hard [16]. To efficiently optimize (5), we treat sum rate maximization as forming NM RSGs by exploiting user CSI correlations such that the CCI within each RSG is minimized.

III. Spectral Clustering Aided Multi-band User Grouping and Scheduling Algorithm

In this section, we will present our proposed solution for problem (5). Specifically, in subsections III-A and III-B, we first address the single-band case of problem (5), where $N = 1$, by leveraging spectral clustering (SC) [17] and propose the SC-aided Single-band Scheduling (SC-SS) algorithm. In subsection III-C, we then extend SC-SS to multi-band systems, where $N \geq 2$, and further propose the SC-aided Multi-band Scheduling (SC-MS) algorithm. Throughout this section, the terms group, subgraph, and RSG are used interchangeably according to different contexts, all of which refer to a set of users allocated to a particular RB.

A. Preliminary User Grouping with Spectral Clustering

We start with the single-band user grouping problem, where the goal is to find M RSGs with low CCI. Intuitively, the pairwise relationship of CSI correlations between users in subband f can be described with an undirected weighted graph $\mathcal{G}^f = (\mathcal{S}, \mathcal{E})$, where $\mathcal{S}$ is the user set and $\mathcal{E}$ denotes the set of weighted edges describing *channel dissimilarity* of users. We define the weight of the edge connecting users i and j as:

$$w_{i,j}^f = \begin{cases} 1-\rho^2(\boldsymbol{h}_i^f, \boldsymbol{h}_j^f) & \rho(\boldsymbol{h}_i^f, \boldsymbol{h}_j^f) \leq \epsilon \\ 0 & \text{otherwise} \end{cases}, \tag{6}$$

where $\epsilon > 0$ serves as a dissimilarity threshold. Note that $w_{i,j}^f \neq 0$ implies the CSIs of users i and j are dissimilar (i.e. low correlation) in subband f and hence they can be assigned to the same RSG, whereas $w_{i,j}^f = 0$ implies the opposite. Although several candidate functions can be used to set the weight value, we adopt these threshold-based weights for simplicity [4]. Based on (6), we can construct the adjacency matrix $\boldsymbol{W}^f = \{w_{i,j}^f\}_{i,j} \in \mathbb{R}^{S\times S}$ of $\mathcal{G}^f$. We then define a diagonal degree matrix $\boldsymbol{D}^f = \{d_{i,j}^f\}_{i,j} \in \mathbb{R}^{S\times S}$ such that

$$d_{i,i}^f = \sum_{k\in\mathcal{S}, k\neq i} w_{i,k}^f \quad \text{and} \quad d_{i,j}^f = 0 \text{ for } i \neq j. \tag{7}$$

The normalized Laplacian matrix of $\mathcal{G}^f$ is given by

$$\boldsymbol{L}^f = (\boldsymbol{D}^f)^{-\frac{1}{2}}\left(\boldsymbol{D}^f - \boldsymbol{W}^f\right)(\boldsymbol{D}^f)^{-\frac{1}{2}}. \tag{8}$$

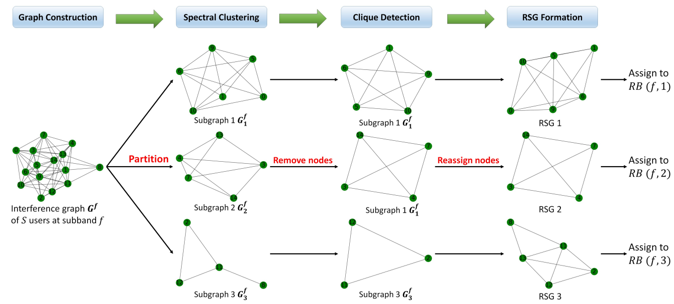

Fig. 2: Illustration of a sample run of our proposed SC-SS with $S = 15$, $Q = 32$, $N = 1$, $M = 3$. We depict each step and its corresponding outputs.

Based on this graph model, the single-band user grouping problem is analogous to partitioning $\mathcal{G}^f$ into M disjoint subgraphs $\mathcal{G}_1^f, \ldots, \mathcal{G}_M^f$ such that $\cup_{t\in\mathcal{M}}\mathcal{G}_t^f = \mathcal{G}^f$ and each subgraph is highly intra-connected. Moreover, this is related to the goal of the k-way NCut problem, which aims to minimize the sum of *cuts* of k partitions normalized by their *volume* [18], and can be formulated by:

$$\min \sum_{t=1}^{k} \frac{Cut(\mathcal{G}_t^f, \widetilde{\mathcal{G}}_t^f)}{Vol(\mathcal{G}_t^f)} = \sum_{t\in\mathcal{M}} \frac{\sum_{i\in\mathcal{G}_t^f, j\in\widetilde{\mathcal{G}}_t^f} w_{i,j}^f}{\sum_{i\in\mathcal{G}_t^f} d_{i,i}}, \tag{9}$$

where $\widetilde{\mathcal{G}}_t^f = \mathcal{G}^f \setminus \mathcal{G}_t^f$ is the complement of $\mathcal{G}_t^f$ in $\mathcal{G}^f$.

In the proposed SC-SS, we adopt spectral clustering [17], which approximates the NCut problem to eigenvalues of the graph Laplacian, to efficiently solves problem (9). The procedure of SC is summarized below:

1) Construct $\boldsymbol{W}^f$, $\boldsymbol{D}^f$ and $\boldsymbol{L}^f$ of the graph $\mathcal{G}^f$ based on user CSI correlation coefficient (4) in subband f.
2) Compute eigenvectors $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_M \in \mathbb{R}^S$ of $\boldsymbol{L}^f$ corresponding to its M smallest eigenvalues.
3) Cluster columns of $\boldsymbol{Y} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_M]^{\mathsf{T}}$ into M separate groups using a given clustering method, e.g. K-means. These groups, corresponding to disjoint M subgraphs $\mathcal{G}_t^f$, form the preliminary RSGs.

The result of SC will then undergo a post-processing procedure described in the next subsection.

B. Post-Processing via Cliques Detection and Reassignment

Although SC is a good solution for community detection, the resulting M preliminary RSGs can be further refined for problem (5). The reason is that these RSGs might still contain user pairs that exhibit high correlation. In other words, disjoint subgraphs obtained with SC are not necessarily *cliques*.

Definition 1. *An undirected weighted graph is a* clique *if it is complete, or fully connected. That is, a graph $\mathcal{G}$, $|\mathcal{G}| \geq 1$, is a clique if all its edge weights $w_{i,j} \neq 0, \forall i, j \in \mathcal{G}$ with $i \neq j$.*

Ideally, we aim to find user cliques so that CSI correlation of *any* user pair in an RSG is low. Hence, our proposed second step in SC-SS is to detect maximal user cliques hidden in the subgraphs $\mathcal{G}_t^f$ obtained from SC. Finding the maximal clique in a graph is a NP-hard problem [19]. We can, however, adopt a heuristic solution. Based on the outcomes of maximal user clique detection, we design a node reassignment scheme to form RSGs and obtain the final scheduling decision. The details are given as follows.

1) Clique Detection: First, we compute the M degree matrices $\boldsymbol{D}_t^f$ of subgraphs $\mathcal{G}_t^f$ $\forall t \in \mathcal{M}$ obtained from SC. For each subgraph, we iteratively remove the node with the lowest degree until the remaining nodes form a clique. The reasoning is that if the degree of a node is low, it is less likely to form a clique than other nodes. We use a buffer set $\mathcal{B}$ to store all nodes removed from the subgraphs. We repeat this process until all M subgraphs become cliques.

2) Node Reassignment: Next, we sequentially remove an user ℓ from the buffer $\mathcal{B}$ and add it to the subgraph t that has the minimum sum of squared CSI correlations between user ℓ and users in $\mathcal{G}_t^f$, i.e. $t = \arg\min_t \sum_{j \in \mathcal{G}_t^f} \rho^2(\boldsymbol{h}_i^f, \boldsymbol{h}_j^f)$, suggesting this RSG has minimum CCI for user ℓ. We repeat this process until $\mathcal{B}$ is empty. The resulting M subgraphs correspond to the final M RSGs, which are mapped to each RB. In this way, each RB contains users from a distinct RSG. These RSG users enjoy low CCI in that RB. Fig. 2 depicts the procedure of SC-SS with an example.

Thus far, we have used index f to describe SC-SS over a single subband f. In a multi-band case where $N \geq 2$, we will have N independent graphs that have the same set of nodes but with different edges and weights. Therefore, we will need further considerations to determine the multi-band scheduling decision, which we present below.

Algorithm 1: SC-aided Multiband Scheduling, SC-MS

Input: CSI matrix $\boldsymbol{H} = \{h_{i,q}^f\}_{i=1,q=1,f=1}^{S,Q,N}$
Output: Scheduling decision $\boldsymbol{X} = \{x_i^{f,t}\}_{i=1,f=1,t=1}^{S,N,M}$
1: Initialize $\mathcal{U} \leftarrow \{1, \ldots, S\}$, $x_i^{f,t} \leftarrow 0$, $\forall i, f, t$
2: **for** $f = 1$ **to** $f = N$ **do**
3: Construct interference graph $\mathcal{G}^f$ of subband f for remaining users in $\mathcal{U}$
4: Partition $\mathcal{G}^f$ into $(N-f)M$ subgraphs by SC
5: **for** $t = 1$ **to** $(N-f)M$ **do**
6: **while** $\mathcal{G}_t^f$ *is not a clique* **do**
7: Remove node $i = \underset{k \in \mathcal{G}_t^f}{\arg\min}\, d_{k,k}$ from $\mathcal{G}_t^f$
8: Sort $\mathcal{G}_1^f, \mathcal{G}_2^f, ..., \mathcal{G}_{(N-f)M}^f$ by their cardinality $|\mathcal{G}_t^f|$
9: **for** $t = 1$ **to** M **do**
10: $\forall i \in \mathcal{G}_t^f$, remove i from $\mathcal{U}$ and set $x_i^{f,t} \leftarrow 1$
11: **while** $\mathcal{U} \neq \emptyset$ **do**
12: Randomly remove a user ℓ from $\mathcal{U}$
13: $x_\ell^{f,t} \leftarrow 1$, where $(f,t) = \underset{f,t}{\arg\min} \sum_{j \in \mathcal{G}_t^f} \rho^2(\boldsymbol{h}_\ell^f, \boldsymbol{h}_j^f)$

C. Extend to Multi-band Systems and Complexity Analysis

We have assumed a frequency-selective wideband channel consisting of N subbands, and independent CSI correlations of users in each subband. Hence, a straightforward multi-band scheduling solution is to construct the N interference graphs $\mathcal{G}^f, \forall f \in \mathcal{N}$ and perform SC with post-processing independently in each graph. However, under constraint C2, RBs in different subbands should be disjoint user sets, and thus we cannot merely treat each subband independently. Hence, we now consider the joint optimization problem

$$\begin{aligned} &\max \sum_{f \in \mathcal{N}} \sum_{t \in \mathcal{M}} |\mathcal{G}_t^f| \,, \\ &\text{s.t.} \quad \text{C1: } w_{i,j}^f \neq 0 \,, \qquad \forall f \in \mathcal{N}, \forall i,j \in \mathcal{G}_t^f \,, i \neq j \,, \end{aligned} \tag{10}$$

where $|\mathcal{G}_t^f|$ denotes the cardinality of graph $\mathcal{G}_t^f$ and thus (10) aims to maximize the sum of user clique sizes over all RSGs obtained *before* node reassignment, over all subbands. By optimizing (10), we enforce a global perspective of the scheduling problem since we are jointly considering user allocation in multiple frequency bands and the CCI of RSGs within each subband. By generalizing the previously discussed SC-SS algorithm, we develop the proposed SC-MS algorithm to effectively solve multi-band user scheduling problem (5).

SC-MS starts by initializing a user pool $\mathcal{U}$ with all S users, and setting indicators $x_i^{f,t}$ to 0. Next, we sequentially select a subband $f \in \mathcal{N}$ and construct its interference graph $\mathcal{G}^f$ based on the CSI correlation of users in $\mathcal{U}$ at subband f. Thereafter, we partition $\mathcal{G}^f$ into $(N-f)M$ subgraphs by SC, where $N-f$ is the number of subbands that have not been selected yet. The rationale behind this is to balance sizes of the subgraphs. If we instead partition $\mathcal{G}^f$ into M subgraphs, the average size of subgraphs in subband f would be greater than subband $f+1$ which is selected after f, leading to size imbalance.

SC-MS then performs the single-band heuristic to extract maximal cliques of $(N-f)M$ subgraphs by iteratively removing nodes from them. Since our goal is to maximize the sum clique sizes, here we select the M subgraphs with the highest clique size as the M RSGs in subband f, and abandon the remaining subgraphs of subband f. We next set the indicators $x_i^{f,t}$ of users within these M cliques to 1 accordingly and remove them from $\mathcal{U}$. We repeat this process until all subbands of the wideband spectrum have been traversed. By then, we will have NM clique subgraphs corresponding to NM RSGs.

Note that at this point, there may still exist residual users in $\mathcal{U}$ that have not been assigned to any RB. For this reason, the final step of SC-MS is to allocate these remaining users to the existing subgraphs following the same idea proposed in the SC-SS. In short, we iteratively remove a random user ℓ from $\mathcal{U}$ and assign it to the subgraph that has the minimum sum of squared CSI correlations between user ℓ and users already in the RSG. The scheduling indicator of user ℓ is then updated by $x_\ell^{f,t} = 1$. This step finishes when $\mathcal{U}$ is empty, terminating the algorithm and yielding $\boldsymbol{X}$, the final scheduling decision. The overall workflow of SC-MS is detailed in Alg. 1.

We now analyze the computational complexity of SC-MS in terms of S, M and N. At line 3, constructing adjacent matrix for graph $\mathcal{G}^f$ has cost $\mathcal{O}(S^2)$. At line 4, performing SC has cost $\mathcal{O}(S^3)$ in general [20]. For line 6, checking the degree of all nodes in $\mathcal{G}_t^f$ to determine if it is a clique has cost $\mathcal{O}(S^2)$. Afterward in line 7, removing the node and updating the graph has cost $\mathcal{O}(S)$. Thus, the total cost for lines 5-7 is $\mathcal{O}(NMS^2)$. For line 8, sorting has cost $\mathcal{O}(S \log S)$. For lines 9-10, removing users from $\mathcal{U}$ and updating their indicators has cost $\mathcal{O}(MS)$. Also, allocating remaining users in $\mathcal{U}$ for line 11-13 has cost $\mathcal{O}(NMS^2)$. Thus, the worst-case overall complexity is $\mathcal{O}(NS^3 + N^2MS^2)$, and for a general case where $NM \leq S$, the **total complexity** of SC-MS is $\mathcal{O}(NS^3)$.

IV. Numerical Experiments

We now evaluate and illustrate the performance of the proposed multi-band user grouping and scheduling algorithms in terms of sum rate and fairness via numerical simulations.

Unless otherwise stated, we consider one BS equipped with $Q = 8$ antennas serving $S = 200$ users. We consider a system of $N = 8$ subbands and $M = 6$ timeslots, resulting in 48 RBs. The CSI 3D-array $\boldsymbol{H} \in \mathbb{C}^{Q \times S \times N}$ is generated randomly, considering both shadowing effect and Rayleigh fading, with elements independent from each other. The channel gains are normalized and we set noise power to $\sigma^2 = 0.01$ (i.e., SNR= 20dB). We use a correlation threshold $\epsilon = 0.07$. We average results over 30 simulations. We compare our proposed SC-MS algorithm with other multi-band scheduling approaches.

1) Heuristic algorithm (HRS) adopted from [8]: This algorithm starts with NM empty RBs. In each iteration, HRS sequentially adds an user i to RB j such that the CCI between user i and existing users in RB j is the minimum among NM RBs, i.e. $j = \arg\min_j \sum_{k \in RB(j)} \rho_{i,k}^2$. Thus, HRS has a lower bound of $(1 - \frac{1}{k})$ times the optimal value for a general MAX k-CUT problem [21], where k is the number of clusters to assign (48 in our setting). **HRS complexity**: $\mathcal{O}(NMS^2)$.

2) Hungarian algorithm (HNG) [22]: This scheme treats the mapping between users and RBs as a bipartite matching problem. It initializes with NM empty RBs. In each iteration, HNG constructs a cost matrix $\boldsymbol{C} = \{c_{i,j}\}_{i,j} \in \mathbb{R}^{S \times (NM)}$ where $c_{i,j}, \forall i \in \mathcal{S}, \forall j \in \{1, \ldots, NM\}$ is the CCI between user i and existing users in RB j. Based on $\boldsymbol{C}$, it then leverages the Hungarian algorithm to *optimally* assign NM users to RBs, minimizing the sum of matching costs. The process repeats until all S users are assigned to an RB, taking $\lceil \frac{S}{NM} \rceil$ iterations. **HNG complexity**: $\mathcal{O}(NMS^3)$.

3) Round Robin (RR): This scheme naïvely assigns an user from the user set, one at a time, to a RB in sequential order, i.e. $\text{RB}(1,1)$, $\text{RB}(1,2)$, $\ldots, \text{RB}(N,M)$, $\text{RB}(1,1)$, $\text{RB}(1,2), \ldots$, until the user set is empty. **RR complexity**: $\mathcal{O}(S)$.

Fig. 3 depicts the box plot of achieved user data ratesOur proposed SC-MS outperforms other benchmark schemes in terms of both average and median data rates. Specifically, SC-MS average data rate outperforms HRS and HNG by 18% and 15%, respectively. Additionally, user data rates achieved by SC-MS are rather consistent and contain fewer outliers, in comparison to HNG and HRS. Overall, the naïve RR leads to the worst performance, and has the highest variability in achieved data rates.

To measure the fairness of data rates between users, we adopt Jain's fairness index [23], which for $\text{RB}(f,t)$ is

$$J_{f,t} = \frac{\left(\sum_{i \in \mathcal{S}} x_i^{f,t} R_i^{f,t}\right)^2}{\left(\sum_{i \in \mathcal{S}} x_i^{f,t}\right) \cdot \sum_{i \in \mathcal{S}} \left(x_i^{f,t} R_i^{f,t}\right)^2}. \tag{11}$$

Fig. 4 illustrates the distribution of Jain's fairness index of all RSGs obtained by each scheme. SC-MS achieves the best fairness among all of the schemes, with a tight distribution close to the best fairness index value of 1. This is possible since SC-MS aims to form RSGs whose subgraphs are highly intra-connected such that the degrees between nodes tend to be very similar. In turn, this implies users within the same RSG tend to have similar CCI levels. Therefore, even if user data rates may vary, as shown in Fig. 1, the users within the same RSG tend to have similar data rates, thus providing improved fairness when compared with other benchmarks.

Fig. 5 depicts the sum rate achieved by the different schemes for varying numbers of users. The sum rate of both HNG and HRS increases with S until $S \geq 300$ before decreasing. This behavior is related to the objective function of (5) and the constraints of each scheme. As long as the CCI among users in each RSG remains low, we can increase the sum rate by adding more users. However, for a given number of RBs (i.e., RSGs), when the number of users reaches a certain point (here, $S = 300$), RSGs become over-populated such that neither HNG nor HRS can find a scheduling decision to separate users with high CSI correlation in different RBs within their design rules. As a result, users' sum rate decreases due to the increased CCI caused by users with high CSI correlation within the RSGs.

On the other hand, both our proposed SC-MS and RR show no decline in sum rates for $S \leq 600$ users, albeit for different reasons. In RR, where RSGs are formed randomly, the resulting RSGs suffer from severe CCI in all cases, even at very low numbers of users. Hence, its achieved sum rate does not show elbow point nor does it change drastically. In contrast, the sum rate of SC-MS increases even beyond $S = 600$ because RSGs are well formed and have not been severely affected by CCI yet. Compared with HNG and HRS, SC-MS has a better global perspective of the problem by leveraging the combination of SC and post-processing. The results show that SC-MS can efficiently find user cliques in different subbands and thus can still generate scheduling decisions to separate users with high CSI correlation when $S \geq 300$, whereas HNG and HRS fail to do so. This illustrates that our SC-MS is more robust for a larger number of users, and also scales better as well. That being said, the sum rate of both SC-MS and RR will eventually start declining when a sufficiently high number of users leads to severe CCI.

Finally, Fig. 6 illustrates the average user data rate under different SNRs. To analyze the behavior of data rate with respect to interference, we also consider a low CCI scenario by generating channels that have pairwise CSI correlation below 0.2, i.e. $\rho(\boldsymbol{h}_i^f, \boldsymbol{h}_j^f) \leq 0.2, \forall i,j \in \mathcal{S}, \forall f \in \mathcal{N}$. The results show that the data rates become CCI-limited even if the SNR value continues to grow, though the data rate limit differs

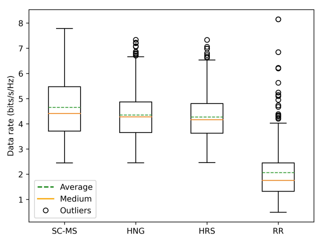

Fig. 3: Distribution of data rate of all users in different schemes.

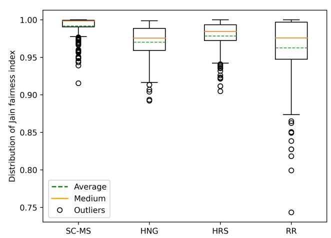

Fig. 4: Distribution of fairness index of all RSGs in different schemes.

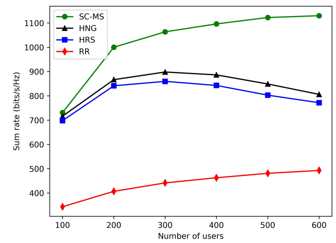

Fig. 5: Average sum rate for different numbers of users in different schemes.

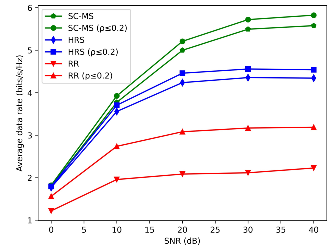

Fig. 6: Average data rate for different SNR values in different schemes.

for each scheme. This phenomenon is quite common because user rate is a function of signal-to-interference-plus-noise ratio (SINR). When the noise power becomes very weak, SINR is dominated by CCI. Positively, the CCI-limited data rate of SC-MS clearly exceeds that of HRS, illustrating the ability of SC-MS scheduling at managing CCI better than other algorithms.

As expected, average rates also grow when we limit CSI correlation. Interestingly, the performance of SC-MS only grew by 4% in the limited correlation setting, implying that most users in the same RSGs formed by our algorithm already have low CSI correlations. In contrast, the average rate of RR increases significantly for limited CSI correlation, because its poor scheduling decisions are less hampered by the strong CCI of the original setting. Overall, SC-MS shows better performance at all SNR values in both scenarios, with even better performance in high SNR regimes.

V. Conclusions and Future Works

This paper addresses the problem of user scheduling and resource sharing in wideband MU-MIMO wireless networks. We form resource sharing groups (RSGs) consisting of low CCI users in a multi-band system. Specifically, we propose a spectral clustering aided multi-band user grouping and scheduling algorithm named SC-MS. Utilizing subband interference graphs, the SC-MS first efficiently finds the largest user cliques based on spectral clustering and post-processing. The process is iterated over all subbands to create as many RSGs as available RBs. SC-MS then assigns unassigned users to RSGs based on minimum CCI. Our numerical results show that SC-MS outperforms other benchmark schemes in terms of user data rates, RSG fairness and sum rates. The SC-MS scales well with the number of users at modest computational complexity, making it appealing for large MU-MIMO networks. In future work, we plan to investigate a more generalized multi-band scheduling problem that also considers power allocation, CSI uncertainty, and individual user rate constraints.

References

[1] J. G. A. et al, "What will 5G be?" *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, p. 1065–1082, 2014.

[2] E. Larsson, F. Tufvesson, O. Edfors, and T. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, p. 185–195, 2014.

[3] Y. Chen, Y. Wu, Y. T. Hou, and W. Lou, "mCore: Achieving sub-millisecond scheduling for 5G MU-MIMO systems," *IEEE 40th Conf. Computer Commun. (INFOCOM)*, pp. 1–10, 2021.

[4] E. Castañeda, A. Silva, A. Gameiro, and M. Kountouris, "An overview on resource allocation techniques for multi-user MIMO systems," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 1, pp. 239–284, 2017.

[5] A. Goldsmith, S. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 5, p. 684–702, 2003.

[6] J. Dai and S. Wang, "Clustering-based spectrum sharing strategy for cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 1, pp. 228–237, 2017.

[7] M. Alkhaled, E. Alsusa, and W. Pramudito, "Adaptive user grouping algorithm for the downlink massive MIMO systems," *IEEE Wireless Commun. Netw. Conf. (WCNC)*, pp. 1–6, 2016.

[8] L. Liang, S. Xie, G. Y. Li, Z. Ding, and X. Yu, "Graph-based resource sharing in vehicular communication," *IEEE Trans. Wireless Commun.*, vol. 17, no. 7, pp. 4579–4592, 2018.

[9] Y. Xu, G. Yue, and S. Mao, "User grouping for massive MIMO in FDD systems: New design methods and analysis," *IEEE Access*, vol. 2, no. 1, pp. 947–959, 2014.

[10] C. Feres and Z. Ding, "An unsupervised learning paradigm for user scheduling in large scale multi-antenna systems," *IEEE Trans. Wireless Commun.*, pp. 1–1, 2022.

[11] Z. Cheng, J. Yang, Z. Wei, and H. Yang, "User clustering and scheduling in UAV systems exploiting channel correlation," *IEEE 30th Annu. Int. Symp. Pers., Indoor Mobile Radio Commun. (PIMRC)*, pp. 1–6, 2019.

[12] R.-F. Trifan, R. Lerbour, G. Donnard, and Y. L. Helloco, "K-means MU-MIMO user clustering for optimized precoding performance," *IEEE Wireless Commun. Netw. Conf. (WCNC)*, pp. 1–5, 2019.

[13] N. Jindal, "MIMO broadcast channels with finite-rate feedback," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, p. 5045– 5060, 2006.

[14] E. Bjornson and E. Jorswieck, "Optimal resource allocation in coordinated multi-cell system," *Found. Trends Commun. Inf. Theory*, vol. 9, no. 2-3, p. 113–381, 2013.

[15] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, no. 10, p. 1458–1461, 1999.

[16] Z. Yang, L. Cai, , and W. Lu, "Practical scheduling algorithms for concurrent transmissions in rate-adaptive wireless networks," *Proc. IEEE 29th Conf. Comput. Commun. (INFOCOM)*, p. 120–128, 2010.

[17] U. von Luxburg, "A tutorial on spectral clustering," *Statist. Comput.*, vol. 17, no. 84, p. 395–416, 2007.

[18] S. M. Z. He, S. Kompella, and A. Swami, "Link scheduling and channel assignment with a graph spectral clustering approach," *IEEE Mil. Commun. Conf. (MILCOM)*, pp. 73–78, 2016.

[19] Q. Wu and J.-K. Hao, "A review on algorithms for maximum clique problems," *Eur. J. Oper. Res.*, vol. 242, no. 3, pp. 693–709, 2015.

[20] D. Yan, L. Huang, and M. I. Jordan, "Fast approximate spectral clustering," in *Proc. 15th ACM SIGKDD Int. Conf. Knowl. Discovery Data Mining*, 2009, p. 907–916.

[21] R. Y. Chang, Z. Tao, J. Zhang, and C. . C. J. Kuo, "Multicell OFDMA downlink resource allocation using a graphic framework," *IEEE Trans. Veh. Technol.*, vol. 58, no. 7, pp. 3494–3507, 2009.

[22] A. A. Khan, R. S. Adve, and W. Yu, "Optimizing downlink resource allocation in multiuser MIMO networks via fractional programming and the hungarian algorithm," *IEEE Trans. Wireless Commun.*, vol. 19, no. 8, pp. 5162–5175, 2020.

[23] R. Jain, D. Chiu, and W. R. Hawe, "A quafntitative measure of fairness and discrimination for resource allocation in shared computer systems," DEC Research, Hudson, MA, USA, Tech. Rep. TR-301, 1984, vol. 38.