LINEAR OR NONLINEAR? RELATING COLLEGE ALGEBRA STUDENTS' COVARIATIONAL REASONING AND GRAPH SELECTION

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We investigate the following problem: How does students' covariational reasoning (Carlson et al., 2002; Thompson & Carlson, 2017) relate to their graph selection on a fully online assessment? In particular, we focus on students' distinction between linear and nonlinear graphs representing the same direction of change in variables. Figure 1 shows two such graphs.

Figure 1: Two graphs relating diameter and height for a fishbowl filling with water

When a student engages in covariational reasoning, they can conceive of relationships between attributes that they view to be capable of varying and possible to measure (Carlson et al., 2002; Thompson & Carlson, 2017). Per the framework from Thompson and Carlson (2017) an early level of covariational reasoning is the "gross coordination of values," which refers to a loose connection between the direction of change in attributes. For either graph in Figure 1, a student reasoning this way can conceive of the diameter increasing then decreasing while the height continues to increase (see Figure 1).

We situate this case study (Stake, 1994) in a larger interview-based validation study of a fully online covariation assessment (Johnson et al., 2021). The assessment contains six items; students view a video animation, then select one of four graphs that best represents a relationship between variables. We report on a case of an early undergraduate student, Maya, who spontaneously wondered how to select between graphs such as the ones shown in Figure 1.

On three items, Maya's covariational reasoning was at least at a level of gross coordination of values, as evidenced by her explanations. Appealing to the direction of change in attributes or a value of an attribute at a single instance, Maya narrowed down the graphs to two choices. Both represented the same direction of change in related attributes. She then selected a graph based on physical aspects of the situation (e.g., "the motion of the cone seems more linear than, than like a wave"). For Maya, either graph was satisfactory. The assessment forced her to make a choice.

Maya's case illustrates how students' covariational reasoning can intertwine with their interpretation of physical aspects of the situation. She demonstrates how students can engage in both figurative and operative thinking (Moore et al., 2019) when interpreting graphs, in ways that are compatible from a student perspective.

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