Towards a Decentralized Metaverse: Synchronized Orchestration of Digital Twins and Sub-Metaverses

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***Abstract*—Accommodating digital twins (DTs) in the metaverse is essential to achieving *digital reality*. This need for integrating DTs into the metaverse while operating them at the network edge has increased the demand for a *decentralized edge-enabled metaverse*. Hence, to consolidate the fusion between real and digital entities, it is necessary to harmonize the interoperability between DTs and the metaverse at the edge. In this paper, a novel decentralized metaverse framework that incorporates DT operations at the wireless edge is presented. In particular, a system of autonomous physical twins (PTs) operating in a massively-sensed zone is replicated as cyber twins (CTs) at the mobile edge computing (MEC) servers. To render the CTs' digital environment, this zone is partitioned and teleported as distributed *sub-metaverses* to the MEC servers. To guarantee seamless synchronization of the sub-metaverses and their associated CTs with the dynamics of the real world and PTs, respectively, this joint synchronization problem is posed as an optimization problem whose goal is to minimize the average *sub-synchronization time* between the real and digital worlds, while meeting the DT *synchronization intensity* requirements. To solve this problem, a novel iterative algorithm for joint sub-metaverse and DT association at the MEC servers is proposed. This algorithm exploits the rigorous framework of optimal transport theory so as to efficiently distribute the sub-metaverses and DTs, while considering the computing and communication resource allocations. Simulation results show that the proposed solution can orchestrate the interplay between DTs and sub-metaverses to achieve a 25.75% reduction in the sub-synchronization time in comparison to the signal-to-noise ratio-based association scheme.**

***Index Terms*—metaverse, digital twins, synchronization, sub-metaverse, optimal transport theory**

I. Introduction

The *metaverse* is perhaps one of the most anticipated technological breakthroughs of the coming decade [1]. In essence, the *metaverse* is a massively scaled and interoperable network of real-time rendered three-dimensional digital worlds. The metaverse will lead to the emergence of a novel suite of hybrid *physical-virtual-digital services* that could revolutionize the interconnection between people, things, and places [2], [3]. In essence, building a limitless metaverse of today's real world has various device (e.g. extended reality devices), communication, computing, and artificial intelligence (AI) challenges. Chief among those challenges is *the end-to-end (E2E) synchronization* of the real world with the metaverse and its components such as digital twins (DTs). Indeed, achieving this transparent replication imposes a set of stringent wireless network demands such as near-zero E2E latency, effective computing, ubiquitous connectivity, and ultra-high data rates [4]. Thus, in an attempt to meet the aforementioned demands, the metaverse must adopt a *decentralized, edge-enabled model*. Instead of limiting the metaverse to a centralized, computationally-draining, and rigid architecture as in the state-of-the-art [5], this shift enables segmenting the metaverse into a decentralized system of interconnected *sub-metaverses* distributed at the network edge [6]. Here, a *sub-metaverse* is defined as a digital replica of a *physical space* in the real world. Furthermore, the decentralization of the metaverse requires decentralizing its key components as well, notably, DTs. DTs are used to coordinate autonomous Internet of Everything (IoE) applications that exist in the real world (e.g., autonomous vehicles) with their digital counterparts in the metaverse [7]. Thus, it is vital for such real-time DTs to replicate the functionalities of the underlying physical applications while meeting the DT synchronization requirements. Consequently, synchronizing both metaverse and DTs is pivotal for guaranteeing a high fidelity replica in the digital world.

This research was supported by the U.S. National Science Foundation under Grants CNS-2210254 and CNS-2007635.

Recent works in [8]–[11] have studied merging DTs and the metaverse with mobile edge computing (MEC) in an effort to meet synchronization demands. The authors in [8] proposed a deep reinforcement learning approach coupled with transfer learning to solve the DT-MEC placement and migration problems while minimizing the DT synchronization delay. The work in [9] studied the resource allocation problem for Internet of Things (IoT) sensing devices with the goal of synchronizing the metaverse by controlling its DTs' synchronization using a game-theoretic framework. In [10], the authors proposed equipping IoT sensing devices with semantic extraction algorithms to minimize the size of the transmitted data in an attempt to achieve enhanced metaverse synchronization. The work in [11] introduced a stochastic semantic resource allocation scheme to enhance the synchronization of virtual transportation networks in the metaverse. However, these prior works are limited in various ways. First, the work in [8] is limited to DT synchronization and completely neglects that DTs are constituents of the metaverse, that in turn should be synchronized as well. Furthermore, the work in [9] assumes that the metaverse synchronization is achieved through the DT synchronization, without strictly differentiating between them as two separate synchronization streams. Meanwhile, synchronization of the metaverse in [10] and [11] does not account for the need to synchronize the metaverse constituents, e.g., DTs. Moreover, despite adopting a MEC framework in [9]–[11], these works still consider a centralized metaverse architecture,

which cannot accommodate the deployment of D
across the edge. Notably, if done in a centralized
a limitless metaverse remains highly intractable.
decomposing the metaverse into sub-metaverses
the edge along with the DT models. *Clearly, th*
of works that investigate the joint synchronizatio
sub-metaverses in a distributed metaverse fram
orchestrating the interoperability between them

The main contribution of this paper is a novel
metaverse framework that distributes DTs and su
over the wireless edge network, while preser
synchronization with their physical counterpart
world. In particular, a system of autonomous p
(PTs) that operate in a physical zone are digitally
cyber twins (CTs) at the network's MEC servers
replicate the physical environment in the digita
zone is equipped with massive sensing abilities.
the zone is partitioned into separate regions that
as sub-metaverses at the MEC servers. To perfectl
the distributed sub-metaverses and their residing CTs with the corresponding physical counterparts, this joint synchronization problem is posed as an optimization problem whose goal is to minimize the average *sub-synchronization time* with the real world, while satisfying the DT *synchronization intensity* requirements. To solve this problem, we propose an iterative algorithm based on *optimal transport theory* to determine the joint association of the sub-metaverses and their corresponding CTs at the MEC servers. We then prove the existence of an optimal solution for the physical region partitioning problem. Subsequently, we find the corresponding map to determine the optimal partitions and DT association to minimize the sub-synchronization time, while considering the communications and computing resource allocations in the network. *To the best of our knowledge, this is the first work that considers the synchronization of both DTs and metaverse by addressing the interplay between them at the edge*. Simulation results show that the proposed solution can provide a tradeoff between associating DTs and sub-metaverses to achieve a 25.75% reduction in sub-synchronization time compared to a baseline association scheme based on the signal-to-noise ratio (SNR).

II. System Model and Problem Formulation

Consider a geographical zone $\mathcal{Z} \subset \mathbb{R}^3$ representing a portion of the real world as shown in Fig. 1. In this zone, there exists a set $\mathcal{B}$ of B MEC servers deployed at the wireless network base stations (BSs) to provide digital teleportation, replication, and fine-tuned rendering services. Given that the real world is massive and limitless, it must be digitally represented as a massive system of decentralized sub-metaverses. As such, B MEC servers must collectively teleport zone $\mathcal{Z}$ into the digital world as a set of decentralized, yet tightly interconnected sub-metaverses. This guarantees that the computationally intensive teleportation and rendering processes within each sub-metaverse are efficiently deployed and distributed across the network's MEC servers. Subsequently, zone $\mathcal{Z}$ is partitioned into a set $\mathcal{A}$ of A non-overlapping regions. Accordingly, each

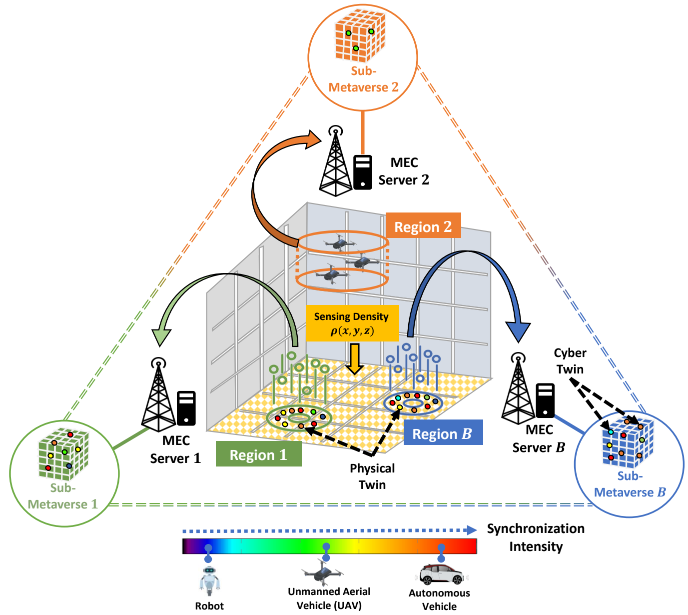


Fig. 1: Illustrative figure showcasing the decentralized sub-metaverses and their associated digital twins distributed at the network edge.

region $a \in \mathcal{A}$ is mapped to a decentralized sub-metaverse $s \in \mathcal{S} \triangleq \{1, 2, \ldots, S\}$, such that each sub-metaverse is associated to a MEC server. Thus, we denote the tuple $\boldsymbol{u}_b = (a_b, s_b)$ that maps every region $a \in \mathcal{A}$ to its corresponding sub-metaverse $s \in \mathcal{S}$ at the designated MEC server $b \in \mathcal{B}$.

Furthermore, there exists a set $\mathcal{K}$ of K of autonomous cyber-physical systems that physically operate in zone $\mathcal{Z}$. To ensure that such systems participate in the metaverse and are autonomously replicated (twinned), a *seamless digital replication* must be performed on each one of them. Thus, each system $k \in \mathcal{K}$ will be operating as a DT. Accordingly, for these DTs, we define the set $\mathcal{P}$ of P PTs. Such PTs are served by the B MEC servers to simulate and render their corresponding set $\mathcal{C}$ of C CTs. Thus, each DT application $k \in \mathcal{K}$ is represented as a tuple $\boldsymbol{v}_k = (p_k, c_k)$ that maps every PT $p \in \mathcal{P}$ to its corresponding CT $c \in \mathcal{C}$, where $K = P = C$. Essentially, the set of PT systems that exist within each region $a \in \mathcal{A}$, have corresponding CT counterparts that reside in the respective digital replica of this region, i.e., sub-metaverse $s \in \mathcal{S}$. Consequently, it is necessary that each MEC server $b \in \mathcal{B}$ *first* replicates each physical region into the digital world as a sub-metaverse, while also, *simultaneously* guaranteeing the synchronization of the PT of each region with its respective CT in its sub-metaverse. It is important to note that the *simultaneity* of these two processes is *a necessary condition that guarantees the duality and twinning between the real world and the metaverse.*

A. Decentralized Metaverse Model

To digitally model the physical regions in the metaverse, with high precision, a massive number of sensors must be deployed to replicate aspects of the real world. Because of this large number of sensors deployed, we assume that the inter-spacing between these sensors is relatively minimal. As such, the sensors will be continuously distributed according to a spatial distribution $g(x,y,z)$ that describes the likelihood localization of the sensors around the 3D objects in zone $\mathcal{Z}$, where x is the longitude unit distance, y is the latitude unit distance, and z is the height unit distance. For instance, areas with a large number of physical objects, have a high number of sensors (e.g. highly urban and crowded areas). Meanwhile, areas with a considerable lower physical density, require a smaller number of sensors to be deployed and replicated (e.g. empty fields in rural areas). Also, given that 3D objects in the real world are described with unique attributes of different dimensions, a unit volumetric *sensing density* $\rho(x,y,z)$ (bps/m^3) is used to describe the flow of data from these sensors [12].

To tractably maintain the metaverse synchronized with the real world, we discretize time into independent time slots of duration Δ. During each slot, the sensory data must be uploaded to the network edge. We assume that Δ is minimal, i.e., Δ will not affect the precision of the teleported sub-metaverse in mimicking reality. Hence, we consider that each sensor $q \in \mathcal{Q} \triangleq \{1, 2, \ldots, Q\}$ forms an independent unit of infinitesimal volume ϵ centered at (x,y,z) such that the rate in which data is generated from this infinitesimal volume is $\epsilon\rho(x,y,z)$. Then, the rate at which sensor q uploads its data to MEC server b is:

$$R_{q,b}(Q_b) = \frac{W_b^s}{Q_b}\log_2\left(1 + \frac{h_{q,b}\xi_q}{\sigma_b^2}\right), \tag{1}$$

where W_b^s is the available bandwidth for uploading sensor data to synchronize sub-metaverse s at MEC server b, Q_b is the number of sensors connected to MEC server b, $h_{q,b}$ is the channel gain between the sensor q and MEC server b, ξ_q is the transmit power of sensor q, and σ_b^2 is the spectral noise power at MEC server b.

Subsequently, the time needed for sensor q to upload its data generated within Δ to MEC server b is:

$$t_{q,b}^{\mathrm{com}}(Q_b) = \frac{\Delta\epsilon}{R_{q,b}(Q_b)}\rho(x,y,z). \tag{2}$$

Furthermore, the computing time needed to render this infinitesimal volume in sub-metaverse $s \in \mathcal{S}$ at its associated MEC server $b \in \mathcal{B}$ can be written as:

$$t_{q,b}^{\mathrm{cmp}}(Q_b,\psi_b^s) = \frac{\Lambda\Delta\epsilon}{\psi_b^s}Q_b\rho(x,y,z), \tag{3}$$

where Λ is a coefficient related to the topological complexity of the regions and ψ_b^s is the number of computing resources assigned for rendering the sub-metaverse s assigned at MEC server b. Then, we define the total delay needed to synchronize the volume unit ϵ of sensor q from the real world with its representative unit in the designated sub-metaverse at MEC server b as:

$$t_{q,b}^{\mathrm{sync}}(Q_b,\psi_b^s) = t_{q,b}^{\mathrm{com}}(Q_b) + t_{q,b}^{\mathrm{cmp}}(Q_b,\psi_b^s). \tag{4}$$

Moreover, replicating region a that is composed of a large number of these infinitesimal sensors requires synchronizing each sensor with its infinitesimal representation in the sub-metaverse. Hence, we define the *sub-synchronization time* as the total delay needed to synchronize region a and its corresponding sub-metaverse s at MEC server b as:

$$T_b(Q_b,\psi_b^s) = \iiint_{a_b} t_{q,b}^{\mathrm{sync}}(Q_b,\psi_b^s)g(x,y,z)\,dx\,dy\,dz. \tag{5}$$

B. DT Model

To perfectly consider a synchronized digital world, there is a need to synchronize each of the constituents of the sub-metaverse as well. In the designated zone, the PTs are spatially distributed according to a distribution $f(x,y,z)$ in the three dimensional plane. These PTs are autonomous systems that execute real-time decisions (e.g., autonomous vehicles, drones, robots, etc). Each DT $k \in \mathcal{K}$ is described with a *synchronization intensity* μ_k that captures the maximum allowable time for the CT to replicate the same action executed by the PT. μ_k is primarily characterized by the time needed for the PT to execute its task. For instance, an autonomous vehicle will have a high synchronization intensity as it executes its decisions in near real-time (the digital counterpart must execute actions critically). Meanwhile, a robot can tolerate a larger time margin with respect to action execution.

Each PT is equipped with a large set of sensors that enable a real-time execution of decisions. Concurrently, the corresponding CT must to mimic the same action performed by its twinned PT while bounded by its intensity limit μ_k. We assume that the inter-region mobility in this system is limited such that each PT remains connected to the same BS; and thus, each CT remains residing in the same sub-metaverse. Hence, each PT p_k must offload its sensory data $\mathcal{D}_k$ collected at each time step to the MEC server that hosts its CT c_k. Subsequently, the achievable uplink rate by p_k and its designated MEC server b will be: $r_{k,b} = W_k\log_2\left(1 + \frac{h_{k,b}\zeta_k}{\sigma_b^2}\right)$, where W_k is the uplink bandwidth for DT k, $h_{k,b}$ is the channel gain between PT p_k and MEC server b, and ζ_k is the transmit power of p_k. Thus, the time needed to upload the data generated from p_k to MEC server b where the CT resides:

$$\tau_{k,b}^{\mathrm{com}} = \frac{D_k}{r_{k,b}}. \tag{6}$$

In addition, the time needed to execute the corresponding CT action at the MEC server b is defined as:

$$\tau_{k,b}^{\mathrm{cmp}}(\phi_b^k) = \frac{\Gamma_k D_k}{\phi_b^k}, \tag{7}$$

where Γ_k is the twin complexity that relates to the accuracy of the CT replication and the intricacy of the underlying physical application, and ϕ_b^k is the computing power allocated to the DT application $\boldsymbol{v}_k$ at MEC server b. Then, the *twinning synchronization time* needed for DT k to map the PT action in the real world to its CT in the sub-metaverse will be:

$$\tau_{k,b}^{\mathrm{sync}}(\phi_b^k) = \tau_{k,b}^{\mathrm{com}} + \tau_{k,b}^{\mathrm{cmp}}(\phi_b^k). \tag{8}$$

C. Problem Formulation

Our goal is to teleport the zone and the existing PT applications into the digital world by distributing them over the network edge. This requires partitioning the regions and mapping them into sub-metaverses at the MEC servers, while accommodating the CTs in their designated sub-metaverses. Thus, our goal to minimize the *sub-synchronization time* between the real world and distributed metaverses, while satisfying the

DT *synchronization intensity* requirements. In addition to this association, accommodating the sub-metaverses and the DTs at the edge requires an efficient allocation of the computational resources at the MEC server between sub-synchronization and DT synchronization. Then, this problem can be formulated as follows:

$$\min_{a_b, b\in\mathcal{B}, \boldsymbol{\psi}, \boldsymbol{\Phi}} \quad \frac{1}{B}\sum_{b\in\mathcal{B}} T_b(Q_b, \psi_b^s), \tag{9a}$$

$$\text{s.t.} \quad \tau_{k,b}^{\text{sync}}(\phi_b^k) \leq \frac{1}{\mu_k} \quad \forall k \in \mathcal{K}, \forall b \in \mathcal{B}, \tag{9b}$$

$$\psi_b^s + \sum_{k=1}^{K} \phi_b^k \leq \Psi_b \quad \forall b \in \mathcal{B}, \tag{9c}$$

$$\sum_{b\in\mathcal{B}} x_{k,b} \leq 1 \quad \forall k \in \mathcal{K}, \tag{9d}$$

$$x_{k,b} \in \{0,1\} \quad \forall k \in \mathcal{K}, \forall b \in \mathcal{B}, \tag{9e}$$

$$\bigcup_{b\in\mathcal{B}} a_b = \mathcal{Z}, \tag{9f}$$

$$a_i \cap a_j = \emptyset \quad \forall i, j \in \mathcal{A}, i \neq j, \tag{9g}$$

where $\boldsymbol{\psi} = [\psi_1^s, \psi_2^s, \dots, \psi_B^s]^T$ is the computing resource allocation vector for sub-synchronization. The computing resource allocation and association matrix for DTs synchronization is given by:

$$\boldsymbol{\Phi} = \begin{bmatrix} \phi_1^1, \dots, \phi_1^K \\ \phi_2^1, \dots, \phi_2^K \\ \dots \\ \phi_B^1 \dots, \phi_B^K \end{bmatrix}. \tag{10}$$

Here, $x_{k,b}$ is the association variable between the MEC server and the DT, and Ψ_b is the maximum computing power of MEC server b.

Problem (9) is challenging to solve as: a) it involves a set of mutually correlated regions, and b) the effective partitioning of the regions depends on the distribution of DTs and their synchronization requirements. Hence, an effective solution that can address both region partitioning and DT association is needed to alleviate the complexity of the problem. Next, we present an iterative algorithm that leverages *optimal transport theory* [13] to solve (9).

III. Optimal Transport Theory for Joint Sub-Metaverse and DTs Association at the Edge

In this section, we introduce a novel iterative algorithm to solve (9) by considering an *optimal transport theory* framework [13]. Optimal transport is suitable here because it provides the optimal mapping, from the sensors to the MEC servers, which determines the region partitions that yield the minimal sub-synchronization time. We first derive the optimal region partitions that minimize the sub-synchronization time based on the metaverse synchronization resources at the MEC servers. Then, the DT-MEC association is determined under given optimal region partitions and the corresponding computing resources required to synchronize each DT are allocated accordingly.

First, we consider finding the optimal region partitions that minimize the sub-synchronization time, under a given computing resource allocation for metaverse synchronization. For that, we define $\boldsymbol{\omega} = (x, y, z)$ as the 3D location of sensor q, and define $\boldsymbol{\kappa}_b$ as the location of the MEC server b. Then, we reformulate $t_{q,b}^{\text{sync}} = L(g(x,y,z))F(\boldsymbol{\omega}, \boldsymbol{\kappa}_b)$, where $L(g(x,y,z)) = Q\Delta\epsilon \iiint_{a_b} g(x,y,z)\, dx\, dy\, dz$, $F(\boldsymbol{\omega}, \boldsymbol{\kappa}_b) = \frac{\rho(\boldsymbol{\omega})}{W_b^s \log_2(1+\frac{\beta(\boldsymbol{\omega},\boldsymbol{\kappa}_b)}{\sigma_b^2})} + \frac{\Lambda\rho(\boldsymbol{\omega})}{\psi_b^s}$, and $\beta_{q,b}(\boldsymbol{\omega}, \boldsymbol{\kappa}_b) = h_{q,b}\xi_q$ as the received power from sensor q at BS b. Hence, problem (9) is reduced to a region partitioning optimization problem under given resource allocation, and can be expressed as:

$$\min_{a_b, b\in\mathcal{B}} \quad \frac{1}{B}\sum_{b\in\mathcal{B}} \iiint_{a_b} L(g(x,y,z))F(\boldsymbol{\omega}, \boldsymbol{\kappa}_b) \times g(x,y,z)\, dx\, dy\, dz, \tag{11a}$$

$$\text{s.t.} \quad \bigcup_{b\in\mathcal{B}} a_b = \mathcal{Z}, \tag{11b}$$

$$a_i \cap a_j = \emptyset \quad \forall i, j \in \mathcal{A}, i \neq j. \tag{11c}$$

Solving (11) is challenging since the optimization variables are a set of continuous and dependent region partitions. Thus, to overcome this challenge, we model it as an *optimal transport theory* problem to derive the optimal region partitions that minimize the sub-synchronization time [14]. In general, optimal transport is a mathematical framework that considers quantifying the minimal cost for transporting the probability mass of a distribution χ_1 to another one χ_2 on $\Omega \subset \mathbb{R}^w$, by finding an optimal transport map Π from χ_1 to χ_2 that minimizes the following function:

$$\min_{\Pi} \int_{\Omega} \mathcal{J}(m,n)\chi_1(m)\ dm; \quad n = \Pi(m), \tag{12}$$

where $\mathcal{J}(m,n)$ is the cost of transporting a unit from point m to point n.

Since the sensing density follows a continuous distribution and the MEC servers can be considered as discrete points of this distribution, it is natural to model our region partitioning problem as a *semi-discrete optimal transport* problem. Thus, we formulate this mapping problem from the infinitesimal sensor to the MEC server as a semi-discrete optimal transport problem that minimizes its sub-synchronization time. In this case, the cost of transportation is considered to be the synchronization time of the infinitesimal volume. Thus, the zone is partitioned into optimal region partitions via this optimal transport map, while noting that $\frac{1}{B}$ is a constant term that does affect the mapping function as it aids in computing the average only. We consider the objective in (11a) equivalent to (12) without the constant term. Next, we prove that an optimal solution for this semi-discrete mapping in (11) exists.

Lemma 1. *The optimization problem in* (11) *admits an optimal solution.*

Proof. Let $\alpha_b = \iiint_{a_b} g(x,y,z)\ dx\ dy\ dz$, $\forall b \in \mathcal{B}$, and define the unit simplex as follows: $E = \{\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_B) \in \mathbb{R}^B, \sum_{b=1}^{B} \alpha_b = 1, \alpha_b \geq 0\}$. Moreover, we define $\chi_1(x,y,z) = g(\boldsymbol{\omega})$ and $\mathcal{J}(\boldsymbol{\omega}, \boldsymbol{\kappa}_b) = L(\alpha_b)F(\boldsymbol{\omega}, \boldsymbol{\kappa}_b)$. Clearly, since $F(\boldsymbol{\omega}, \boldsymbol{\kappa}_b)$ is continuous, and noting that $L(\alpha_b)$ is differentiable, we can see that $\mathcal{J}(\boldsymbol{\omega}, \boldsymbol{\kappa}_b)$ is continuous.

Algorithm 1: Optimal Transport Algorithm for Joint Sub-Metaverse and DT Association

Input: $g(x,y,z)$, $\rho(x,y,z)$, $f(x,y,z)$, Q, μ_k, $\forall k \in \mathcal{K}$, $\beta_{q,b}$, $\forall b \in \mathcal{B}$, $\forall q \in \mathcal{Q}$.
Output: a_b^*, T_b^*, $\forall b \in \mathcal{B}$, $\boldsymbol{\psi}^*$, $\boldsymbol{\Phi}^*$.
Initialize: Set iterations $\nu = 0$ and $\psi_b^{s^{(0)}} = \Psi_b, \forall b \in \mathcal{B}$.
Generate initial region partitions $a_b^{(0)}$ and calculate $\alpha_b^{(0)}$.
repeat
  $\nu \leftarrow \nu + 1$.
  Generate region partitions $a_b^{(\nu)}$ using $\alpha_b^{(\nu-1)}$ and $\psi_b^{s^{(\nu-1)}}$ according to (14).
  Update $\alpha_b^{(\nu)} = \iiint_{a_b^{(\nu)}} g(\boldsymbol{\omega})\, d\boldsymbol{\omega}$
  for *each region* a_b **do**
    Associate the DTs in region a_b to MEC server b.
    for *each DT* k *connected to MEC server* b **do**
      Compute $\phi_b^{k^{(\nu)}} = \frac{\Gamma_k}{\frac{1}{\mu_k D_k} - \frac{1}{r_{k,b}}}$.
    Update $\psi_b^{s^{(\nu)}} = \Psi_b - \sum_k \phi_b^{k^{(\nu)}}$.
until *the convergence of* (12);
Determine $a_b^* = a_b^{(\nu)}$, $\boldsymbol{\psi}^* = \boldsymbol{\psi}^{(\nu)}$, $\boldsymbol{\Phi}^* = \boldsymbol{\Phi}^{(\nu)}$.
Compute the average sub-synchronization time as (12) $\times \frac{1}{B}$.

Subsequently, $\mathcal{J}(\boldsymbol{\omega}, \boldsymbol{\kappa}_b)$ is also a lower semi-continuous function. Hence, considering g as a continuous distribution and $\lambda = \sum_{b\in\mathcal{B}} \alpha_b \delta_{\boldsymbol{\kappa}_b}$ as a discrete distribution, with the lower semi-continuous cost function, there theoretically exists an optimal transport map Π from g to λ. Then, for any $\boldsymbol{\alpha} \in E$, problem (11) admits an optimal solution. Since E is a unit simplex that is a compact set and non-empty, then problem (11) admits an optimal solution over the entire set E. Thus, the lemma is proved. □

Furthermore, we characterize the transport map Π which relates each sensor to its corresponding sub-metaverse as follows:

$$\left\{ \Pi(\boldsymbol{\omega}) = \sum_{b=1}^{B} \boldsymbol{\kappa}_b \mathbb{1}_{a_b}(\boldsymbol{\omega});\ \iiint_{a_b} g(\boldsymbol{\omega})\, d\boldsymbol{\omega} = \alpha_b \right\} \tag{13}$$

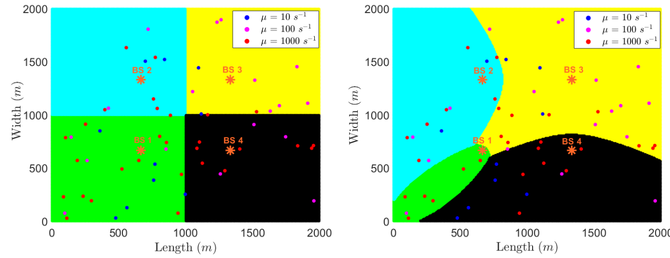
In the following proposition, we set up the solution space for problem (11), which yields the optimal region partitions that minimize the sub-synchronization time.

Proposition 1. *The optimal region partitioning is given by the following map:*

$$a_b^* = \left\{ \boldsymbol{\omega} = (x,y,z) |\ \alpha_b F(\boldsymbol{\omega}, \boldsymbol{\kappa}_b) \leq \alpha_j F(\boldsymbol{\omega}, \boldsymbol{\kappa}_j), \forall j \neq b \in \mathcal{B} \right\}. \tag{14}$$

Proof. The complete proof is omitted due to space limitations. Essentially, Proposition 1 can be proved by extending Theorem 1 in [15] to a 3D scenario, while modifying the cost function to include both communication and computing resources at the BS and converting it to an uplink scenario. □

According to Proposition 1, we can see that a_b and α_b are correlated. Hence, to divide the zone into optimal partitions, it is necessary to first initialize region partitions from an arbitrary partitioning scheme (e.g., Voronoi diagrams). After that, the partitions are iteratively updated based on (14). Moreover, after the partitions in each iteration are determined, the DTs residing in each region are accommodated in the corresponding sub-metaverse, whereby each DT should have its synchronization requirement satisfied. This will require providing them with sufficient computing resources at the MEC server. Note that the region boundaries impact the number of PTs within each region, and hence, the DT computing power that the MEC server can provide for each DT application. After determining the remaining computing resources at each MEC server, this mapping procedure is iterated until convergence. Hence, we can reach the optimal association of the regions and DT applications that can simultaneously guarantee upmost synchronization of the metaverse and its DT components. This iterative algorithm is summarized in Algorithm 1.



(a) (b)

Fig. 2: Region Partitioning and DT association according to the a) SNR method and b) proposed optimal transport method.

IV. Simulation Results and Analysis

For our simulations, we consider a cross-sectional grid zone having dimensions of 2 km × 2 km, in which $B = 4$ BSs are deployed to provide digital teleportation services. The sensing density is modeled as a truncated Gaussian distribution with mean values centered at (750 m, 750 m) and having a standard deviation σ. This model is suitable to model hotspots of sensors. We consider that the MEC servers have a computing power of $\Psi_b = [8, 8, 16, 16]$ GHz and a bandwidth for metaverse synchronization $W_b^s = [10, 10, 20, 20]$ MHz. We consider three types of DT applications having $\mu_k = [10, 100, 1000]$ s^{-1} and $\zeta_k = [100, 200, 300]$ mW, respectively. Also, the PTs are distributed according to a Gaussian mixture model. Unless otherwise stated, we set the following simulation parameters: $Q = 25{,}000$, $\Delta = 1\,\mathrm{ms}$, $\epsilon = 0.01\,\mathrm{m}^3$, $\xi_q = 1\,\mathrm{mW}$, $\sigma_b^2 = \sigma_o^2 = -170\,\mathrm{dBm/Hz}$, $\rho(x,y,z) = 50\,\mathrm{bps/m}^3$, $\Lambda = 5000\,\mathrm{cycles/bit}$, $D_k = 10\,\mathrm{Mb}$, $W_k = 1$ MHz, and $\Gamma_k = 10^4$cycles/bit. As a benchmark, we compare our proposed optimal transport solution to an SNR-based association scheme. In the SNR scheme, the sensors are associated to the BS having the best channel conditions. Accordingly, the PTs in the resulting region are associated to the same BS to execute their twinning processes.

Fig. 2 compares the partitioning technique adopted by the SNR-based approach in Fig. 2a, and our proposed partitioning technique in Fig. 2b. Here, the number of DTs in the network is considered to be $K = 60$. Fig. 2a clearly shows that the regions are equally partitioned based on the SNR method. Given that our approach considers the DTs' operation and its corresponding computing and communication resources, the regions are unequally divided. Moreover, owing to the fact that region 1 has the highest sensing density, it was associated

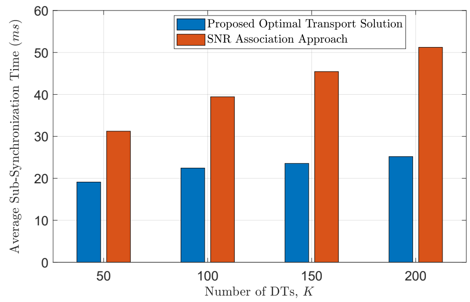


Fig. 3: The average sub-synchronization time as a function of number of DTs.

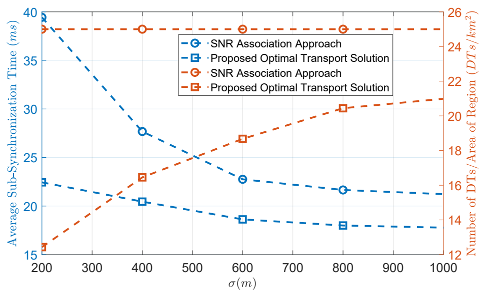


Fig. 4: The average sub-synchronization time and the regional density of DTs versus the standard deviation of the sensing density.

with the smallest region portion and the least number of DTs. Notably, the other regions characterized with less sensing density are larger in size and encompass a higher number of DTs. Hence, this figure showcases that our approach provides a *tradeoff between DTs and sub-metaverses* to guarantee the least sub-synchronization time, unlike the SNR-based method.

In Fig. 3, the average sub-synchronization time for different number of DTs in the system is evaluated for the SNR-based association and our proposed optimal transport approach. The proposed method clearly outperforms the SNR-based association for all the numbers of DTs. Fig. 3 shows that, in our proposed approach, the sub-synchronization time is robust to changes in the number of DTs. That is, the sub-synchronization time merely increases from 19.11 ms to 25.17 ms while increasing the number of DTs from $K = 50$ to $K = 200$. Meanwhile, when adopting the SNR based association, the average sub-synchronization time varies from 31.23 ms to 51.23 ms for an increase of $K = 50$ to $K = 200$.

Fig. 4 showcases the average sub-synchronization time and the regional density of DTs versus the standard deviation of sensing density distribution σ. First, we can see that the SNR-based association does not take into account the number of DTs nor the available resources. Thus, the regional density of DTs remains constant despite changes in the sensing density. Meanwhile, in our proposed approach, as σ increases, the number of DTs becomes more uniformly distributed across regions. Thus, the density of DTs per region increases to asymptotically reach a plateau. Given that a high σ characterizes a more uniformly sparsed distribution of sensors, we can see that the regional density reaches a plateau close to that of the SNR-based approach. With regards to the sub-synchronization time, we can see that our proposed approach has a $25.75\,\%$ lower sub-synchronization time for all values of σ. Fig. 4 also shows that the gap between the two methods decreases as σ increases. This results from the fact that the number of DTs becomes more uniformly distributed across regions, which should asymptotically lead to that of the SNR method.

V. Conclusion

In this paper, we have proposed a novel framework that decentralizes the metaverse and distributes it along with its DT constituents at the edge. In particular, we have digitally replicated a physical zone containing autonomous PT systems as sub-metaverses that encompass CTs at the MEC servers. To perform an E2E synchronization that utilizes the overall computing and communication resources, we have formulated an optimization problem whose goal is to minimize the sub-synchronization time, while satisfying the DT synchronization intensity requirements. This requires partitioning the zone into regions that are associated along with their PTs as CTs in their respective sub-metaverses. To solve this problem, we have presented an iterative algorithm based on optimal transport theory to determine the optimal region mapping and DT association that minimizes the sub-synchronization time and synchronizes the DT applications. Simulation results show the superiority of our approach compared to SNR-based associations that fail to consider the DTs' operation and its synchronization resources.

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