Preservice Teacher Noticing of Students' Mathematical Thinking in a Technology-Mediated Learning Environment

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The teaching practice of noticing students' thinking includes both attention to students' mathematical strategies and interpretation of their mathematical understanding. This practice is further complicated when students are engaged with mathematical action technologies while learning mathematics content because, in these contexts, teachers must also notice how the students' engaged with the technology to develop their mathematical understanding. For this cross-institutional study, we focus on how preservice secondary mathematics teachers (PSMTs) coordinated their noticing of students' mathematical thinking and engagement in a technology-mediated environment. The PSMTs viewed artefacts of high school students working with a Desmos task focused on rational functions, specifically locating vertical asymptotes, and noticed the students' mathematical thinking. Results showed it was easier for PSMTs to attend to and interpret the students' spoken and written responses than for them to attend to and interpret the students' engagement with the technology. Further, it was even more difficult for the PSMTs to coordinate the students' spoken and written responses with their technology engagement.

1. INTRODUCTION

Mathematics teachers are responsible for paying attention to their students' mathematical thinking and for making instructional decisions based on their students' understandings. The practice of teacher noticing has recently received prominence in the field of mathematics teacher education as a skill focused on students' mathematical thinking that can be purposely developed and learned by both preservice and inservice teachers. Researchers have found that the practice of noticing when focused on students' mathematical thinking can influence both teacher and student learning. Jacobs and Spangler explain, "When teachers explore students' reasoning, they benefit by gaining a window into students' reasoning, which can be mathematically powerful…Students benefit because they…have opportunities to articulate and reflect on their [own] reasoning." (2017, p. 767). This becomes especially important and powerful when students are working in technology-mediated learning environments. Technology-mediated learning environments are those in which students use mathematical action technologies to interact with digital objects in mathematically defined ways (Dick & Hollebrands, 2011). Math action technologies (e.g., virtual manipulatives, graphing calculators, dynamic geometry programs) provide ways for students to communicate their mathematical ideas through their interactions with the technology and thus affording new avenues for teachers to elicit evidence of students' mathematical thinking (Dick & Hollebrands, 2011). In what follows, we will explore what it means for teachers to notice students' thinking in a technology-mediated learning environment.

In such spaces, in addition to students' verbalising and recording their thinking on paper, students can articulate their thinking through their interactions with the technology. As a result, the practice of noticing student thinking in such an environment requires not only considering what students say or record, but also the ways in which they engage with the technology. Given the important role that technology-mediated learning environments can play in teachers facilitating high quality instruction and developing students' mathematical understanding, there is a need for preservice secondary mathematics teachers (PSMTs) to be provided with opportunities to notice students' thinking in such environments. Thus, the purpose of this study was to examine the ways in which PSMTs notice students' mathematical thinking in technology-mediated learning environments. In this paper, we share the framework that guides the study, discuss the findings, and share next steps for improving PSMTs' noticing in technology-mediated learning environments.

2. BACKGROUND LITERATURE

As we consider PSMT noticing in technology-mediated learning environments, we situate this study in relevant literature related to teacher noticing. First, we define this study's construct of teacher noticing. We then review literature on teachers' noticing in technology-mediated learning environments.

Teacher noticing

In their 2017 review of work surrounding teaching practices, Jacobs and Spangler identified teacher noticing as a "hidden core practice of mathematics teaching" (p. 771). With origins in the work of researching one's own teaching practice (Mason, 2002), teacher noticing can be described as the act of paying attention to and making sense of the complexities that occur in the classroom. For PSMTs, teacher noticing is a skill that needs to be purposely developed as "teachers can be responsive only to what has been noticed" (Jacobs & Spangler, 2017, p. 772). When studying what teachers' notice, the object of the noticing should be defined and can vary from noticing teacher actions (e.g., Osmanoglu et al., 2015) to noticing children's participation (e.g., Wager, 2014). For this paper, we are concerned with mathematics teacher noticing explained by Philipp et al. (2020) as "how teachers interact with a mathematical instructional situation" (p. 46). We refer to PSMTs' noticing students' mathematical thinking as conceptualised by Jacobs, Lamb, and Philipp (2010) as consisting of three interrelated components: attending to students' strategies, interpreting students' understandings, and deciding how to respond on the basis of students' understanding. For the first component, when examining students' work, attending to students' strategies involves noticing mathematically significant details. The second component involves teachers "generating plausible interpretations of students' work" (Goldsmith & Seago, 2011, p. 170). The final component presented by Jacobs et al. (2010) is deciding how to respond in light of students' mathematical thinking which involves synthesising what was learned and making educated decisions about how to proceed with instruction based on the analysis.

Teacher noticing in technology-mediated learning environments

With one-to-one device environments becoming more prevalent in secondary classrooms (Zheng et al., 2016), technology-mediated learning environments are often used to develop students' mathematical understanding. Research has shown that the ways in which students engage with such technologies mediates their sense-making about the object of their investigation (e.g., Arzarello et al., 2002; Baccaglini-Frank & Mariotti, 2010; Doerr & Zangor, 2000; Lee et al., 2010; Lopez-Real & Leung, 2006; Trouche & Drijvers, 2010). Moreover, careful observation of students' engagement with these technologies can reveal how they are thinking about the mathematics. For example, Arzarello and colleagues (2002) examined how students engaged with dynamic geometry technologies and noted that students used the technology to achieve different goals; the students' engagement provided insight into their cognitive processes. The same has been found across studies of students' use of graphing calculators (e.g., McCulloch, 2011; Doerr & Zangor, 2000) and computer algebra systems (e.g., Artigue, 2002; Kieran & Saldanha, 2005). The research on students' doing mathematics while using technology-mediated learning environments shows time and again that attending to and interpreting students' engagement with these technologies provides insight to their mathematical thinking. In fact, Arzarello and colleagues (2002) explicitly call for the importance of teacher noticing student engagement with technologies stating, "Such an analysis is a powerful tool to investigate the cognitive processes of pupils through visible actions" (p. 69).

In 2011, prior to the introduction of the noticing construct, Wilson, Lee, and Hollebrands engaged PSMTs with examining students' work through their solving statistical problems using a dynamic statistical tool (i.e., TinkerPlots). They found that PSMTs could describe students' actions with the tool but found little evidence of the PSMTs connecting the students' actions with the technology to the students' mathematical thinking. Overall the PSMTs drew on their own mathematical content knowledge to interpret the students' thinking, which often hindered their ability to unpack the students' understandings. More recently, Chandler (2017) compared PSMTs' noticing students' mathematical thinking on geometry tasks presented in two different mediums: written work and technology-mediated work using The Geometer's Sketchpad. She found both groups of PSMTs focused their noticing on the students' mathematical thinking, but struggled to provide evidence to back up their interpretations; there was not much variation in how the PSMTs noticed the students' thinking between the two mediums.

McCulloch et al. (2019) designed a multi-part lesson to engage PSMTs with noticing researcher-selected video and written artefacts of middle school students engaging with an interactive applet (Lovett et al., 2020) designed to introduce the function concept. For a noticing assignment, PSMTs completed a written reflection in which they were asked to attend to how the students used the applet to determine the defining characteristics of function and to interpret the students' understanding of function. Findings from the analysis of how the PSMTs noticed the students' thinking showed that when the PSMTs were attending and interpreting, they tended to describe either the students' engagement with the technology or to describe their mathematical understanding of function. It was more difficult for the PSMTs to coordinate the students' engagement with the technology with their understanding. Due to this difficulty and the limited research in this area, we agree with Thomas who in 2017 posited, "it may be wise to further explore noticing development in technology-centered contexts" (p. 512).

3. CONCEPTUAL FRAMEWORK

We adopt Jacobs et al.'s (2010) teacher noticing of students' mathematical thinking construct with a focus on tool-engagement. As described above, Jacobs and colleagues' (2010) teacher noticing construct (i.e., attend, interpret, and decide how to respond) focuses on the decision-making process teachers' use when evaluating students' responses. As we consider attending to students' mathematical strategies, we recognize that insight into those strategies is often limited to what students say (their spoken words and associated gestures) and what they record in writing (Jacobs, 2017). In the context of students working in a tool-mediated learning environment (including digital technology tools), attending to students' understanding requires not only focusing on students' spoken and written work, but also on their engagement with the tool. We have found that taking a semiotic meditation perspective (e.g., Bartolini Bussi & Mariotti, 2008; Jones, 2000; Mariotti, 2000; 2013) of tool use provides a way to make sense of teacher noticing of student thinking in tool-mediated learning environments.

The premise that students' understandings are shaped by the tools that they use and by their internal relationship with those tools is consistent with socio-cultural theories of learning (e.g., Vygotsky, 1978). When discussing tools, Vérillon and Rabardel (1995) made a distinction between a tool as an artefact and a tool as an instrument, the former being the man-made material object, and the latter being a psychological construct. Specifically, an instrument includes the tool and all of the ways a person thinks about using it (i.e., their utilisation schemes). Mariotti (2000) explains when a student "uses the artefact, according to certain utilization schemes, in order to accomplish the goal assigned by the task; in doing so the artefact may function as a semiotic mediator where meaning emerges from the subject's involvement in the activity" (p. 36). In other words, when a tool is used as a psychological tool, it is an instrument of semiotic mediation, a mediating artefact between the learner and the mathematics (Jones, 2000). Furthermore, it is important that when choosing to use a tool that teachers have considered the semiotic potential of the tool, e.g. the "potentiality that the use of a specific artefact has in fostering mathematical learning" (Mariotti, 2013, p. 442).

Since utilisation schemes are internal, they are not observable. However, teachers can observe techniques–the observable interactions between the user and the artefact (Drijvers et al., 2010). Bartolini Bussi and colleagues (2008) note that "personal meanings are related to the use of the artefact, in particular in relation to the aim of accomplishing the task; on the other hand, mathematical meanings may be related to the artefact and its use" (p. 754). In other words, in the context of attending to student thinking, attending to students' techniques provides insight to their thinking. It follows then that when interpreting students' understandings in a tool-mediated learning environment, it is necessary to coordinate students' spoken and written responses with the ways in which they engage with the tool (i.e., their techniques) to fully interpret their understandings. If a teacher only relies on part of that information, it is possible to miss important aspects of a student's growing understanding.

Our conceptualisation of teacher noticing of students' mathematical thinking in a tool-mediated learning environment, referred to as the Noticing in a Tool-mediated Environment (NITE) Framework, is shown in Figure 1. While we acknowledge that all components of noticing are interrelated by their nature (Jacobs et al., 2010), which is indicated by the use of vertical double-headed arrows, we have chosen to separate both attention to and interpretation of students' spoken and written mathematical thinking from attention to and interpretation of the students' engagement with the tool (i.e., techniques) to emphasise its importance. Though separated in the figure, they are interrelated which is indicated with horizontal double-headed arrows.

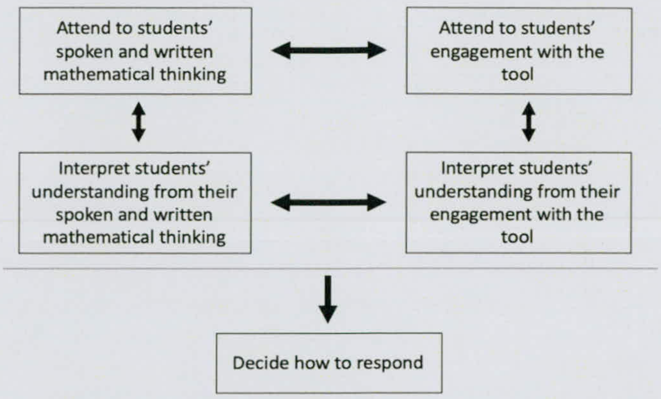

Figure 1. Teacher noticing of students' work in a tool-mediated learning environment [The NITE Framework]

The purpose of noticing student thinking is to make informed decisions about what to do next in supporting students' learning. When attending to and interpreting student thinking in a tool-mediated learning environment, the teacher is noticing the "unfolding of the semiotic potential" (Mariotti, 2013, p. 443) of the artefact (i.e., the emergence of students' mathematical meaning making in relation to the artefact). Thus, our conceptualization involves both the horizontal coordination of attention and interpretation as well as the vertical integration of both.

We have separated decide how to respond from the other components of noticing to balance the importance of focusing on both spoken and written mathematical thinking, and tool-engagement prior to making instructional decisions; if a teacher focuses on one more than the other, then they may not be fully informed when making an instructional decision. Here is where the semiotic potential of the tool is especially important. After attending to and interpreting the unfolding of the semiotic potential so far in the students' work (i.e., the personal signs that have emerged), one must decide how to respond in order to guide students to produce mathematical signs—what Mariotti (2013) refers to as "exploiting the semiotic potential of the artefact" (p. 443). As such, when deciding how to respond to a student working in a tool-mediated learning environment, the teacher must consider how to position the tool (or not) in their response to support the student in moving forward. For this reason, deciding how to respond does not necessarily include students' further engagement with the tool.

For our conceptualisation, like Jacobs et al. (2010), we emphasise "that the ability to effectively integrate these three component skills is a necessary, but not sufficient, condition for responding on the basis of children's understandings" (p. 197). Hence integration of the three noticing components while coordinating attend and interpret is the goal of this complex teaching practice. Given the complexity, we developed the NITE framework as an analytical operationalisation to help us understand the ways in which PSMTs' attend to and interpret students' mathematical thinking in a technology tool-mediated environment, hereafter referred to as a technology-mediated learning environment.

4. METHODS

Sherin and colleagues (2011) described three approaches to studying teacher noticing: 1) engaging teachers with researcher-selected artefacts and sharing their noticing about the artefacts; 2) teachers retrospectively sharing their noticing about their own teaching; and 3) researchers observing teachers' instruction and inferring what teachers noticed. For our cross-institutional study, we designed a technology-mediated task, aligned with the first approach, to engage PSMTs with researcher-selected artefacts chosen to focus their noticing on students' mathematical strategies and interpreting their mathematical understandings. This decision was made, in part, because we wanted to be able to compare teacher noticing across PSMTs which only common artifacts will allow (Jacobs & Spangler, 2017).

We employ the domain specific approach to the study of teacher noticing (Jacobs & Spangler, 2017), meaning we are analysing teacher noticing within a particular mathematical domain—function. Specifically, the technology-mediated environment under study for this paper was designed to develop early understandings of the relationship between the structure of a rational function, and the existence and location of a vertical asymptote. The details of this study in which we explored PSMTs' noticing of students' thinking in a technology-mediated learning environment are described in the following sections.

Participants

A total of 23 PSMTs from three public universities in the southeast U.S. participated in this study in Spring 2020. All PSMTs were enrolled in a secondary mathematics education methods course. The earlier portion of the courses were carried out in person, and the latter were carried out remotely due to COVID-19. While this transition in the middle of the semester did change the structure of some of the course materials, it did not change the content of the materials nor plans for data collection. The methods courses, though at different universities, all focused on materials and methods for developing PSMTs' mathematical knowledge for teaching secondary mathematics. As such, each course kept students' mathematical thinking at the forefront of lessons and assignments. However, the practice of noticing student thinking in technology-mediated learning environments had not been explicitly introduced prior at any of the institutions.

Context of the study

In each of the methods courses the PSMTs first engaged extensively with a Desmos activity titled, Introduction to Vertical Asymptotes. This activity is designed to be a first experience for secondary students with vertical asymptotes in the context of rational functions. The first few pages of the activity show various graphs of rational functions with constant numerators and linear denominators (e.g., $f(x) = \frac{3}{x+1}$) with prompts asking students to describe the domain and range and asking what they notice when comparing across the graphs. Then the term "vertical asymptote" is introduced as shown in Figure 2.

The next page of the activity provides an opportunity to investigate the relationship between the structure of the function and the existence and location of any vertical asymptotes using sliders (Figure 3). Notice the vertical asymptote is shown in the graph along with the function. As sliders are changed, the graph of the function and the function in the directions at the top dynamically change accordingly, as does the vertical asymptote. The next prompt in the activity is "If you were going to describe to a friend how to find the vertical asymptote given the function rule, $f(x) = \frac{k}{ax+b}$, what would you tell them?" The activity continues with more complex rational functions (e.g., constant numerator and linear denominator, linear numerator and quadratic denominator).

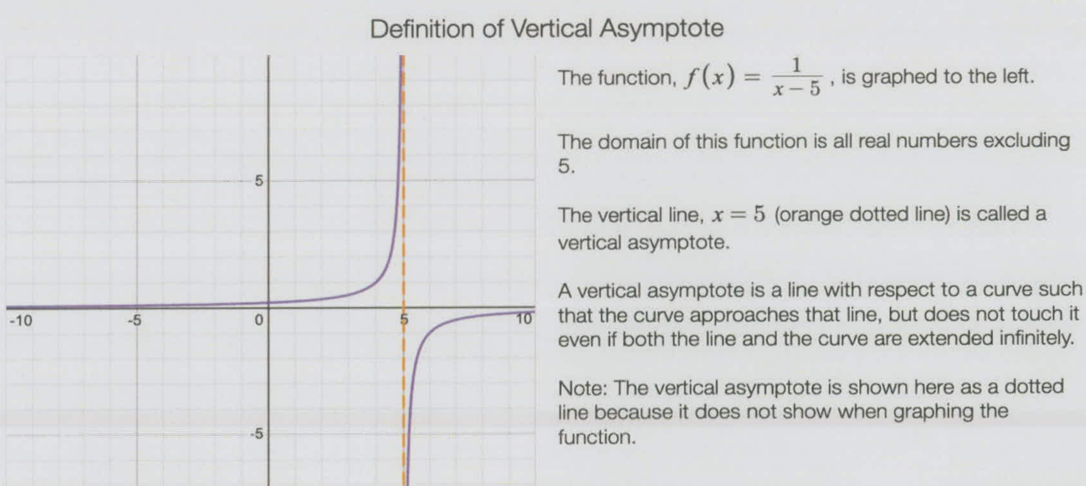

Figure 2. Screenshot of Desmos activity where vertical asymptote is defined.

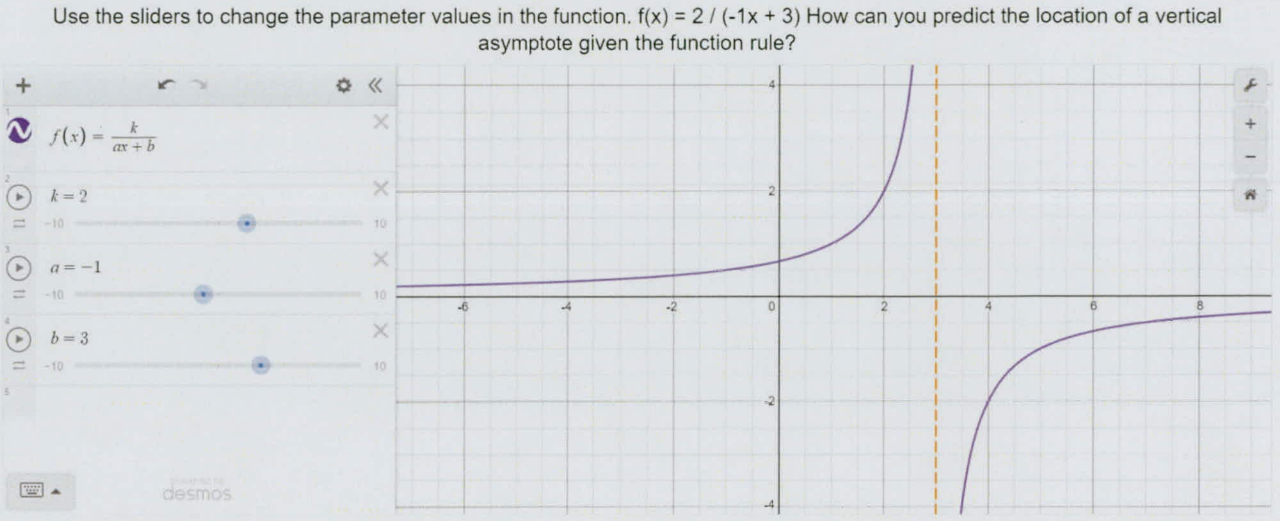

Figure 3. Screenshot of Desmos activity with sliders to investigate the location of the asymptote.

After engaging with the Introduction to Vertical Asymptotes activity as a learner, PSMTs were asked to individually complete a noticing assignment for homework. They viewed a three-minute clip (https://www.youtube.com/watch?v=LTh095tfD2Y&feature=youtu.be) that showed a pair of secondary students exploring the rational function with a constant k in the numerator and $ax + b$ in the denominator with sliders for parameters a, b, and k (Figure 3). In this video clip the students engage with all of the sliders, deciding that only a and b affect the asymptote. A detailed transcript of the video clip is provided in Figure 4.

After PSMTs watched the clip, they responded to two noticing prompts. The question prompts accompanying the videos were guided by the NITE framework. One prompt focused on attending to the students' spoken and written mathematical thinking and engagement with the technology: "Attend to (i.e., describe in detail) how the students determined the location of the vertical asymptote of the form$f(x) = \frac{k}{ax+b}$". The second prompt focused on interpreting the students' understanding of vertical asymptotes: "Interpret the students' current understanding of vertical asymptotes. Provide evidence from the video to support your claims". For this study, we consider the following research questions: When considering students' work on a task designed to introduce vertical asymptotes using dynamic graphing technology,

1. In what ways do PSMTs attend to the students' spoken and written mathematical thinking and engagement with the technology?
2. In what ways do PSMTs interpret the students' spoken and written mathematical thinking and engagement with the technology?
3. In what ways do PSMTs coordinate their attention and interpretation of the students' spoken and written mathematical thinking and their engagement with the technology?

McKenzie:	The asymptote isn't based on... just those numbers alone. Because look, if you do it there... it's not going to be exactly on that number. [Student drags slider b to change the value from 5 to 4 which moves the graph to the right on the x axis]
Eden:	Yeah...
McKenzie:	So… how can you predict the location of a vertical asymptote given the function rule? I think... wait… I don't know… I'm confused because 'a' and 'b' do the same thing… wait move 'a' [Student drags slider a to change the value from 5 to 2 which results the graph moving to the left 3 places and the graph stretches horizontally]
McKenzie:	I feel like it makes it bigger, or is that just me?
Eden:	Oh oops! What did I just do?
McKenzie:	Go to zero. [Student drags slider to change the value to 0 which results in a horizontal line at y=1]
Eden:	But it's still three.
McKenzie:	Go to zero for 'b'. [Student drags slider b to change the value to 0 which results in the function no longer appearing in the graphing window; the asymptote is still visible]
McKenzie:	That moves up and down because… this doesn't move vertical asymptote. [Student drags slider k to change the value from 1 to 12 which does not affect the vertical asymptote but results in the graph stretching vertically.]
McKenzie:	This moves your vertical asymptote, so something with that and this one is your vertical asymptote. [Student drags slider a to test values from -1 to 4 which results in the horizontal asymptote moving from left to right]
McKenzie:	So I think that whatever 'b' is your vertical asymptote.
Eden:	I think it's...
McKenzie:	But it has something to do with 'a' too, though.
Eden:	I think it's… I think it's… um… [groaning] divided by 'a'. Yeah, I think it's, I think it's 'b' divided by 'a' cause, cause look two divided by four is what?
McKenzie:	Two divided by four is one half but two divided by negative four is negative one half.
Eden:	I know, I think it's just, I think it's just one of those weird flippy thingy with that graphs do.
McKenzie:	Let's try this number and… oh and I need to go down to this number. Hold on one… point five. [Student drags the sliders and sets the values at k = 0, a = -10, b = 5. K = 0 results in the graph not moving even when the student changes the a value from -10 to 0.]
McKenzie:	Oh! You're right cause that's point five.
Eden:	I think it's one of those weird flippy thingy. That doesn't really make sense and yeah
McKenzie:	Or... This [laughing] or...
Eden:	Two… two and a four
McKenzie:	We can do two in here and four. [Student drags sliders a and b and sets the value of a to 2 and the value of b to 8 which changes the vertical asymptote from x=-.5 to x=-4]
Eden:	Yeah.
McKenzie:	Yeah, it's one of those flip things. And then that's...
Eden:	So… if I were to define the vert... I would tell my friend to… divide... 'b', whatever 'b' is, by 'a' and then make it equals… okay, 'b' divided by 'a'.
McKenzie:	One sec, I have to write this out.
Eden:	Equals negative 'x' or whatever.
McKenzie:	Okay.

Figure 4. Transcript of video clip

Data Sources

Data included each PSMT's written work on the noticing assignment. This included their responses to the prompts asking them to attend to and interpret the students' work. As noted above, PSMTs were asked questions focused on attending and interpreting the students' work. Thus, there were 23 attend and 23 interpret statements.

Data Analysis

We utilised a data analysis process similar to Jacobs et al. (2010). To establish our codebook, we started by identifying the mathematically significant details of the students' strategies in the video clips—both the strategies they said or wrote and their engagement strategies with the technology (i.e., use of the sliders and what they see in response to their use). The mathematically significant details were what we expected PSMTs to identify in a robust response to the assignments prompts. We then identified the understanding of vertical asymptotes as related to the function structure that could be interpreted from the students' work. With the full list of significant details expected for both the attend and interpret prompts, we created a three-level coding scheme for each component: robust evidence, limited evidence, and lacking evidence. Details are shown in Figures 5 and 6. When reading and coding PSMTs' responses to the noticing prompts, we adopted the approach of Castro Superfine and colleagues (2017) in acknowledging that "attending and interpreting are inextricably linked" (p. 422); thus, we coded across the PSMTs' written statements to both assignment questions. For example, if a response to the interpret question included elements of attention, it was coded as attend.

Once the codebook was established, five researchers used the process described by DeCuir-Gunby et al. (2011) to establish reliability in our code application. Specifically, a randomly selected subset of data was coded by all researchers. This process was repeated until the codes were being applied consistently. Once this reliability was met, all data were coded by two researchers and any disagreements were discussed until a consensus was met. The researchers then considered all code categories (i.e., robust evidence, limited evidence, and lacking evidence for each of the four attend or interpret components) to identify emergent themes and summaries of each theme were created (Creswell, 2017). Finally, the researchers looked across the four components for emergent themes related to coordination of the attend and interpret statements. Summaries of these themes were created.

5. RESULTS

We present PSMTs' evidence of attention to and interpretation of the students' understanding of vertical asymptotes in the context of rational functions with a constant numerator and linear denominator. We do this by focusing on the PSMTs' noticing of the students' spoken and written responses, their technology-engagement, and then PSMTs' coordination of the two. The breakdown of PSMTs' levels of evidence when attending to and interpreting the students' spoken and written responses, and their engagement with the technology is displayed in Table 1. Each will be discussed in detail below.

PSMTs' noticing: Attend

The breakdown of PSMTs' levels of evidence when attending to students' spoken and written responses, and their engagement with the technology is displayed in Table 1. The frequencies suggest that PSMTs' were far better at attending to students' spoken and written responses with robust evidence than attending to students' technology-engagement with robust evidence. In what follows, we share exemplars for each of the levels of evidence related to attending, first for spoken and written responses and then for technology-engagement. We then share overall themes that emerged in the PSMTs' coordination of the two.

	Spoken & Written Attend	Tech Engage Attend	Spoken & Written Interpret	Tech Engage Interpret
Robust Evidence	5	1	4	1
Limited Evidence	12	11	2	0
Lacking Evidence	6	11	17	22

Table 1. Noticing component frequencies

Attend: Spoken & written responses

As shown in Table 1, five (23%) of the PSMTs attended to the students' spoken and written responses at the robust evidence level, meaning they discussed all of the aspects (Figure 5). For example, PSMT U1-02 attended, "

> Whatever b is, is your vertical asymptote, but a has something to do with a too, I think it's $b/$a' Then the students tried examples with numbers that divide evenly like -10 and 5. They let a equal -10 and b equal 5 and discovered the asymptote was [at] 0.5. They then realized it "had to be one of those flippy thingys" referring to the sign of the asymptote. "So divide b by a equals -x." The students realized that the asymptote was the x-value given to them representing the orange dotted line. Then they concluded, through trial and error, that the asymptote value is going to be $-b/a$.

In this statement, the PSMT clearly identified all aspects of a robust attend statement related to the students' spoken and written responses; this PSMT used specific evidence of the students' statements as a way to describe the students' thinking.

In contrast, the 12 (52%) PSMTs who showed limited evidence of attention left out aspects present in the video-case. Notably, 11 of the 12 PSMT statements classified as limited attention on student thinking did not mention the students' discussion of the "weird flippy thing" as their explanation for the asymptote being located at $-b/a$. For example, PSMT U2-06 attended,

> They first figured out that it was not impacted by k. Then, once they figured that out they moved on to a and b. They knew from the graph that both a and b changed the vertical asymptote, it was just determining how they were connected. They figured out that it was b/a and then changing the sign which is the equivalent to setting it equal to zero and solving the equation, they just didn't know to solve the equation that way.

Not attending to evidence of the connection the students made to their previous knowledge of "weird flippy things that graphs do" may affect the PSMTs' interpretation of the students' understanding of this mathematical relationship.

Verbal Spoken & Written Mathematical Thinking	Robust	• Students recognize that only *b* and *a* affect the location of the vertical asymptote • Students recognize that rather than $\frac{b}{a}$ it is the opposite ($-\frac{b}{a}$) **and** explain it by saying it is one of those "weird flippy thingys that graphs do" or some paraphrase of this student language • Students write x = -*b/a* (or -x = *b/a)* as the location for a vertical asymptote
	Limited	• Two of the bullets above
	Lacking	• One or none of the bullets above or incorrect
Technology Engagement	Robust	• The students change each of the sliders and watch / discuss how they do (or do not) affect the location of the vertical asymptote • The students set ***K***=0 so that the function is no longer rational **OR** they state that k has no effect • The students **change** ***b*** and then **move** **a** and notice both have something to do with the asymptote. (e.g. "I think whatever ***b*** is is your vertical asymptote, but it has something to do with **a** too.") • They test the conjecture with multiple values of ***a*** and ***b*** on the sliders • Once they conjecture that the vertical asymptote is located at ***x = -b/a*** **AND** then test the conjecture with **additional** values of ***a*** and ***b*** on the sliders
	Limited	• Three or four of the bullets above • Exception: If one of the 3-4 includes noting that k=0 and they are no longer looking at a rational function, it should be scored as robust.
	Lacking	• Two or fewer of the bullets above or incorrect

Figure 5. Necessary elements for each of the levels of attend for spoken & written responses, and technology engagement

Verbal Spoken & Written Mathematical Thinking	Robust	• The students understand that the location of a vertical asymptote can be determined by ***x = -b/a*** (***-x = b/a***) (i.e., they have a procedure for locating the vertical asymptote) • The students have not yet connected their rule why it makes sense in the context of a rational function (they have not connected **a** and **b** to the denominator of the rational function and the fact that it can't be 0 **OR** to continuity). • The students have not yet connected their rule to setting the denominator of the function equal to 0 to solve to explain why the vertical asymptote is located at ***x = -b/a*** rather than ***x = b/a***
	Limited	• Two of the bullets above
	Lacking	• One or none of the bullets above or incorrect interpretation
Technology Engagement	Robust	• The way the technology allowed them to just see the asymptote and not tie it to rational function (the function is not rational when they are determining and testing their rule) - connected to rational functions being undefined • The way the technology allowed them to test and make sense of the "weird flippy thing", specifically related to the negative value • The way that they used the sliders to change and make sense of the location of the asymptote, but have not explained what a vertical asymptote is (connecting to continuity or why the denominator matters)
	Limited	• Two of the bullets above
	Lacking	• One or none of the bullets above

Figure 6. Necessary elements for each of the levels of interpret for both spoken and written responses and technology engagement

Finally, six of the 23 (26%) PSMTs attended to the students' spoken and written responses at the lacking evidence level. These PSMTs either only discussed that the students determined that only *a* and *b* affect the location of the vertical asymptote or did not reference any of the elements of attention to spoken and written language. PSMT U1-01 is an example who only attended to *a* and *b* affecting the location of the vertical asymptote, "They seem to believe that *a* and *b* should be divided and [are] not really worried about *k*." Even though this response is vague in terms of evidence, we see it as the

PSMT attending to the fact that the students determined that only a and b affect the location of the vertical asymptote. PSMT U3-01 typifies the other PSMTs' attention statements that showed no evidence of attending the students' written and verbal responses, "The students determined the asymptote by dividing b by a." The PSMT did not accurately state the students' final procedure of finding the location of the vertical asymptote.

Attend: Tech-Engagement

As shown in Table 1, only one PSMT (4%) attended to the students' technology engagement at the robust evidence level. PSMT U1-06 robustly attended,

> The students started by using examples to try and find a pattern for all cases. They found out that k did not move the vertical asymptote, a and b did. They then continued to play with the a and b variables to try and find the pattern. The students then found that when you divide b by a, the vertical asymptote would appear at the whatever that equalled to with the positive and the negative switched. … They said, "I think it's b divided by a." One of the partners then quizzed the other with an example. They realized the number was right, but the sign was opposite of what it needed to be. "2 divided by 4 is one half, but this is 2 divided by -4, which is negative one half," her partner responded, "it's just one of those weird flippy things graphs do." With what they said, they know the formula to find where the vertical asymptote will be. They said b divided by a, then flip the sign.

This PSMT clearly attended to all of the ways the students' used the technology to make and test conjectures as they worked towards their determination of how to find the location of the vertical asymptote.

Eleven (48%) PSMTs attended to the students' technology engagement at the limited evidence level, leaving out one or two of the five components (Figure 5). Specifically, every PSMT who scored at the limited attention to technology engagement level left out noticing that the students "changed b and then moved a" and/or that "the students tested their conjecture with additional values". For example, PSMT U1-05 did not carefully attend to the order the students' moved the sliders, but did discuss the students' testing their conjecture:

> First, they moved all three k, a and b. Then, just a and b changed the vertical asymptote. They noticed that the vertical asymptote is negative x of b divided by a.
> if b=8, a =4, b/a = 8/4 =2. Then we take negative of 2 = -2 is the vertical asymptote. If b=-8, a=4, b/a = -8/4 = -4. Then we take negative of (-4) = 4 is the vertical asymptote … While moving all three k, a and b, they know that changing k does not change the vertical asymptote. Then, they know it has something to do with a and b. Then, they find out that when b divided by a equal negative x is the vertical asymptote.

In contrast, PSMT U3-05 attended to the order the students' moved the sliders, but did not discuss the students returning to the technology to test their final conjecture:

> The students changed the values of k, a, and b to see what occurred. They noticed when they changed the values of a and b was when the vertical asymptote moved/changed location. The students decided that k did not play a role in the location of the vertical asymptote. They determined this also by playing around with different values of k.

In each of these examples, the PSMTs attended to the ways the students engaged with the technology, but failed to provide evidence of their attention to all the ways the students used the technology.

The final eleven (48%) PSMTs attended to the students' technology engagement at the lacking evidence level (two or less components from Figure 5). Of these, three PSMTs did not mention the students' engagement with the technology at all. For example, PSMT U2-08 wrote, "The students decided that a vertical asymptote could be determined by dividing b by a. Then that produced a $-x$ which they took as where the asymptote was located." In this statement, the PSMT attended to the result of the students' exploration with the technology; however, there was no discussion of how the students determined the location of the asymptote. The remaining eight PSMTs' attend statements mentioned that the students used the sliders in some manner to make their determination about the location of the asymptote. Of these eight PSMTs, three only mentioned one of the components, the students using the sliders. For example, PSMT U3-02 stated, "The students dragged the points representing a and b and assessed their effect on the vertical asymptote." However, five of these eight PSMTs included both a mention of the students' use of sliders as well as one additional attend component. For example, PSMT U2-07 mentioned the students using the sliders via "trial and error" as well as how the students tested the "patterns" (i.e. their conjecture) with additional values. However, despite having these two components, the PSMT did not attend to the other three components. Specifically concerning for many of these PSMTs was the absence of the students recognizing the effect of parameter k.

Coordination of attention

When considering the PSMTs' evidence of coordination of attending to students' spoken and written responses and their technology engagement, 11 PSMTs coordinated their responses. These can be seen in the three combinations of cells (denoted with an *) without a lacking designation in Table 2; we do not consider PSMT statements scored at the lacking level either in noticing spoken and written responses or in noticing technology engagement to be considered as a coordination. These 11 PSMT statements fell into three categories: 1) Coordination with limited evidence of both, 2) Coordination with robust attention to students' spoken and written responses and limited attention to technology-engagement, and 3) Coordination with robust evidence of both.

	Technology engagement attend		
Spoken & Written Attend	Lacking evidence	Limited evidence	Robust evidence
Lacking evidence	5	1	0
Limited evidence	6	6*	0
Robust evidence	0	4*	1*

Table 2. PSMT coordination of attend statements

Six (26%) PSMTs coordinated their attend statements with limited evidence for both attention to the students' spoken and written responses and their engagement with the technology, thus showing some coordination of their noticing. For example, U2-06 attended,

> They first figured out that it was not impacted by k. Then, once they figured that out they moved on to a and b. They knew from the graph that both a and b changed the vertical asymptote, it was just determining how they were connected. They figured out that is was b/a and then changing the sign which is the equivalent to setting it equal to zero and solving the equation, they just didn't know to solve the equation that way. I think they are close to a good understanding of how to find the vertical asymptote of a rational function. They said that they think the vertical asymptote is $-b/a = x$. They noticed that when both a and b was positive, the asymptote was negative, so they assumed that the fraction had a negative associated with it (or a negative out front). I think this is a good understanding for just having started working with rational functions.

The first three sentences of the PSMT's attend statement shows how the students' engagement with the technology (i.e. changing slider values) led the students to the spoken determination that k had no effect on the location of the asymptote. However, because the PSMT did not attend to all aspects of either the students' spoken and written responses nor their technology engagement, it is not considered full coordination.

Four PSMTs fell into the second coordination category of the PSMTs' attend statements. Their statements showed coordination with robust attention to students' spoken and written responses and limited attention to technology-engagement. For these statements, PSMTs fully attended to all components of the students' spoken and written responses which included attending to the students' noticing of the affect a and b had on the location of the vertical asymptote but the PSTMs did not attend to all of the ways in which the students engaged with the technology to make those determinations. For example, PSMT U1-02 attended,

> The students continually played with the asymptote manipulator to figure out a pattern in the relationship between a and b. The students went back and forth between the three different manipulators to see what each individual manipulation did. Once they figured out the individual purposes, they continued to find relationships. "Whatever b is, is your vertical asymptote, but it has something to do with a too." "I think it's b/a" Then the students tried examples with numbers that divide evenly like -10 and 5. They let a equal -10 and b equal 5 and discovered the asymptote was [at] 0.5. They then realized it "had to be one of those flippy thingys" referring to the sign of the asymptote. "So divide b by a equals $-x$." The students realized that the asymptote was the x value given to them representing the orange dotted line. Then they came to the conclusion, through trial and error, that the asymptote value is going to be $-b/a$.

In this attend statement, the PSMT was vague in their description of how students engaged with the sliders but very thorough in attending to the exact language students used in their exploration. Thus, there was a coordination among robust evidence of the students' spoken and written words with limited evidence of their technology-engagement, with the PSMT placing greater emphasis on what was said and written than on the students' sensemaking with the technology.

Finally, one PSMT coordinated both their robust attention of the students' spoken and written responses as well as the students' engagement with the technology (referred to as full coordination). U1-06 fully coordinated,

> The students started by using examples to try and find a pattern for all cases. They found out that k did not move the vertical asymptote, a and b did. They then continued to play with the a and b variables to try and find the pattern. The students then found that when you divide b by a, the vertical asymptote would appear at the whatever that equalled to with the positive and the negative switched. They said, "I think it's b divided by a." One of the partners then quizzed the other with an example. They realized the number was right, but the sign was opposite of what it needed to be. "2 divided by 4 is one half, but this is 2 divided by -4, which is negative one half," her partner responded, "it's just one of those weird flippy things graphs do."

This PMST clearly noted the interplay between the ways the students engaged with the technology and their verbal responses reacting to what they saw; and coordinated all aspects of attention to both the students' spoken and written responses as well as to their engagement with the technology.

PSMTs' noticing: Interpret

The breakdown of PSMTs' levels of evidence when interpreting students' understanding of vertical asymptotes in the context of rational functions with a constant numerator and linear denominator was previously shown in Table 1. The frequencies suggest that PSMTs were somewhat better at providing evidence of interpreting students' spoken and written responses than interpreting their technology-engagement. In what follows, we share exemplars for each of the levels of evidence related to the PSMTs' interpretations, first for spoken and written responses and then for technology-engagement.

Interpret: Spoken & written responses

Only four (17%) of the PSMTs' interpret statements included robust evidence of interpretation of the students' understanding about vertical asymptotes based on their spoken and written responses. For example, PSMT U2-03 interpreted,

> The students seem to have a good understanding of how to calculate the location of a vertical asymptote. They came up with a definitive method to tell their friend, that could be used with a given function. However, there are two concepts regarding vertical asymptotes that they still need to understand. They knew from experience that the answer was the opposite sign of what you would think initially, but they were unsure of exactly why. This can be algebraically explained by the use of the full equation: $ax + b = 0$. Another concept that may be absent from the students' understanding of vertical asymptotes is why the location is only dependent on a and b. Based off the video, you cannot say for sure that they know when the denominator in the function is zero, the value is undefined. Eden and McKenzie understand how to find vertical asymptotes, but not why they exist. Their understanding of vertical asymptotes would increase given a concrete definition and explanation.

This PSMT draws on the students' spoken and written responses and uses that information to make a claim about the students' understanding about how to determine the location of the vertical asymptote. The PSMT also notes the missing evidence related to how a vertical asymptote is related to the structure of a rational function and its location in the denominator of the function.

The two (9%) PSMT interpretations that included limited evidence justified the students' understanding of how to determine the location of a vertical asymptote based on their spoken and written responses and noted how the students' rule for determining the location was not related to finding a zero of the expression in the denominator of the function, but they did not connect this to the overall structure of a rational function. For example, PSMT U2-05 stated,

> Eden and McKenzie had no real understanding that the vertical asymptote represents where a function is undefined; they are just looking for a connection between a, b, and the x-value of the vertical asymptote. The student who first discovered the solution even states that she thinks having to change the sign is "one of those weird flippy thingies that doesn't really make sense in math." If they had been setting the denominator equal to zero, they would have clearly understood why the sign was reversed.

This interpretation was lacking the connection to the structure of a rational function.

Looking across the PSMTs' interpretations based on the students' spoken and written responses we see that most are lacking evidence (17 or 74%), with the majority of the PSMTs only interpreting the students' understanding of how to locate a vertical asymptote. PSMT U3-02's interpret statement was typical of those coded as lacking evidence. PSMT U3-02 interpreted,

> The students understand how to find the vertical asymptote as the video shows them figuring out it is -b/a for the given function. However, they do not understand why. Therefore, if they were given a function of any other form, they wouldn't be able to find the asymptote correctly.

In this example the PSMT focused exclusively on the students' understanding of a rule for finding a vertical asymptote without interpreting their understanding of the relationship between the rule and the structure of rational functions.

Interpret: Tech-Engagement

None of the PSMTs' interpretations of students' understanding of vertical asymptotes included robust or limited evidence related to their technology engagement. Those that were classified as robust evidence based on students' written and spoken responses, did not mention the students' engagement with the technology at all in their interpretations. For example, PSMT U1-03 wrote,

> The students were definitely on the right track in starting to understand the relationship of the variables in the function with the vertical asymptote of the function. The last thing they said at the end of the video is $-x = b/a$. If they had looked at the function rule again, they may have seen that $ax + b = 0$ gives us the relationship that x would be equal to $-b/a$. Thus, the vertical asymptote would be a value of x where the denominator of the function is equal to zero and is undefined.

While PSMT U1-03 has interpreted the students' understanding of vertical asymptotes related to the structure of the rational function, the fact that they have not noted the role of the students' technology engagement at all is problematic. One reason is after students had set *k=0*, they were not looking at the graph of a rational function anymore while making sense of the location of the asymptote, which limits their sensemaking opportunities with respect to the connection between the location of the asymptote and the structure of the function.

In considering the 23 PSMT statements lacking evidence of interpretation of students' understanding of function related to their technology-engagement, all were similar to U1-03 above in that there was no mention of the students' engagement with the technology as part of the interpretation of the students' understanding.

Coordination of interpret

Given the lack of acknowledgement of the role of the technology in the PSMTs' interpretations, as shown in Table 3, it follows that there was no evidence of coordination between their interpretation of the students' spoken and written responses and their engagement with the applet.

	Technology Engagement interpret		
Spoken & Written Interpret	Lacking evidence	Limited evidence	Robust evidence
Lacking evidence	17	0	0
Limited evidence	2	0	0
Robust evidence	4	0	0

Table 3. PSMT coordination of interpret statements

6. DISCUSSION

Central to the enactment of high-quality mathematics instruction is the ability for mathematics teachers to notice students' mathematical thinking. In tool-mediated learning environments, students' thinking is not only expressed via spoken and written language but also in the ways in which students interact with the tool itself. This makes noticing in environments in which students are utilising tools (including digital technology tools) markedly different than when they are not. The results of this study suggest that noticing in technology tool-mediated environments is not only different, but more difficult for PSMTs. In the following paragraphs we answer the research questions and discuss the implications of these findings for mathematics teacher educators and researchers.

Our results showed that a large percentage of PSMTs in our study could attend to students' spoken and written responses at a robust level; however, it was more difficult for the PSMTs to attend to the students' technology-engagement. Like Chandler (2017) and Teuscher and colleagues (2017), we found the PSMTs were stronger at attending than at interpreting. Interpreting students' understanding as a whole was difficult for the PSMTs. While not prevalent, it was more common for PSMTs to demonstrate robust interpretations of the students' mathematical understanding based on their spoken and written responses than from their technology-engagement.

In terms of coordination, since 11 (48%) of the PSMT statements provided at least some coordination of their attention to the students' spoken and written responses and their technology-engagement, our findings show that this is an accessible skill for PSMTs. However, no PSMTs coordinated their interpretations of the students' understanding (since none interpreted their technology engagement). Given the insight that noticing technology-engagement has been shown to provide to students' cognitive processes (e.g., Arzarello et al. 2002), the lack of coordination evident in PSMTs' interpretation statements is disconcerting. These findings suggest that PSMTs need to be explicitly directed to consider how the use of technology impacts students' mathematical thinking.

As others have noted, it is possible that the PSMTs indeed noticed more about the students' engagement with the technology than they included in their written responses. Jacobs (2017) discussed this, noting, "Researchers should be concerned not only with what teachers notice but also with what they fail to notice ... However, identifying what teachers miss is challenging methodologically" (p. 278). We must allow for the possibility that the PSMTs failed to report their noticing related to the students' use of technology, not that they failed to notice it. Perhaps the PSMTs' minimal references to the students' use of the technology was that they felt it was not necessary to verbalise what, to them, may have been obvious. The directions for the PSMTs did not explicitly ask them to discuss the students' technology-engagement, thus they may not have determined it was important to discuss whether the students, for example, used the sliders to check multiple values. Hence, some of the PSMT responses could be explained by their lack of thoroughness, not their lack of ability to notice the students' technology-engagement.

Asking PSMTs to notice students' thinking in a technology-mediated learning environment was a new practice for them. Given that the ways in which students use digital technologies is related to their mathematical thinking, we now believe that explicitly asking PSMTs to focus on students' engagement with technology when noticing students' mathematical thinking is important. Thus, when working with PSMTs on developing the skill of attending and interpreting students' thinking in technology-mediated environments, we posit it is important to explicitly ask them to notice students' technology-engagement as well as to their spoken and written responses. In addition, in the future noticing prompts should explicitly ask PSMTs to coordinate both in their justifications. Given the PSMTs' lack of attention and interpretation of the students' engagement with technology, we believe that PSMTs need to review multiple pairs of students working on the same digital mathematics task to understand the important role that technology plays as students develop their mathematical understandings.

Future research is needed in several areas to better understand how to prepare PSMTs to notice students' mathematical thinking in technology-mediated learning environments. Since this study, instead of only viewing the NITE framework as an analytical operationalisation, we have begun to examine the use of the NITE framework as a guiding framework for PSMT coursework. Preliminary research has shown that explicitly introducing PSMTs to the NITE framework and incorporating specific prompts asking for PSMTs to notice both students' spoken and written responses as well as their technology engagement improved PSMTs' coordination of students' spoken and written responses, and their engagement with technology (Bailey et al., In Press). Another important area to explore is the relationship between PSMTs' specialised content knowledge related to their own understanding of vertical asymptotes and their attending to and interpreting the students' understanding of vertical asymptotes (Dick, 2017). Lastly, we think it is crucial to examine the relationship between PSMTs' own technological pedagogical content knowledge (TPACK; Neiss, 2005) and their attending to and interpreting the students' understanding of mathematics in a technology tool-mediated learning environment. We hypothesise that if PSMTs do not understand the semiotic potential of the technology tool, then they will have difficulty determining how student engagement with the technology provides a window into their mathematical understanding.

Overall, findings from this study suggest that coordinating noticing of student thinking in a technology-mediated environment is not trivial. Our findings point to the importance of providing PSMTs with explicit opportunities to notice students' mathematical thinking in such contexts.

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BIOGRAPHICAL NOTES

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