EM-Based Radar Signal Processing and Tracking of Maneuvering Targets

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***Abstract*—The accuracy of radar tracks depends strongly on the variances of the measurements, and those variances are inversely proportional to the signal-to-noise (SNR) produced the hardware and signal processor. The signal processor uses matched filter processing, and the efficiency of that depends on knowledge of the kinematics of the target. In particular, the matched filter performance depends heavily on range rate and range acceleration. Traditionally, the predicted state of the target from the track filter is used for matched filter processing, but the predicted kinematic state can have rather large errors, and those errors result in match filter loss. This loss can be very large for maneuvering (*i.e.,* accelerating) targets. In this paper, an expected-maximization (EM) approach is taken to jointly address signal processing and tracking. The signal processor maximizes the SNR using the predicted state and produces measurements. The state estimator (e.g., Kalman filter) uses those measurements to produce expected values of the kinematic state (*i.e.* the nuisance parameters). The signal processor then maximizes the SNR using the new state estimates. This process continues until the maximum likelihood values of the measurements are achieved. In this paper, the Interacting Multiple Model (IMM) estimator is introduced for the tracking function better address sudden maneuvers. The EM-Based approach to join signal processing and tracking are presented along with a discussion of the real-time computing. Monte Carlo simulation results are given to illustrate a 6 dB improvement in SNR and enhanced tracks for a maneuvering target.**

***Index Terms*—Signal Processing, Target Tracking, Expectation-Maximization, Sensor Processing, Real-Time Computing**

I. Introduction

Optimal radar signal processing requires knowledge of the target's range, range rate, and radial acceleration. On practice, the truth values of these kinematic parameters are not known, but are estimated in real time from the tracking function. The tracking function estimates the kinematic state (*i.e.,* position, velocity, and acceleration) of the target through range and angle measurements, whose precision is determined by the signal-to-noise ratio (SNR) at the output of the signal processor. Measurement variance is inversely proportional to the SNR and directly impacts tracking performance. The SNR is maximized by minimizing the matched filter loss due to uncertainties in the radial velocity and acceleration of the target. When a target maneuvers and the estimates of the target kinematic state are imperfect, the target echo will exhibit range walk (RW) over the radar's coherent processing interval (CPI) [13]. If the range walk and nonlinear slow-time phase changes due to radial acceleration are not addressed prior to coherent processing over slow-time, a loss in SNR will occur. Compensating for RW and radial acceleration requires accurate estimates of the target's range rate and any radial acceleration that is present. Also, when a target maneuvers by suddenly accelerating or decelerating, the estimates of radial velocity and acceleration will lag the true values, and significant losses in SNR are likely to occur. Thus, accurate responsive estimates of the kinematic state are needed during target maneuvers to reduce the SNR loss. In this research, the Interacting Multiple Model (IMM) Estimator [1] is introduced to the tracking function to provide better kinematic state estimates. An expectation-maximization (EM) approach [6] is taken to signal processing and tracking to reduce the SNR loss when tracking maneuvering targets.

Traditional radar signal processing attempts to maximize the SNR given estimates of the kinematic state of the target from the track filter. The signal processor produces maximum likelihood (ML) estimates of the measurements of the target state, and the predicted kinematic state of the target are nuisance parameters. Since the kinematic state estimates are predicted from the previous measurement, the errors are amplified by the prediction, and the SNR losses are excessive. This research builds upon the existing EM algorithm [14] [8] [12] [9] that involves an iterative solution of a nonlinear ML estimation problem. The EM algorithm involves updating the nuisance parameters with their expected values on each iteration. In this new EM-Based radar signal processing, the traditional signal processing based on the predicted state of the target is performed as in the traditional approach. Then, the predicted kinematic state of the target is updated with the measurements with a Kalman filter to produce the conditional mean estimate of the state. This E-step significantly reduces the errors in the estimate in the kinematic state for signal processing. The signal processing is repeated with the new estimates of the kinematic state. If the observed SNR increases, the predicted state is updated with the newest measurements, and the signal processing step, M-step, is performed a third time. If SNR increases meaningful value again, another cycle is performed. If, at any point, the observed SNR fails to increase or decreases, the EM process is exited, and the measurements produced under the highest SNR are passed to the target tracker. A challenge to the successful implementation of the EM algorithm in real-time is convergence to an accurate ML estimate in a timely manner. Typically, one or at most two EM iterations have been found to provide near-optimal results.

In this paper, the Kalman filter of [10] is replaced by an IMM Estimator, and the simulation studies were performed in full Cartesian space tracking with radar measurements of range and two angles. Drawing on the results of [5] on the use of nearly constant velocity (NCV) versus nearly constant acceleration (NCA) motion models, an IMM estimator with two NCV models are utilized for tracking in three dimensional Cartesian space, while an IMM Estimator with NCV and NCA models are used for tracking in range to support the signal processing. Also, The details of the EM processing of [10] in refined in this paper and tracking performance is full Cartesian space is studied. Simulation results and analysis demonstrate that the EM-Based scheme efficiently compensates for the signal processing loses associated with maneuvering targets.

This paper is organized as follows. Section II formulates the problem and provides background on traditional signal processing and tracking. Section III describes the implementation of the EM-Based signal processing scheme in terms of the iterative E-step and M-step. Section IV provides the results of Monte Carlo simulations to demonstrate the reduction of SNR loss and improvement in track filter performance while tracking a maneuvering target with a pulse Doppler radar. Concluding remarks and future directions are given in SectionV.

II. Background

Traditional signal processing, traditional full Cartesian tracking, and range tracking are reviewed in this section.

A. Traditional Signal Processing

Given the potential for relative radial motion between a target and a pulsed Doppler radar system, range walk (RW) over a coherent processing interval (CPI) is of concern and must be addressed in the waveform design (e.g., short CPIs) and signal processing. If the range walk is not addressed prior to coherent processing, the target response may exhibit a movement through one or more range bins resulting in an SNR loss and a smearing of the target response across range-Doppler (RD) space. If the radial motion includes an acceleration component over the CPI, then a slow-time quadratic phase (SQP) results. If SQP is not removed, it will result in a smearing of the Doppler response at the output of the slow-time discrete Fourier transform (DFT). Higher-order radial components (e.g., jerk) could be present, but these are not addressed here. However, the proposed EM algorithm would support compensation for higher-order terms if those components were included in the kinematic estimate of the state estimator and the measurement quality and rate supported meaningful estimation of those components [5]. RW and SQP lead to a loss in SNR and a broadening of the target's response in RD or Doppler space, respectively. Signal processing functions that proceed with coherent processing include detection and parameter estimation, both of which are degraded by the loss of SNR and response broadening.

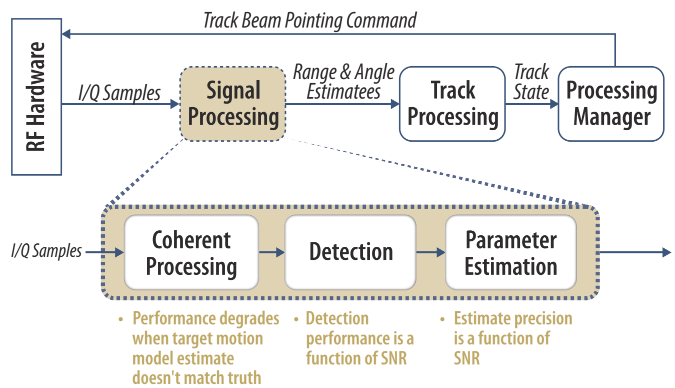

Fig. 1. Traditional Radar Signal and Track Processing

Figure 1 gives a diagram of a common pulsed Doppler signal processing chain. Having received the in-phase and quadrature (IQ) samples collected over a CPI, the IQ data is presented to the RW and SQP correction modules. Typically, the correction is based on the predicted radial components of velocity and acceleration at a time corresponding to the current CPI. RW correction is applied to the return from each pulse by applying a linear phase ramp in the frequency domain. The slope of the phase ramp is proportional to the desired time-delay correction. Often, the RW correction is applied to align the target returns to a time delay consistent with the true location of the target at the center of the CPI. When employing pulse compression waveforms (e.g., intra-pulse phase or frequency modulation), the application of the matched filter is implemented via fast convolution in the frequency domain. In this case, applying the linear phase ramp in the frequency domain is trivial. If fast-convolution is not applied, a transformation to and from the frequency domain is required to apply the phase taper. The SQP correction consists of a slow-time dependent phase correction across the range (fast-time). Since the phase correction is constant for a given pulse, the phase correction may be applied in either the time or frequency domains. The Doppler shift across fast-time may also degrade the range response at the output of the matched filter. For Doppler-tolerant waveforms such as a linear frequency modulated (LFM) pulse, the ambiguity surface exhibits range-Doppler coupling and a gradual loss in SNR as a function of Doppler shift. For Doppler-intolerant waveforms such as bi-phase codes, the ambiguity surface resembles a thumbtack. The range-Doppler coupling is no longer present. However, the SNR loss may be significant, thus degrading or preventing detection. For any waveform, an estimate of the radial velocity of the target may be applied via a linear phase ramp in fast-time to remove the Doppler-induced phase rotation. In this case, a narrowband waveform is assumed and wideband effects, which include an expansion or compression of the pulse, are ignored. The effects of fast-time Doppler are most acute for long pulses operating in the presence of large Doppler shifts. In the subsequent examples and the simulation results, the effects of fast-time Doppler are negligible and are not addressed in the processing architecture, while they could be easily incorporated.

Once RW/SQP correction, along with pulse compression (if appropriate), has been applied to the CPI data, the IQ samples are presented to the Doppler processing module, where a DFT is applied across slow-time to form a range-Doppler map. The RD map is presented to the detection module, which consists of a rectifier (e.g., square law detector) and a detection thresholding scheme (e.g., a constant false alarm rate detector). With RW, the spreading of response in RD may yield a number of detections provided a sufficient SNR is achieved to satisfy the threshold. A clustering scheme (e.g., DBScan) is then applied to group detections associated with a single target [7] [2]. The detections associated with a given target are then passed to the parameter estimation module in order to estimate the range and angles for the target. In this paper, the radar system employs monopulse processing for angle estimation. Given that the target response may be spread in range, a centroiding scheme is applied to estimate the target's range. Other range estimators may be employed depending on the severity of the response distortion. A quadratic curve fit would be appropriate, given minimal or no range response distortion. The monopulse angle estimate is derived from the sum and difference channels associated with the RD bin containing the largest detection amplitude. A weighted average across all detections within the cluster is another option. A target's range and angle estimates are provided to the Tracker, where measurement-to-track association and track filter updates are performed. In addition to the range and angle measurements, an estimate of the resultant SNR is generated and used in combination with the contributions of other error sources to estimate the measurement precision. The precision of the range and angle estimates are limited by the Cramer-Rao Lower Bound (CRLB) for an unbiased estimator under additive Gaussian noise. In general, the variance of the estimate is given by

$$\sigma_{\text{est}}^2 = \kappa \frac{\delta x}{SNR} \tag{1}$$

where κ is a proportionality constant that is dependent on the estimated domain (e.g., range, angle, or Doppler), and δx is the radar resolution in a given domain (e.g., range resolution, beam width, Doppler resolution). As noted in (1), the variance is inversely proportional to the SNR. Thus, mechanisms such as RW and SQP that reduce SNR also degrade the estimate precision in both range and angle. RW also affects the numerator of (1) by degrading the effective range resolution as the point response is broadened. The proposed EM algorithm is intended to provide an enhanced correction for RW and QSP, thus supporting finer measurement precision as a result of a higher observed SNR and a more compressed range response. The observed SNR loss is a function of the degree of uncompensated RW and SQP over a CPI and the single pulse ambiguity function. Curves illustrating the loss are shown for an X-band radar (10 GHz) with a 0.1 s CPI assuming an LFM pulse (10 μs pulse width and 10 MHz bandwidth) and a 10 kHz PRF. The SNR loss due to uncompensated radial velocity of 100 m/s is about 0.5 dB. A plot of the SNR loss as a function of uncompensated radial acceleration is provided in Figures 2. While the effects of RW and SQP on SNR loss are not separable, the loses are plotted as independent loss mechanisms. Figure 3 contains the resultant RD map for a target located at zero relative range with an uncompensated radial velocity of 20 m/s and uncompensated radial acceleration of 6 m/s/s. The combined SNR loss is approximately 4.47 dB. In this example, the dominant source of SNR loss is uncompensated acceleration. The observed loss closely matches the loss predicted in Figure 2.

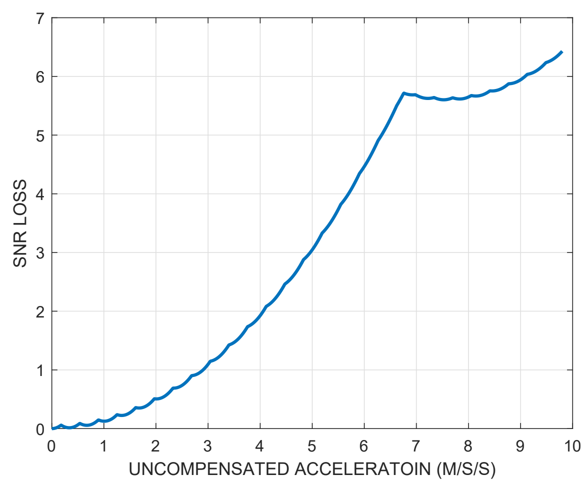

Fig. 2. SNR Loss Versus Uncompensated Radial Acceleration for a 0.1 s CPI

B. *Traditional Tracking for Maneuvering Targets*

Let X_k denote the kinematic state vector of the target at time t_k, and it typically contains the position, velocity, and possibly acceleration of the target as well as other variables used to model a time-varying acceleration. The kinematic model commonly assumed for a maneuvering target in track [1] is given by

$$X_{k+1} = F_k(\theta_{k+1})X_k + G_k(\theta_{k+1})v_k(\theta_{k+1}) \tag{2}$$

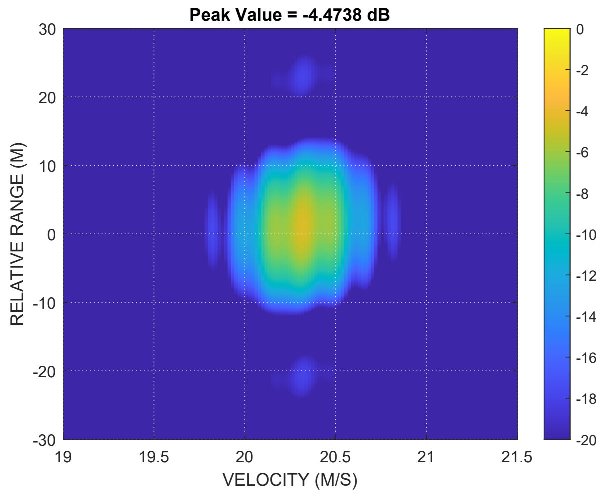

Fig. 3. RD Map for an Uncompensated Radial Velocity of 20 m/s and Uncompensated Radial Acceleration of 6 m/s/s

where $F_k(\theta_{k+1})$ defines the linear constraint on the target kinematics between times k and $k+1$, and $v_k(\theta_{k+1}) \sim N(0, Q_k(\theta_{k+1}))$ is the white process noise errors that account for uncertainty in the linear dynamics. The θ_{k+1} is a pointer to one of N models that describes the motion of the target. The θ_{k+1} is treated as a finite state Markov chain with probability p_{ij} of switching between Model i and Model j. The model for radar measurements is a nonlinear model of the target state in Cartesian coordinates and given by

$$Y_k = h_k(X_k, \theta_k) + w_k \tag{3}$$

where Y_k is the typical radar measurement of range and angles for the target and $w_k \sim N(0, R_k)$ is the white noise observation errors. Both w_k and $v_k(\theta_{k+1})$ are assumed to be independent "white" Gaussian error processes. For this paper, the radar measures range, bearing, and elevation, and the covariance R_k will includes variances of the measurements in the range σ_r^2, bearing σ_b^2,, and elevation σ_e^2.

The IMM estimator is well accepted as the best approach for estimating the state of a Markovian switching system defined by (2) and (3), when the computational cost is considered [1]. For surveillance-level radar tracking, two nearly constant velocity (NCV) models are commonly used in the IMM Estimator for tracking targets in three dimensional Cartesian space due to the larger cross-range errors making a nearly constant acceleration (NCA) model ineffective [5]. When tracking in range only as discussed next, an NCA model provides value. The two NCV models with Discrete White Noise Acceleration (DWNA) errors [1] are used in an extended Kalman filter (EKF) for processing the nonlinear radar measurements. The process noise variance for Model 1 is set to $\sigma^2_{ncv,1} = 1.0$ m/s/s for regimes of flight in which the target is not maneuvering. The process noise variance for Model 2 for tracking through target maneuvers is selected as specified in [4] for a target with a maximum acceleration of A_{max} according to

$$\sigma^2_{ncv,2} = \frac{1}{3}(\kappa_1^{pos} A_{max})^2 \tag{4}$$

where

$$\kappa_1^{pos} = 1.70(0.66)^{\bar{\Gamma}_D(r)}(1.02)^{\bar{\Gamma}^2_D(r)} \quad \text{for} \quad 0.001 \le \Gamma_D(r) \le 10 \tag{5}$$

$$\Gamma_D(r) = \frac{T^2 A_{max}}{\sqrt{3}\; r \max\{\sigma_b, \sigma_e\}} \tag{6}$$

T is the measurement period, r is the target range and $\bar{\Gamma}_D(r) = \log 10(\Gamma_D(r))$. For this research, the Markov switching probabilities for the IMM estimator are defined by

$$p_{11} = 0.9 + 0.1e^{-\frac{T}{2}} \tag{7}$$

$$p_{22} = 0.8 + 0.2e^{-\frac{T}{2}} \tag{8}$$

with $p_{12} = 1 - p_{11}$ and $p_{21} = 1 - p_{22}$.

C. *Range Tracking for Maneuvering Targets*

In support of the signal processing function, range is tracked separately from the filter estimating the three dimensional Cartesian state. A separate filter is used because the order of the filter and the filter design parameters differ significantly between the range only estimator and the three dimensional estimator. The smaller measurement errors in range allow for effective estimation of acceleration in range [5]. Thus, an NCA filter is used in the IMM estimator for enhanced estimation. Furthermore, accurate estimation of range acceleration of the target is important for reducing the signal processing loss due to a target maneuver. The NCV and NCA models [1] are used in a Kalman filter for processing the range measurements in the IMM estimator. In order for the NCV model to match the target motion when it is flying with constant velocity motion in Cartesian space, pseudo accelerations are required in the time update of the state. The pseudo accelerations are applied to range and range rate in the prediction step of both the NCV and NCA filters. The pseudo accelerations at time k is computed by

$$\ddot{r}_{k,pseudo} = \frac{s^2_{k|k} - \dot{r}^2_{k|k}}{r_{k|k}} \tag{9}$$

where $s_{k|k}$ is the speed estimate of the target at time k based on the state estimate of the three dimensional filter, $\dot{r}_{k|k}$ is the range rate estimate, and $r_{k|k}$ is the range estimate at time k. When the range state estimate is reported to the signal processing function, the pseudo acceleration is added to the range acceleration estimate produced by the IMM Estimator.

The process noise variance for the NCV model is set to $\sigma^2_{ncv,1} = 1.0$ m/s/s for regimes of flight in which the target is not maneuvering. The process noise variance for the NCA model for tracking through target maneuvers is selected as specified in [3] for a target with maximum acceleration of A_{max} according to

$$\sigma^2_{nca,2} = \frac{1}{3}(\kappa_3^{max} A_{max})^2 \tag{10}$$

where

$$\begin{aligned}\kappa_3^{max} &= .6(1.62)^{\bar{\Gamma}_D(r)}(.921)^{\bar{\Gamma}_D^2(r)}(.922)^{\bar{\Gamma}_D^3(r)}(.983)^{\bar{\Gamma}_D^4(r)} \\ &\quad \text{for} \quad 0.001 \le \Gamma_D(r) \le 10 \qquad (11) \\ \Gamma_D(r) &= \frac{T^2 A_{max}}{\sqrt{3}\,\sigma_r} \qquad (12)\end{aligned}$$

The mode switching probabilities as specified in (7) and (8) are also used in the range estimation.

III. EM-Based Signal Processing and Tracking

For EM-based signal processing and tracking, the initial range and angle measurements derived from the current CPI are provided to an IMM Estimator contained within the notional boundaries of the signal processor instead of the tracker. The IMM estimator uses the new measurements and the previous model-conditioned estimates of the IMM estimator to produce a new estimate of the mean of the target state. Hence, E-step of the EM algorithm. In this paper, the pulse Doppler radar uses traditional monopulse for angle estimation. Thus, digital beam resteering is not available. Hence, only the IMM estimator for range is included in the signal processing function. If digital beam forming with an array antenna was employed, the IMM estimator in Cartesian space would also be included in the signal processor in support of beam resteering. The new signal processing paradigm employing an EM based approach [10] to RW/SQP correction is depicted in Figure 4, where the initial measurements are provided to the IMM estimator contained in the box labeled “Expectation Update.” The updated estimate of the range state is provided for a new iteration of the signal processing. By providing an update of the range state estimate of target, the random errors in the state estimates are reduced and changes in the target kinematic states (*i.e.*, velocity and acceleration) due to maneuvers between CPIs are observed. The RW/SQP correction and Doppler processing combine to provide the maximization component of the algorithm leading to improved SNR and reduction in response spreading. Hence, the M-step of the EM algorithm. If the SNR increases in the new signal processing iteration by more than 3 dB, the newest measurements with a higher SNR (*i.e.*, smaller variances) are provided to the IMM Estimator for another expectation update of the original range state estimates provided by the tracker. The range state estimate from the first expectation update are disregarded as they already include measurements from this current CPI. These newest range state estimates are provided to the signal processor for another iteration. If the SNR in the next iteration increases less than 3 dB, the newest measurements are provided to the tracker. If in any signal processor iteration the SNR decreases, the measurements from the previous iteration of the signal processor are provided to the tracker. Hence, measurements with the highest SNR are provided to the tracker. Note that the measurement-to-track association problem has been omitted from Figure 4 and this discussion.

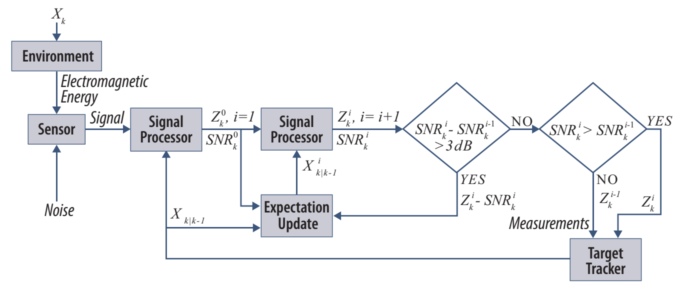

Fig. 4. EM Approach to Joint Signal Processing and Tracking

IV. Simulation and Evaluation Results

To illustrate the benefits of the EM-Based RW/SQP correction processing, an X-band pulsed Doppler, monopulse radar system was modeled and simulated. A 10 GHz center frequency was selected. The pulses are linear frequency modulated (LFM) with a bandwidth of 10 MHz and a pulse width of 10 μs. The CPI consists of 1024 pulses at a pulse repetition frequency (PRF) of 10 kHz. The CPI duration is 0.1024 s. The CPI duration may be long for some applications, but the goal was to demonstrate the ability of the EM-Based RW/SQP correction to address significant RW conditions. An LFM pulse was employed versa a bi-phase code to mitigate the impacts of fast-time Doppler, while as noted previously, the approach may be used to correct for similar effects on the compressed range response. A symmetric antenna with a 1.9 degree beamwidth employing monopulse processing on receive was instantiated. The standard deviations of the measurements at an SNR of 18 dB are approximately 0.5 m in range and 2.5 mrad in bearing and elevation. Targets are tracked at 1 Hz.

Targets that maneuver with a maximum acceleration of 30 m/s/s are expected. In the scenario studied here, the target starts at a range of 14 km, bearing of 0, and elevation of 0, flying at a speed of 230 m/s. This scenario has two 30 m/s/s maneuvers. At 18 s, the target performs a constant speed, horizontal turn with an acceleration of 30 m/s/s for a 90 degree in 12 s. At 42 s, the target performs a constant speed, horizontal right turn with an acceleration of 30 m/s/s for a 90 degree in 12 s. The target trajectory ends at 60 s with a range of about 9 km and a bearing of -45 degrees.

To evaluate the iterative EM-Based algorithm benefit compared to the traditional tracking loop pipeline execution, a Monte Carlo simulation with five runs was conducted with the computer model executing the EM-Based signal processing developed for this research. Then, five runs executing the scenario in the traditional mode was conducted. Figure 5 shows that the EM-Based signal processing improves the measurement quality of the Tracker with an average increase in SNR of 5.1 dB over the 60 s scenario and 7.9 dB during the maneuvers. The primary benefit of this research is the significant reduction in SNR loss at the start and end of the maneuvers. The EM-Based signal processing directly results in an improvement in the tracking. The Root-Mean-Square Error (RMSE) in the estimates of Cartesian velocity, range rate, and range acceleration were studied. The EM-Based signal processing provides an improvement in RMSE of the velocity estimates of the Cartesian tracker by an average of 4.4 m/s over the entire scenario and 6.6 m/s during the maneuvers. The peak improvement at 54 s is 15.1 m/s. The average improvement in the range rate over the entire scenario is 1.3 m/s, while the average improvement is 2.0 m/s during the maneuvers. The two peak improvements are at 44 s of 7.1 m/s during the beginning of the second maneuver and 9.1 m/s at the completion of that maneuver. The RMSEs in the range acceleration estimates had an average improvement of 1.5 m/s/s over the entire scenario and 2.4 m/s/s during the maneuvers. The most significant improvement of 10.3 m/s/s was at 55 s near the completion of the second maneuver. The model probabilities of the IMM estimator used for range tracking are slightly improved by the EM-Based signal processing.

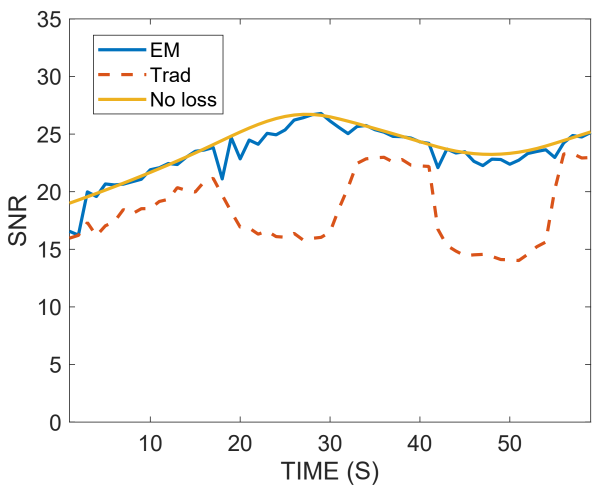

Fig. 5. SNR versus time for traditional and EM-Based signal processing

V. Concluding Remarks

EM-Based approach to radar signal processing and tracking with an IMM estimator was introduced to reduce the signal processing loss due to RW and SQP over a CPI and improve the measurement quality for tracking maneuvering targets. Errors in the estimates of the target kinematic state negatively impact measurement quality, and those errors are most significant when targets maneuver. In the EM-Based approach to radar signal processing and tracking, the measurement produced by the traditional signal processing is used to update the kinematic state estimate, and the signal processing is modified to reflect the new, enhanced kinematic state estimate and produce measurements with an enhanced SNR. In the pulse Doppler radar example and track scenario considered in this paper, the SNR was increased by approximately 6 dB by the EM processing. The SNR-enhanced signal processing produce better measurements in range and angle, and those improve the track quality. Furthermore, better measurements reduce the time line required by the radar to achieve an objective track quality. Numerous applications of this EM-Based radar signal processing and tracking are envisioned. First, when tracking targets in the presence of false alarms, the EM-Based signal processing and tracking should increase SNR for the target-originated measurements while decreasing SNR for the false alarms. Second, when initiating tracks in the presence of false alarms, the application of the EM-Based signal processing to the second measurement should show increased SNR for true target detections and decreased SNR for false tracks. Third, the EM-Based signal processing and track can be extended to include resteering of the beam for digital arrays. Enhanced beam steering could easily increase the SNR of the measurements by 3 dB. Fourth, the EM-Based processing will be used to enhance the assignment of measurements to tracks for multiple closely-spaced targets.

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